Robust Tuning Rules for Series Elastic Actuator PID Cascade Controllers

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Abstract: This paper deals with the control of a collaborative robot manipulator with series elastic actuators. In particular, robust tuning rules for cascade control of the joints are presented. Both the motor velocity and link position control loops are considered. The proposed tuning rules allow the online computation of robust control parameters to cope with the different link reflected inertia. Experimental results show the effectiveness of the method in real applications.

Keywords: Collaborative robots; Series elastic actuator; Robust control; PID; Tuning rules.

1. INTRODUCTION

Standard industrial robots are not typically designed to co-exist with humans in the same working environment. In fact, industrial robots are usually employed in several tasks in an isolated workspace by means of walls or fences. They are designed to provide high accuracy and precision, but the achievement of such a high level performance makes them characterized by a rigid and heavy structure. These features do not allow robots to reach the safety conditions that are necessary to share a working environment with humans without expensive tools like sensitive skin.

Due to the increased demand of human-robot interaction, a paradigm shift is changing the structural design of the robot to reduce the impact forces during possible collision. In recent years, a class of lightweight industrial robot has been presented (Guiochet et al., 2017) where the impact forces are reduced by limiting the mass, and therefore the momentum of the robot, and the maximum exerted forces. However, many tasks do not require the precision provided by an industrial robot and therefore impact forces reduction can be obtained by introducing elastic elements in the robot chain (Verstraten et al., 2016). Series Elastic Actuators (SEAs) are explicitly designed to pursue this idea. In fact, an elastic element is inserted in series to the actuator thus reducing the part of the mass that directly impacts the human, and therefore the overall impact force. Moreover, springs allow estimating the exerted forces without the use of expensive force/torque sensors.

However, this type of structure makes the control of the link position more difficult (Maleki et al., 2016; Paine et al., 2014; Ragonesi et al., 2011; Calanca and Fiorini, 2017; Calanca et al., 2017; Dos Santos and Siqueira, 2014). Indeed, the actual position of the link is given by the position of the gearbox output shaft plus the spring deformation. The elasticity implies the presence of a resonance and antiresonance behavior in the frequency response, coupled with the typical small damping factor of metallic spring elements. Moreover, there are multiple sources of model mismatches: the reflected inertia strongly depends on the robot configuration and on the (possibly unknown) payload. Further, the spring characteristic can be nonlinear, aging can increase hysteresis and backlash, static and viscous friction terms strongly depend on temperature (Simoni et al., 2017), and the rigid body assumption could not be an accurate hypothesis for lightweight links. These model mismatches can imply significant detriment in model-based control strategies which do not take them into account. In this case, it is reasonable to employ a robust PID-based control architecture instead of an adaptive model-based algorithm. One limit of this solution is the computational burden of an ad-hoc robust tuning optimization, which is not compatible with modular design (Maurtua et al., 2016). In fact, the robot presents a modular design, thus its overall inertia may change based on the setup of the mechanical structure. Moreover, in many applications, the usage of a single controller for the entire robot workspace can lead to very poor performance, requiring the retuning of the controllers. In this case, a gain-scheduling approach is proven to be an effective solution. Also in this case, effective robust tuning rules can be used to tune the controller without the need of an external dedicated computer (Misgeld et al., 2017).

The approximation of robust problem solutions with tuning rules is a common strategy in process control (Åström and Hägghund, 2004), but there are no tuning rules that address the control of robotic mechanical systems, as far as the authors know. The aim of this paper is to cope with this issue by proposing robust tuning rules for series elastic actuators. In this way, it is possible to guarantee a robust performance by limiting the maximum sensitivity in link position and motor velocity controllers for different values of the reflected inertia and different damping levels.

The paper is organized as follows: in Section 2 is described the dynamic model of the SEA. In Section 3 is proposed the optimization problem performed to obtain specific tuning rules, able to guarantee robust performance.
Fig. 1. Illustrative scheme of the SEA model.

The experimental results are described in Section 4, they have been obtained by means of two trajectories on a three degree-of-freedom series elastic actuated robot.

2. SERIES ELASTIC ACTUATOR MODEL

A series elastic actuator manipulator can be modeled as an open kinematic chain consisting of a rigid link and lumped elastic part coaxial with the joint rotational axis. The proposed approach considers a decentralized model, where the coupling effects are considered as model mismatches. This model takes into account two inertias: the first one represents the motor and the gearbox, while the second one represents the inertia of the link and of the downstream part of the kinematic chain for the actual joint configuration. Thus, the state variables are the motor position θ, the motor velocity θ̇, the spring deflection δ and the spring velocity ˙δ. Other interesting variables are the link position q = θ + δ and its velocity ˙q = θ̇ + ˙δ, which represent the real motion of the link. A joint model can be derived by dynamic laws as:

\[
\begin{aligned}
J_m \ddot{\theta} &= h\dot{\delta} + k\delta - f_m \dot{\theta} + \tau - C_m \text{sign} \dot{\theta} \\
J_l (\dot{\theta} + \dot{\delta}) &= -h\dot{\delta} - k\delta
\end{aligned}
\]  

(1)

where \( J_m \) is the motor inertia, \( J_l \) is the link (and the downstream part of the kinematic chain) inertia, \( \tau \) is the motor torque, \( f_m \) is the motor viscous friction (between motor and stator), \( C_m \) is the motor Coulomb friction (between motor and stator), \( k \) is the elasticity (between link and motor), and \( h \) is the viscosity (between link and motor). An illustrative scheme of the model is shown in Figure 1.

Equation (1) can be linearized by neglecting the Coulomb friction terms, so that the resulting transfer function between the motor torque and the motor velocity is:

\[
P_{\theta, \tau}(s) = \frac{\theta(s)}{\tau(s)} = \frac{J_m s^2 + h s + k}{J_m s^2 + (J_m + J_l) s + (J_m + J_l) k + f_m s + f_m k}
\]  

(2)

while the transfer function between the motor torque and the link position is:

\[
P_{q, \tau}(s) = \frac{q(s)}{\tau(s)} = \frac{h s + k}{(J_m s^2 + (J_m + J_l) s + ((J_m + J_l) k + f_m s + f_m k))}
\]  

(3)

It is worth stressing that the link inertia \( J_l \) strongly depends on the robot configuration and on the (possibly unknown) payload, while several possible model mismatches can occur in real setup. For example, the spring characteristic can be nonlinear, and aging can increase hysteresis and backlash. Finally, static and viscous friction terms might strongly depend on temperature (Simoni et al., 2017). Those model mismatches can imply significant detriment in model-based control strategies which do not take them into account.

2.1 Reduction of model parameters

In order to simplify the robust tuning rules expression, a model similarity is used to reduce the number of model parameters.

Rewriting (1) in the Laplace domain it is possible to obtain:

\[
J_m s^2 \dot{\theta} = h s \delta + k \delta - f_m s \dot{\theta} + \tau
\]  

(4)

\[
J_l s^2 (\dot{\theta} + \delta) = -h s \delta - k \delta
\]  

(5)

Then, by defining the following quantities

\[
s = \sqrt{\frac{k}{J_m}} \dot{s} \iff \dot{s} = \sqrt{\frac{J_m}{k}} s,
\]  

\[
\dot{\theta} = k \theta
\]  

(6)

\[
\delta = k \delta, \quad J_l = \frac{J_l}{J_m},
\]

it is possible to rewrite (4) as

\[
\dot{s}^2 \dot{\theta} = h s \dot{\delta} + \delta - f_m s \dot{\theta} + \tau
\]  

(7)

and (5) as

\[
\dot{J}_l \dot{s}^2 (\dot{\theta} + \delta) = -h s \dot{\delta} - \dot{\delta}
\]  

(8)

The ratio \( \dot{J}_l \) is the most important parameter, in fact, it causes a decrement of the resonance and antiresonance frequencies as shown in Figure 3 and, at the same time, it increases their peaks. The effect of \( h \), shown in Figure 4, is mainly on resonance and antiresonance peaks, while their frequencies present a slight shift. Finally, the motor viscous friction parameter changes the low frequency behavior and it reduces the resonance peak.

It is worth stressing that \( h \) has normally a small value for mechanical elastic elements (like torsional, coned-disc and coil springs) without dampers, while \( f_m \) has not a significant impact on the crossover frequency range. Finally, it is important to note that the motor velocity frequency response is less sensible to parameter changes for frequencies higher than the resonance frequency.

3. ROBUST OPTIMAL TUNING

The tuning procedure starts by considering the inner loop and continues with the outer one. The process of the inner loop has a transfer function defined between \( \tau \) and \( \theta \). Once the tuning parameters of this loop have been detected, the velocity closed loop obtained will be seen as the process of the outer loop, which has a transfer function defined between \( \theta \) and \( q \).
3.1 Motor velocity control

A PI controller regulates the motor torque to reduce the speed trajectory error. The controller is described by the following transfer function:

\[ C_v(s) = K_{p,v} + \frac{K_{i,v}}{s} \]  

(9)

where \( K_{p,v} \) is the proportional gain and \( K_{i,v} \) is the integral gain. The controller parameters can be rewritten by following (6) as

\[ \hat{J}_l = 0.5, \hat{h} = 0.2, \hat{f}_m = 0. \]

Magenta line: \( \hat{J}_l = 1, \hat{h} = 0.2, \hat{f}_m = 0. \)

Ocher line: \( \hat{J}_l = 1, \hat{h} = 0.3, \hat{f}_m = 0. \)

Violet line: \( \hat{J}_l = 4, \hat{h} = 0.2, \hat{f}_m = 0. \)

Fig. 3. Effect of the parameter \( \hat{J}_l \) on frequency responses \( P_{\theta,v}(s) \) (upper plot), \( P_{\eta,v}(s) \) (lower plot). Blue line: \( \hat{J}_l = 0.5, \hat{h} = 0.2, \hat{f}_m = 0. \) Magenta line: \( \hat{J}_l = 1, \hat{h} = 0.2, \hat{f}_m = 0. \) Ocher line: \( \hat{J}_l = 2, \hat{h} = 0.2, \hat{f}_m = 0. \) Violet line: \( \hat{J}_l = 4, \hat{h} = 0.2, \hat{f}_m = 0. \)

\[ \hat{K}_{p,v} = K_{p,v} \frac{\sqrt{k/J_m}}{k}, \hat{K}_{i,v} = \frac{\hat{K}_{i,v}}{k}, \]  

(10)

\[ K_{p,v} = \hat{K}_{p,v} \frac{\sqrt{k/J_m}}{k}, \]

\[ K_{i,v} = \hat{K}_{i,v} k, \]

The controller parameters are then computed by solving the following optimization problem:

\[ \min (\hat{\omega}_{c,v} - \hat{\omega}_{d,v})^2 - \lambda_v \phi_m \]  

(11)

subject to:

\[ \phi_m \geq \phi_{m,min} \]

\[ M_s \leq M_{s,max} \]

\[ |L_v(\hat{\omega})| \leq L_{h,v,max}, if \hat{\omega} \geq \hat{\omega}_h \]

\[ |L_v(\hat{\omega})| \geq L_{d,v,min}, if \hat{\omega} \leq \hat{\omega}_d \]

where \( \hat{\omega} \) is the normalized angular frequency, \( \hat{\omega}_{c,v} \) and \( \hat{\omega}_{d,v} \) are, respectively, the cutoff frequency and its desired value, \( \lambda_v > 0 \) is a weighting factor, \( \phi_m \) and \( M_s \) are the phase margin and the maximum sensitivity, \( L_v(\hat{\omega}) = C_v(\hat{\omega})P_{\theta,v}(\hat{\omega}) \) is the loop transfer function, \( L_{h,v,max} \) is the maximum allowed value of \( |L_v(\hat{\omega})| \) for frequencies greater than \( \hat{\omega}_h \), and \( L_{d,v,min} \) is the minimum allowed value for frequencies lower than \( \hat{\omega}_d \). In the tuning procedure, those values has been set equal to:

\[ L_{h,v,max} = -40dB, \quad \hat{\omega}_h = 100\hat{\omega}_d, \]

\[ L_{d,v,min} = 40dB, \quad \hat{\omega}_h = 0.01\hat{\omega}_d, \]

\[ M_{s,max} = 1.4, \quad \phi_{m,min} = 60^\circ \]

\[ \lambda_v = 10^{-2} \]

(12)

It is worth stressing that the presence of the antiresonance zeros can cause multiple crossing of the 0dB axis, and therefore the system has more than one phase margin. In this case, the most critical value is taken into account in the optimization algorithm. Due to these multiple cutoff frequencies, the maximum sensitivity \( M_s \) is therefore of utmost importance to guarantee the required level of robustness. The term \( -\lambda \phi_m \) of the cost function discards between solutions with the same cutoff frequency by selecting the one with the biggest phase margin.
3.2 Link position control

A PD controller addresses the link position control by computing the target motor speed signal \( \dot{\theta}_l \). The controller is described by the following transfer function:

\[
C_p(s) = \frac{K_{p,p}}{s} + \frac{K_{d,p}s}{sT_{f,p} + 1}
\]

(13)

where \( K_{p,p} \) is the proportional gain, \( K_{d,p} \) is the derivative gain, and \( T_{f,p} \) is the filter time constant. Controller \( C_p(s) \) acts on the process made by the secondary loop \( P_{\theta_l,q}(s) \) as shown in Figure 2. The target motor speed signal \( \dot{\theta}_l \) is given by the controller output plus a feedforward action equal to the velocity profile provided by the link motion planner. Integral action is not employed since \( P_{\theta_l,q}(s) \) has a pole at the origin and external disturbances are managed by the secondary loop.

The controller parameters can be rewritten by following (6) as

\[
\begin{align*}
K_{p,p} &= K_{p,p} \sqrt{\frac{M_{l,p}}{J_{m}}}, \\
K_{d,p} &= K_{d,p}, \\
\dot{K}_{f,p} &= T_{f,p} \sqrt{\frac{k}{J_{m}}}, \\
\dot{K}_{f,v} &= \dot{T}_{f,p} \sqrt{\frac{k}{J_{m}}},
\end{align*}
\]

As in Section 3.1, the controller parameters are computed by solving the following optimization problem:

\[
\begin{align*}
\min & \omega_{d,p} - \omega_{d,v}\nonumber \\
\text{subject to:} & \phi_m \geq \phi_{m,\text{min}}, \\
& M_l \leq M_{l,\text{max}} \\
& L_p(\omega) \leq L_{h,p,\text{max}}, \text{ if } \omega \geq \omega_h \\
& L_p(\omega) \geq L_{l,p,\text{min}}, \text{ if } \omega \leq \omega_l
\end{align*}
\]

(15)

where \( \omega_{d,v} \) and \( \omega_{d,v} \) are, respectively, the cutoff frequency and its desired value, \( L_p(\omega) = C_p(\omega)P_{\theta_l,q}(\omega) \) is the loop transfer function, \( L_{h,p,\text{max}} \) is the maximum allowed value of \( |L(\omega)| \) for frequencies greater than \( \omega_h \), while \( L_{l,p,\text{min}} \) is the minimum allowed value for frequencies lower than \( \omega_l \) and \( \lambda_v > 0 \) is a weighting factor. In the tuning procedure, those values have been set equal to:

\[
\begin{align*}
L_{h,p,\text{max}} &= 40dB \quad \omega_h = 100\omega_l, \\
L_{l,p,\text{min}} &= 20dB \quad \omega_l = 0.3\omega_l, \\
M_{l,\text{max}} &= 1.4, \quad \phi_{m,\text{min}} = 60^\circ, \\
\lambda_v &= 10^{-2}
\end{align*}
\]

3.3 Tuning rules

During the robot working activities, the link inertia can significantly change, and therefore a gain scheduling approach is highly recommended. However, solving (11) and (15) online is not possible due to computational burdens. For this reason, the development of an approximate solution by means of tuning rules has been performed, in order to allow the online computation of the robust controller for a given value of \( J_l \). The motor velocity controller tuning is performed by imposing the desired cutoff frequency to be part of the set \( \omega_{d,v} \in \{3, 4, 5, 6, 7, 8, 9, 10\} \). In order to find the tuning rules, the optimization problem (11) has been repeated 2000 times by varying \( h \) in the interval \([0.01, 0.5]\), \( f_m \) in the interval \([0.0, 0.5]\), and \( J_m \) in the interval \([0.5, 2]\) for each value of cutoff frequencies.

The resulting controllers parameters are approximated by means of the following tuning rules:

Table 1. Matrices coefficients of the motor velocity tuning rules when \( \omega_{d,v} = 3 \)

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Matrix Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{K_p} )</td>
<td>[2.531878, -0.187629, 0.766849, 0.176141, 2.360994]</td>
</tr>
<tr>
<td>( M_{K_i} )</td>
<td>[0.063666, 0.590771, -1.587520, 0.125880, -2.663157]</td>
</tr>
<tr>
<td>( M_{K_p} )</td>
<td>[0.009777, -0.295008, 0.786927, -0.124231, 0.948668]</td>
</tr>
</tbody>
</table>

Table 2. Matrices coefficients of the motor velocity tuning rules when \( \omega_{d,v} = 4 \)

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Matrix Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{K_p} )</td>
<td>[3.698848, -0.104410, 0.377411, 0.032056, 1.437171]</td>
</tr>
<tr>
<td>( M_{K_i} )</td>
<td>[0.011565, 0.280642, -0.653636, 0.130805, -1.496274]</td>
</tr>
<tr>
<td>( M_{K_p} )</td>
<td>[0.003181, -0.116833, 0.292964, -0.071785, 0.498541]</td>
</tr>
</tbody>
</table>

Table 3. Matrices coefficients of the motor velocity tuning rules when \( \omega_{d,v} = 5 \)

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Matrix Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{K_p} )</td>
<td>[4.754888, -0.113563, 0.302272, 0.032533, 0.976489]</td>
</tr>
<tr>
<td>( M_{K_i} )</td>
<td>[0.003699, 0.189066, -0.436630, 0.068670, -0.936631]</td>
</tr>
<tr>
<td>( M_{K_p} )</td>
<td>[0.003717, -0.074892, 0.188993, -0.035957, 0.298819]</td>
</tr>
</tbody>
</table>

Table 4. Matrices coefficients of the motor velocity tuning rules when \( \omega_{d,v} = 6 \)

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Matrix Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{K_p} )</td>
<td>[5.850311, -0.174389, 0.379250, 0.005614, 0.796468]</td>
</tr>
<tr>
<td>( M_{K_i} )</td>
<td>[-0.025062, 0.249436, -0.541724, 0.102136, -0.787528]</td>
</tr>
<tr>
<td>( M_{K_p} )</td>
<td>[0.010887, -0.094186, 0.217908, -0.048836, 0.201223]</td>
</tr>
</tbody>
</table>

Table 5. Matrices coefficients of the motor velocity tuning rules when \( \omega_{d,v} = 7 \)

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Matrix Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{K_p} )</td>
<td>[6.979575, -0.263059, 0.537100, -0.048779, 0.787911]</td>
</tr>
<tr>
<td>( M_{K_i} )</td>
<td>[-0.054282, 0.386998, -0.835283, 0.186529, -0.833085]</td>
</tr>
<tr>
<td>( M_{K_p} )</td>
<td>[0.020995, -0.156084, 0.344335, -0.08107, 0.301129]</td>
</tr>
</tbody>
</table>

The resulting values for each \( \omega_{d,v} \) are in Tables 1-8. For all the values of \( \omega_{d,v} \) the maximum sensitivity is much smaller than 1.4, as shown in Figure 6, which demonstrates the effectiveness of the tuning rules to ensure the required robustness.

The link position controller tuning rules are also computed by solving 2000 times the optimization problem (15) by using the motor velocity tuning rules with \( \omega_{d,v} = 3 \), even if it can be used also with faster secondary controlled systems. The primary loop cutoff frequency is set to \( \omega_{d,p} = 1 \) but, differently to the secondary loop case, the constraint \( M_s = 1.4 \) limits the bandwidth in the range \([0.3, 1]\).
Table 6. Matrices coefficients of the motor velocity tuning rules when $\hat{\omega}_{d,v} = 8$

\[
M_{K_p} = \begin{bmatrix}
7.946372 & -0.410546 & 0.836845 & -0.142072 & 0.832658 \\
-0.101432 & 0.656619 & -1.444846 & 0.373792 & -1.22394 \\
0.049538 & -2.894846 & 0.621562 & -0.175203 & 0.440879 \\
-0.011640 & 7.488284 & 0.159922 & 0.031979 & 0.016734 \\
0.886549 & -3.804287 & 4.914911 & -0.004192 & 0.022654 \\
-0.027762 & 0.613100 & -1.055465 & 0.002601 & -0.008212 \\
\end{bmatrix}
\]

Table 7. Matrices coefficients of the motor velocity tuning rules when $\hat{\omega}_{d,v} = 9$

\[
M_{K_p} = \begin{bmatrix}
8.996517 & -0.605644 & 1.256875 & -0.259264 & 0.988444 \\
-0.163220 & 1.035979 & -2.314674 & 0.615981 & -1.54295 \\
0.080999 & -0.405935 & 1.022770 & -0.291915 & 0.659466 \\
1.177551 & -4.797556 & 5.980280 & -0.001422 & 0.034360 \\
-0.054543 & 0.871900 & -1.428947 & 0.002580 & -0.013569 \\
\end{bmatrix}
\]

Table 8. Matrices coefficients of the motor velocity tuning rules when $\hat{\omega}_{d,v} = 10$

\[
M_{K_p} = \begin{bmatrix}
10.054296 & -0.854067 & 1.833401 & -0.489032 & 1.374578 \\
-0.257920 & 1.549434 & -3.540797 & 1.105627 & -2.438608 \\
0.111419 & -0.676386 & 1.601485 & -0.522677 & 1.091334 \\
0.034812 & 7.481134 & 1.048431 & 0.030974 & -0.033009 \\
1.009584 & -5.778026 & 6.918657 & -0.022352 & 0.059535 \\
-0.082965 & 1.149977 & -1.799310 & 0.011330 & -0.025308 \\
\end{bmatrix}
\]

Fig. 6. Nyquist plot of the motor velocity open loop transfer function ($\hat{\omega}_{d,v} = 10$). The dashed circle indicates the value $M_s = 1.4$.

The proposed tuning rules provide the fastest primary loop. The tuning rules are expressed by means of the following equation:

\[
K_p = \begin{bmatrix} 1 & \hat{f}_m & \hat{j}_m & \hat{h} & \hat{k} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 & \hat{J}_l & \hat{J}_d \\ 0 & 0 & 1 & \hat{J}_l \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_{K_p} \\ M_{K_d} \end{bmatrix}
\]

\[
\hat{T}_f = \frac{1}{5}
\]

The resulting values are in Table 9.

0.6 depending on the values of $\hat{h}$, $\hat{f}_m$, and $\hat{j}_l$. Thus, the proposed tuning rules ensure the required robustness.

Figure 7 shows that the maximum sensitivity is limited to 1.4 demonstrating the effectiveness of the tuning rules to ensure the required robustness.

4. EXPERIMENTAL RESULTS

The proposed tuning rules are tested on a three degree-of-freedom series elastic actuated robot, designed as a part of the FourByThree European project (FourByThree, 2018). The actuators are composed by a brushless DC motor, an harmonic drive and an elastic element made by a series of coil springs. The actuator has two encoders: one measures the motor position after the harmonic drive while the second measures the angle after the spring element. The current loop is implemented inside a FPGA board and it is considered as a unitary gain for the position and velocity controllers. The torque reference signal is sent to the FPGA board by using the ROS operating system framework (ROS, 2018).

Joint 1 has been used to show the effectiveness of the proposed method during a trajectory tracking task. The dynamics parameters of the robot are in Table 10. Moreover, there is a static friction of $C_m = 2.3$ [Nm] and a backlash of 0.015 [rad] (around 1 [deg]) which are considered as model mismatches, as well as the coupling effect caused by the other links.

The control performance are tested by means of two trajectories. In particular, a trapezoidal motion profile from position 0 [rad] to 1.5708 [rad] followed by another trapezoidal motion profile from 1.5708 [rad] to 0 [rad].
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reduced value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_l$ (no payload)</td>
<td>0.650 [kgm$^2$]</td>
<td>0.736</td>
</tr>
<tr>
<td>$J_l$ (m= 0.5 [kg])</td>
<td>0.740 [kgm$^2$]</td>
<td>0.8385</td>
</tr>
<tr>
<td>$J_l$ (m= 1 [kg])</td>
<td>0.786 [kgm$^2$]</td>
<td>0.900</td>
</tr>
<tr>
<td>$J_l$ (m= 2 [kg])</td>
<td>0.924 [kgm$^2$]</td>
<td>1.047</td>
</tr>
<tr>
<td>$J_l$ (m= 5 [kg])</td>
<td>1.514 [kgm$^2$]</td>
<td>1.714</td>
</tr>
</tbody>
</table>

Table 11. Link inertia values with disk payloads.

<table>
<thead>
<tr>
<th>Payload</th>
<th>$|\hat{\Delta}e|_p$</th>
<th>$|\hat{\Delta}e|_c$</th>
<th>$c_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_l$ (no payload)</td>
<td>0.0014 [rad]</td>
<td>0.0548 [rad/s]</td>
<td>0.3055 [Nm]</td>
</tr>
<tr>
<td>$J_l$ (m= 0.5 [kg])</td>
<td>0.0019 [rad]</td>
<td>0.0471 [rad/s]</td>
<td>0.3073 [Nm]</td>
</tr>
<tr>
<td>$J_l$ (m= 1 [kg])</td>
<td>0.0018 [rad]</td>
<td>0.0720 [rad/s]</td>
<td>0.4243 [Nm]</td>
</tr>
<tr>
<td>$J_l$ (m= 2 [kg])</td>
<td>0.0012 [rad]</td>
<td>0.0498 [rad/s]</td>
<td>0.3466 [Nm]</td>
</tr>
<tr>
<td>$J_l$ (m= 5 [kg])</td>
<td>0.0020 [rad]</td>
<td>0.0648 [rad/s]</td>
<td>0.3992 [Nm]</td>
</tr>
</tbody>
</table>

Table 12. Performance indexes for the different $J_l$ values.


has been considered. The motor velocity cutoff frequency has been chosen equal to $\omega_{d,p} = 3$, which corresponds to $\omega_{d,p} = 86.5735$ [rad/s]. In order to demonstrate the robustness of the controller, four different disk payloads have been attached to the robot flange, changing $J_l$ as shown in Table 11.

For each test, the performance are evaluated by computing the link position and the torque trends for each $J_l$ values. It is possible to note the effectiveness of the proposed methodology on a three degree-of-freedom series elastic actuated robot.

5. CONCLUSIONS

Robust tuning rules for Series Elastic Actuator control have been presented. The tuning rules are designed for a PD-PI cascade control and allow the online computation of the robust control parameters for each value of the link inertia. Experimental results show the effectiveness of the methodology on a three degree-of-freedom series elastic actuated robot.

ACKNOWLEDGEMENTS

The work is partially supported by FourByThree Project H2020-FoF-06-2014-737095.

REFERENCES


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