Comparing filtered PI, PID and PIDD\(^2\) control for the FOTD plants

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Abstract: The aims of the paper are: (a) to extend the 2DOF PI and PID controller design for the first order time-delayed (FOTD) plant by the multiple real dominant pole method to the 2DOF PIDD\(^2\) control, (b) to modify for this controller augmented by an nth order series binomial filter required for the derivative action implementation and measurement noise attenuation the simple integrated tuning procedures known already for the PI and PID control. (c) to align all the filtered controllers as for the guaranteed stability range in case of unstable plants, and (d) to compare the performance limits expressed in terms of the integral of absolute error (IAE) and (e) to discuss the corresponding closed loop robustness by a simple test based on comparing impacts of “exact” and simplified tunings based on the integral + dead time (IPDT) models.

Keywords: PI control, PID control, PIDD\(^2\) control, filtration.

1. INTRODUCTION

PI and PID control represent the most frequently used control technology (Åström and Hägglund, 2006), the first order time delayed (FOTD) systems the most commonly used approximations for their tuning. Recently, a great deal of attention has been paid to design of appropriate filters for attenuation of the measurement noise (Isaksson and Graebe, 2002; Leva and Colombo, 2004; Hägglund, 2012; Micle and Matausek, 2014; Fiser et al., 2017). A new design of nth order binomial filters (Huba, 2015) has shown that an appropriately tuned filtered PID control may yield faster closed-loop transients by producing a less excessive control effort than an optimally tuned PI control. Thereby, the controller design based on the multiple real dominant pole method (MRDP) (Vitečková and Viteček, 2010, 2016) has been extended also to the filter design, whereby new interesting development areas appeared. These made it possible to deal, for example, with controllers using higher order derivative actions and to show them attractive also in control of the time-delayed systems (Korobičnik et al., 2017; Huba, 2018). Physically, PIDD\(^2\) controllers offer position, velocity and acceleration feedback (Siciliano et al., 2009) useful in dealing with systems not allowing rapid output changes, when the loop behavior depends significantly on the previous control history. Since an analytical optimal design of four parameters of a PIDD\(^2\) controller, which, in addition, requires appropriate implementation filters represents a highly complex problem, different alternative approaches as, for example, the particle swarm optimization (Oliveira et al., 2014; Sahib, 2015) have been tested. In comparison with the much simpler PI control, which still attracts attention of the contemporary research (Mercader and Banos, 2017), the design is yet more complicated also due to the fact that an increased speed of transients exhibits all modeling and tuning imperfections.

This paper completes the derivation of an integrated tuning of FPI and FPID controllers in Huba (2015, 2016); Huba and Bisták (2016) by its extension to the FPIDD\(^2\) control. All controllers are designed by the MRDP method and compared with respect to the integral of absolute error (IAE), the noise attenuation, the achievable stability regions in control of the unstable FOTD plants and robustness in a simplified controller tuning based on integral plus dead time (IPDT) plant models.

The rest of the paper is structured as follows. Section 2 introduces the control problem and performance measures for FOTD systems. Sections 3, 4 and 5 are devoted to the FPI, FPID and FPIDD\(^2\) controller design by using the MRDP method. Section 6 discusses the obtained results, which are finally summarized in Conclusions.

2. FOTD PLANT’S CONTROL

All the considered controllers will be applied to the first order time-delayed (FOTD) plant model

\[
S(s) = \frac{Y(s)}{U(s)} = \frac{K_{sm}e^{-T_{dm}s}}{s + a_m} \quad \text{or for } a_m \neq 0, T_m = 1/a_m, K_m = K_{sm}/a_m \tag{1}
\]

For the reference setpoint \(w\), the efficiency of the tracking and control performance will be evaluated by

\[
\text{IAE} = \int_{0}^{\infty} |e(t)| \, dt \quad ; \quad e = w - y \tag{2}
\]

applied to the setpoint (\(IAE_s\)) and the input disturbance (\(IAE_d\)) step responses. Firstly, the model parameters are supposed to be known and the model index in their symbols will be omitted.

3. INTEGRATED TRDP FPI TUNING

For a PI controller \(C(s)\) with a prefilter \(F_p(s)\) (Huba, 2016)

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Fig. 1. Considered control structure, δ- measurement noise

\[
C(s) = \frac{K_c}{1+Ts} \quad \text{with} \quad \frac{bT_{i+1}}{sT_{i+1} + 1} \quad \text{or} \quad \frac{K_c + Ki}{s} 
\]

the loop (Fig. 1) is described by the transfer functions

\[
F_s(s) = \frac{Y(s)}{W(s)} = \frac{K_cK_r(T_{i+1})}{Ts(s+a)^2 + K_cK_r(T_{i+1})} 
\]

\[
F_d(s) = \frac{D(s)}{I(s)} = \frac{Ts}{Ts(T_{i+1})}T_{i+1} \quad \text{or} \quad \frac{K_cK_r(T_{i+1})}{sT_{i+1} + 1} 
\]

It yields the characteristics quasi-polynomial

\[
P(s) = T_{i+1}e^{Ts}(s + a) + K_cK_r(T_s + 1) 
\]

A triple real dominant pole (TRDP) \( s_0 \) of the characteristic quasi-polynomial \( P(s) \) (Vitečková and Viteček, 2010) satisfying \( P(s_0) = 0 \), \( \dot{P}(s_0) = 0 \) and \( \ddot{P}(s_0) = 0 \), the “optimal” parameters \( K_{co}, T_{do} \) with the prefilter tuning \( b_0 \) canceling one of the dominant poles \( s_0 \) are calculated from the formulas

\[
s_0 = -\frac{A^2 + S}{2T} \quad A = aT_d \quad S = \sqrt{A^2 + 8} 
\]

\[
K_0 = K_{co}K_rT_d = (S - 2)e^{(S - A - 4)/2} 
\]

\[
\tau_o = \frac{2T}{K_0} = \frac{A^2 + 2A + 28 - (4 + 10)S}{(S - 2)(S - 4)} 
\]

\[
b_0 = \frac{1}{\tau_oT_{so}} \quad \text{under the assumption of a constant control error sign} 
\]

\[
\text{AIE}_s = IE_s = T_s(1 - b) + aT_s/(K_cK_r) 
\]

\[
\text{IAME}_s = T_s/K_c = 1/K_c 
\]

For the integral systems with \( a = 0 \)

\[
\text{IAE} = (4 + 3\sqrt{2})/2 = 4.12T_d 
\]

\[
\text{IAE} = (7 + 5\sqrt{2})e^{(S - 1)/2} = 12.64K_0T_d^2 
\]

With the binomial filter in the feedforward path

\[
Q_n(s) = 1/(T_j s + 1)^n \quad n = 1, 2, \ldots 
\]

for \( T_d = 0 \) the open-loop transfer function becomes

\[
F_o(s) = K_c + T_s \quad \text{for} \quad \frac{T_{i+1}}{Ts(s+a)} \quad (s + a)^n \quad (1 + T_j s)^n 
\]

For the closed loop characteristic polynomial

\[
P(s) = T_s(s + a)(1 + T_j s)^n + K_cK_r(T_s + 1) 
\]

the triple real dominant pole (TRDP) \( s_0 \) is

\[
s_n = -\frac{n(A - S_n) + 4}{2(n+2)T_j} \quad A = aT_j \quad S_n = \sqrt{A_j^2 + 8 + 4\frac{1 - A_j}{n(n+1)}} 
\]

For a simple evaluation of the filtering properties, it is necessary to keep the closed loop dynamics nearly constant by a constant position of the dominant closed loop poles in (6) and (12), when

\[
\frac{A - S_n + 4}{2T_j} = \frac{n(A - S_n) + 4}{2(n+2)T_j} \quad A = aT_j \quad S = \sqrt{A_j^2 + 8 + 4\frac{1 - A_j}{n(n+1)}} 
\]

This allows an introduction of the so-called equivalent dead time \( T_e \) for approximation of the \( Q_n(s) \) delay. It may be used to characterize \( Q_n(s) \) impact on both the noise filtration and closed loop performance. Solving (13) for \( T_j \) with \( T_d = T_c \) yields

\[
T_j, \text{PID} = \frac{T_c}{(n+1)(4 - S) - \sqrt{n(A + 4(1 + 4(n + 1)/(S + 8)(S + 1)(S + 2))} \quad (1 + n)(4 + 4A + 4B + 2(n + 1))} 
\]

This may mean that an equivalent dead time \( T_e \) has the same impact on the dominant poles determining the closed loop performance as the \( Q_n(s) \) with the time constant \( T_j \). Therefore, the controller (6) is tuned according to

\[
T_d = T_{do} + T_c 
\]

\( T_c \) and \( n \) represent the tuning parameters for modifying the noise attenuation respected by the tuning (6).

4. INTEGRATED QRDP FPID TUNING

For a 2-DOF PID controller with a prefilter \( F_p(s) \)

\[
C(s) = \frac{K_c}{1+Ts} \quad \frac{bT_{i+1}}{sT_{i+1} + 1} \quad (14) 
\]

the “optimal” parameters \( K_{co}, T_{do} \) to a quadruple real dominant pole (QRDP) are given by

\[
s_0 = -\frac{A^2 + S}{2T_d} \quad A = aT_d \quad S = \sqrt{A^2 + 8} \quad K_0 = K_{co}K_rT_d = 0.5(S + A + 12) - (A^2 + 2A + 36) e^{(S - A - 6)/2} \quad \tau_o = T_{io}/T_d = 2(36 + 2A + A^2 - (A + 12)S) \quad \text{S} = 2 - \frac{A^3 + 12A^2 + 36A + 288 - (A + 12)S}{2 - S} \quad \text{A}_{T_d} = \sqrt{A^2 + 2A + 36 - (A + 12)S} \quad (17) 
\]

The optimal prefilter tuning will be determined to cancel two of the dominant poles \( s_0 \), which yields optimal values

\[
b_0 = -\frac{A^3 + 12A^2 + 36A + 288 - (A + 12)S}{2 - S} \quad (18) 
\]

\[
B_0 = A^3 + 12A^2 + 36A + 288 - (A + 12)S \quad (19) 
\]

Formally, the IAE values are given by (7). The limit figures for an integral systems with \( a = 0 \) and \( T_f \rightarrow 0 \)

\[
\text{IAE}_s = 2.1547T_d \quad \text{IAE}_c = 4.7626K_rT_d^2 
\]

show with respect to the PI control (8) a significant improvement. It is, however, to note that the performance limits will always be increased by the necessary filtration.

For a loop with \( T_d = 0 \) and \( Q_n(s) \), a QRDP \( s_n \) of the characteristic polynomial \( P(s) \)

\[
P(s) = T_s(s+a)(1 + T_j s)^n + K_cK_r(T_s + 1) \quad (20) 
\]

is given by

\[
s_n = -\frac{A - S_n + 4}{2T_j} \quad A = aT_j \quad S_n = \sqrt{A_j^2 + 8 + 4\frac{1 - A_j}{n(n+1)}} 
\]

For (15), a chosen \( T_c \) and \( n \), \( T_f \) is tuned by a requirement of a fixed closed loop poles position according to

\[
T_{f, PID} = \frac{T_e}{(n + 1)(4 - S) - 2A + (n - 1)(S + 8)(S + 1)(S + 2)} \quad \text{S} = 4 - 4A + (S - 1)/(S - 1) 
\]

5. INTEGRATED QNRDP FPID TUNING

Consider a PIDD2 controller extended by a prefilter \( F_p(s) \) with weighting coefficients \( b, c \) and \( d \).
\[ C(s) = \frac{K_c}{1+sT_i s + s^2T_i D s + s^3T_i D^2} \]
\[ F_p(s) = \frac{d}{s^3T_i D s + s^2T_i D s + sT_i} \]

The loop is characterized by the transfer functions
\[ F_{d}(s) = \frac{Y(s)}{W(s)} = \frac{K_T s}{s^2 + sT_i + sT_i D s} \]
\[ F_{d}(s) = \frac{Y(s)}{u(s)} = \frac{K_T s}{s^2 + sT_i + sT_i D s} \]

The quintuple real dominant poles (QnRDP) \( s_o \) of the characteristic quasi-polynomial \( P(s) \)
\[ P(s) = T_i s^3 + s + K_c K_s (1 + sT_i s + sT_i D s + s^2T_i D^2) \]
and the corresponding optimal PIDD controller parameters \( K_{co}, T_{Do}, T_{D20} \) and \( T_o \) may be given in the form of normed (dimensionless) coefficients
\[ s_o = \frac{-S - A - S}{2T_d}, \quad A = aT_d, \quad S = \sqrt{A^2 + 16} \]
\[ K_o = K_c K_s T_d = \frac{e^{(2 - \sigma - 1)(A^2 + 4S + 112)} - 416 - 44A - 16A^2 - A^3}{4} \]
\[ \tau_o = \frac{T_P}{T_d} = \frac{6.5S(A^2 + 14S + 112) - 64 - 44A - 16A^2 - A^3}{2(A^2 + 14S + 112)} \]
\[ \tau_{Do} = \frac{T_D}{T_d} = \frac{14S(A^2 + 14S + 112) - 64 - 44A - 16A^2 - A^3}{2(A^2 + 14S + 112)} \]
\[ \tau_{D20} = \frac{T_{D2}}{T_d} = \frac{6S(A^2 + 14S + 112) - 64 - 44A - 16A^2 - A^3}{2(2A^2 + 34A + 144 + 144 - 14A^2)} \]

Formally, the IAE values are again given by (7). The figures for integral systems with \( a = 0 \) and \( T_f = 0 \) are
\[ IAE_o = 1.5000 T_d; \quad IAE_i = 2.7709 K_s T_o \]

With respect to the PI control (8) and the PID control (19) they again represent a significant improvement which yet, however, does not include the necessary filtration.

Furthermore, these figures are still significantly higher than the absolute performance limits \( IAE_{e,\text{min}} = T_d \) and \( IAE_{e,\text{min}} = 0.5K_s T_o^2 \) (Huba et al., 2016), which motivates to deal also with higher order derivative actions. Hence, before using some faster (and obviously more complex) solutions, it is necessary to explore in detail the implementation conditions of all the above controllers.

The first results are related to implementation of the derivative actions in the PID and PIDD controllers and to the requirement of a systematic integrated and sufficiently simple controller + filter design in all the above mentioned situations (including the PI control)\(^1\).

For the FPIDD controller considering the filter (9) the input-disturbance-to-output transfer function is
\[ F_{d}(s) = \frac{K_c T_D (1+sT_i s)^3}{s^3T_i D s + s^2T_i D s + sT_i} \]

A quintuple real dominant pole (QRDP) \( s_n \) of
\[ P(s) = T_i s^3 + s + K_c K_s (1 + sT_i s + sT_i D s + s^2T_i D^2) \]
is given by
\[ s_n = \frac{2(-2n - 2 + \sqrt{n^2 - n - 2})}{(2 + 3n + n^2)T_f} \]

The requirement \( s_o = s_n \) yields now the equation

\(^1\) as e.g. mentioned in Åström and Hågglund (2006), in practice the derivative action is frequently not used because of the lack on reliable tuning methods
Fig. 2. Performance limits of considered controllers

Fig. 3. Impact of a simplified controller tuning in dependence on the derivative action degree

with $a_m = a \neq 0$ and IPDT models increases. This effect makes it possible to work with $a_m = 0$, which may partially compensate the complexity of the multi-parameter PIDD$^2$ controllers. For the relatively large $|T_d/T|$, where the differences in IAE for $a_m = a > 0$, $a_m = a < 0$ and $a_m = 0$ increase, one has to expect an increased sensitivity also from the “preciser” FOTD models. These expectations are confirmed by the setpoint and disturbance step responses in Figs 4-6. Whereas in Fig. 4 with the minimal order $n = 2$ required by the FPIDD$^2$ controller the impact of a measurement noise is significant, FPI$_n$ and FPID$_n$ show with this filter much smoother transients. However, for a simplified tuning with $a_m = 0$ these controllers are more sensitive. In this sense, FPIDD$^2$ is apparently more robust, although it yields a slight overshooting. The use of the filter order $n = 4$ (Fig. 5) offers for the FPIDD$^2$ controller a much better performance - the simplified FPI$_n$ controller is yet more unstable than with $n = 2$. As it was predicted by Fig. 3, in case of stable systems with the same $|aT_d|$ the impact of simplified controller tunings is much less disturbing than for unstable systems. The more detailed presentation enables to note the simulation imperfection (non-smooth shapes of transients) in the setpoint steps which may be observed also for other time delayed systems with faster transients (Huba and Žáková, 2017; Huba and Bélaí, 2018). This gives the motivation to test the developed control by real time experiments and to come round the numerical imperfection by discrete time solutions.

The paper has evaluated the impact of a 2nd order derivative term used for modifying the PI and PID to PIDD$^2$ control. Together with a simple tuning method for the introduced $n$th order binomial filters, the simply applicable FPI$_n$ and FPID$_n$ control have been extended by a FPIDD$^2_n$ control. All they offer a third degree of freedom devoted to the measurement noise filtration, whereas it is possible to keep a nearly constant speed of transients. Since the additional control effort may be significantly reduced, the use of derivative action may be used to speed up transients without increasing the noise sensitivity. Besides that, the simplified robustness analysis forecast robustness increasing with higher order derivative action.

REFERENCES


Fig. 4. Setpoint and disturbance step responses for different derivative action degrees with a nominal (a) and a simplified controller tuning (0), \( a = -0.2, K_s = 1, T_d = 1, T_e = 0.9, n = 2 \); measurement noise with the amplitude \( |\delta| < 0.1 \) generated in Matlab/Simulink by the “Uniform Random Number” block.

Fig. 5. Setpoint and disturbance step responses for different derivative action degrees with a nominal (a) and a simplified controller tuning (0), \( a = -0.2, K_s = 1, T_d = 1, T_e = 0.9, n = 4 \); measurement noise with the amplitude \( |\delta| < 0.1 \) generated in Matlab/Simulink by the “Uniform Random Number” block.


