Robust High-Gain Generalization of PID Controllers with Anti-Windup Compensation *

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Abstract: Generalization of PID controllers depending on the plant relative degree is addressed in the paper. The consecutive compensator approach is redesigned in the state-space representation and augmented with the integral loop and anti-windup scheme. The stability of the closed-loop system is proved. Efficiency of the proposed approach is illustrated by the experiments carried out using the 2-DOF indoor quadcopter testbed.

Keywords: Robust control, Output regulation, Saturation control, Windup, Popov criterion

1. INTRODUCTION

Development of robust high-gain control algorithms with trivial parameters adjustment for wide range of applications is of interest to scientists and engineers from all over the world. There is a number of research works, where various problem statements are addressed using the consecutive compensator approach (for example, see Bobtsov (2005); Bobtsov and Nikolaev (2005)), which is based on the passification principle Fradkov (2003). However, the majority of such works assume unbounded control signals, while the input saturation effect together with the consecutive approach and internal model principle still had not been properly studied.

The input saturation together with the integral term in the controller structure might lead to the so-called integral windup. In turn it expresses in overshoot and settling time increase, which at critical values can lead to self-oscillations and loss of stability in real systems, especially unmanned aerial vehicles. The issue of the input saturation lies in the fact that integral component accumulates error value when the control signal exceeds the saturation limits.

Anti-windup approaches based on various principles are used to take into account input saturation and eliminate its negative effects. Setpoint Limitation principle assumes specification of admissible areas, where the reference values can be given so, that the control signal never approaches the saturation limits. As a rule, such approach restricts the control quality and does not solve the issue under conditions of external disturbances. Conditional Integration deactivates the integral loop using the conditional algorithm, when the control signal reaches the saturation limits. Back Calculation assumes choosing an auxiliary signal as an error between the saturated control and the original one and sending it to the integrator input to compensate error accumulation within the saturation zone. An advantage with respect to the previous one is that the deactivation of the integral term is done dynamically instead of the conditional algorithm Åström and Hägglund (2006).

Anti-windup compensation is used extensively in a wide range of applications. For example, the effect of integral windup is addressed in the flight control issues in An-drievsky et al. (2013, 2012, 2015); Leonov et al. (2012). The problem of the discrete-event control of manufacturing systems using the anti-windup procedure is considered in Van Den Bremer et al. (2008).

This study represents the continuation of the quadcopter control research published in Borisov et al. (2016, 2017); Tomashevich et al. (2017b). The preliminary theoretical study on quadcopter stabilization using the consecutive compensator approach with the integral term and anti-windup scheme is presented in Borisov et al. (2016). The control law for quadcopter stabilization is updated with the adaptation law in Borisov et al. (2017). The work Tomashevich et al. (2017b) focuses on the experimental verification of the designed control algorithms using 2-DOF quadcopter testbed “KOMEX-1”. The current study provides a novel state-space representation of the robust output high-gain control approach. The integral loop with anti-windup compensation scheme is implemented to the controller structure following the internal model principle.

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The detailed stability analysis of the closed-loop system is provided. The proposed control algorithm is implemented to the quadcopter tested and results of the experimental study are given in the paper.

The paper is organized as follows. The addressed problem is stated in Section 2. The mathematical model of the plant is transformed in Section 3 to extract its zero dynamics. The control law and stability proof are given in Section 4. The proposed control algorithm is applied to the quadcopter model in Section 5. The experimental results of the dual-mode stabilization of the quadcopter are provided in Section 6.

2. PROBLEM STATEMENT

Consider the SISO system

\[
\begin{align*}
\dot{x} &= Ax + bu + Rw, \\
y &= cx,
\end{align*}
\]

where \(x \in \mathbb{R}^n\) is the state vector, \(u \in \mathbb{R}\) is the control signal, \(w \in \mathbb{R}^{n_R}\) is the disturbance vector, \(y \in \mathbb{R}\) is the measurable output signal, \(A, R, b, c\) are matrices and vectors of corresponding dimensions.

In general the disturbance vector \(w\) can be considered as

\[
w = Sw,
\]

where \(S\) is the state matrix of the corresponding dimension.

The control signal \(u\) satisfies the saturation condition

\[
u = \text{sat}(v) = \begin{cases} u_{\text{max}} & \text{if } v \geq u_{\text{max}}, \\
u & \text{if } u_{\text{min}} < v < u_{\text{max}}, \\
u_{\text{min}} & \text{if } v \leq u_{\text{min}}, \end{cases}
\]

\(u_{\text{min}}\) and \(u_{\text{max}}\) are the input saturation limits, \(v\) is the nominal control signal generated by the linear regulator.

Assumption 1. The zero dynamics of the plant (1), (2) is stable.

Assumption 2. The relative degree of the plant (1), (2) \(\rho \geq 1\) is known.

Assumption 3. The output of the disturbance generator (3) is a static signal, i.e. \(S = 0\) and \(w(0) \neq 0\).

Assumption 4. The input limits \(u_{\text{min}}\) and \(u_{\text{max}}\) satisfy

\[
u_{\text{min}} = -u_{\text{lim}} < 0, \quad u_{\text{max}} = u_{\text{lim}} > 0,
\]

where \(u_{\text{lim}} > 0\) is the saturation limit.

Assumption 5. The disturbance is bounded \(w \in L_\infty\) and the nominal control signal \(u_0\) needed for its compensation at steady state satisfies

\[
u_{\text{min}} \leq |u_0| \leq u_{\text{max}}.
\]

The purpose is to design the control law \(u\) based only on the measurements of the output signal \(y\) such that the following relation holds

\[
\lim_{t \to \infty} y(t) = 0
\]

under conditions of external disturbances and saturated input (4).

3. ZERO DYNAMICS EXTRACTION

Perform change of coordinates to extract the stable zero dynamics. The input-output representation of the plant model (1), (2) is

\[
y(s) = \frac{b(s)}{a(s)}u(s) + \frac{r(s)}{a(s)}w(s),
\]

where the polynomial \(b(s)\) is Hurwitz due to Assumption 1.

Transform the model (5) as follows

\[
a(s)b(s)y(s) = u(s) + \frac{r(s)}{b(s)}w(s),
\]

which rewrite as

\[
z(s) = \frac{d(s)}{b(s)}y(s) - \frac{r_1(s)}{b(s)}w(s),
\]

\[
y(s) = \frac{1}{c(s)}(u(s) - z(s)) + \frac{r_2(s)}{c(s)}w(s),
\]

which is equivalent to the state-space representation of the plant (1), (2)

\[
\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_2 \end{bmatrix} u + \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} w,
\]

\[
y = \begin{bmatrix} 0 \\ c_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix},
\]

where the matrix \(A_{11}\) is Hurwitz due to Assumption 1, \(b_2 = [0 \ldots 0 b_0]^T\), \(c_2 = [1 0 \ldots 0]^T\).

4. CONTROL DESIGN AND STABILITY ANALYSIS

Choose the control law

\[
u = -\kappa (c_T^\top \xi + y) - \gamma \eta,
\]

\[
\dot{\xi} = A \xi + b_2 \eta,
\]

\[
\dot{\eta} = \kappa (c_T^\top \xi + y) + \nu \varphi(v),
\]

\[
\varphi(v) = v - \text{sat}(v),
\]

where \(\varphi(v)\) is the nonlinear anti-windup signal, \(\kappa > 0\), \(\gamma > 0\), \(\nu > 0\), the matrix \(A_q\) and vectors \(b_q, c_q\) are of the form

\[
A_q = \begin{bmatrix} -q_0^\sigma & 0 & \ldots & 0 \\ -q_{\rho-1}^\sigma & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -q_0^{\sigma_{\rho-1}} & 0 & \ldots & 0 \end{bmatrix}, \quad b_q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_\rho \end{bmatrix}, \quad c_q = \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix},
\]

where \(\sigma > 0\), \(q_i^\sigma (i = 1, \ldots, \rho)\) are chosen for the system (9) to be Hurwitz, \(q_i \in (1, \rho)\) are coefficients of an arbitrary Hurwitz polynomial of the form \(q(s) = q_0 s^{\rho-1} + \cdots + q_2 s + q_1\).

Combine the plant (6), (7) and the control law (8)-(11) and obtain the model of the closed loop system (12).
The change of coordinates \( \chi = z_2 - \xi \) transforms the closed-loop model (12) in the form (13), where

\[
I_0 = A_q + b_q T^2 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix}.
\]

The next change of coordinates \( \zeta = (b_2^T b_2)^{-1}b_2^T z_2 + \eta \) transforms the closed-loop model (13) in the form of (14), where

\[
b_2^T = (b_2^T b_2)^{-1} b_2^T = b_0^{-2} b_2^T = b_2^T = \begin{bmatrix}
0 & \ldots & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad b_0 \neq 0.
\]

Due to Assumption 1 and chosen basis of the model (6), (7) matrix \( A_1 \) is Hurwitz. Note that the block matrix

\[
A_1 = \begin{bmatrix}
A_{11} & 0 \\
A_{12} & -\gamma
\end{bmatrix}
\]

is Hurwitz, since choice of \( \gamma > 0 \) allows to shift the eigenvalues of the block matrix \( A_1 \) to the left from the imaginary axis.

It can be shown that there exists the number \( \kappa^* \) such that for \( \forall \kappa \geq \kappa^* \) the block matrix

\[
A_2 = \begin{bmatrix}
A_{11} & 0 \\
A_{12} & -\gamma
\end{bmatrix}
\]

is Hurwitz. Indeed, the block element \( A_{12} - \kappa b_2 c_q^T + c_2^T \) depends on the parameter \( \kappa \). Its sufficiently large increase shifts eigenvalues of the block \( A_2 \) to the left from the imaginary axis without effecting the remaining block elements.

Due to the structure of the matrix \( A_q \) it can be similarly shown, that the overall state matrix of the closed system is Hurwitz. Indeed, the parameter \( \sigma \) together with the vector \( c_q \) defining the stable dynamics of \( A_q \) and consequently the overall block \( A_q + \kappa b_2 c_q^T \) is not included in the remaining block elements. Its increase allows to shift eigenvalues of the matrix \( A \) to the left from the imaginary axis.

As a result, due to choice of parameters \( \gamma, \kappa, \sigma \) in the system (14) Hurwitzness of the all diagonal block elements of the matrix \( A \), from which Hurwitzness of the overall matrix follows.

Temporarily assume \( w = 0 \) and consider the case without external disturbances. Write the closed-loop system (14) in the compact form as the feedback interconnection of the plant (1),(2) and the control law (8)–(11)

\[
A = \begin{bmatrix}
A_q & 0 \\
A_{12} & -\gamma
\end{bmatrix}
\]

\[
x = A x + b \zeta(v),
\]

\[
\zeta = \begin{bmatrix}
0 \\
-b_2
\end{bmatrix}
\]

where

\[
\begin{bmatrix}
x^T \\
\zeta^T
\end{bmatrix} = \begin{bmatrix}
A_{11} & 0 \\
A_{12} & -\gamma
\end{bmatrix} x = \begin{bmatrix}
A_q & 0 \\
A_{12} & -\gamma
\end{bmatrix} x.
\]

Proposition 6. The closed-loop system consisting of the linear part (15)–(16) and static nonlinearity \( \zeta(v) \) is absolutely stable with all initial conditions.

Proof. According to the Popov criterion Khalil (2002), the closed-loop system consisting of the linear part (15)–(16) with Hurwitz state matrix \( A \) and the memoryless nonlinear function \( \zeta(v) \) is absolutely stable for all initial conditions if there exists a constant \( \varpi \geq 0 \) such that \( (1 + \lambda \varpi) \neq 0 \) for each eigenvalue \( \lambda \) of the matrix \( A \) and the transfer function \( W(s) = 1 + (1 + s \varpi) W_i(s) \) strictly positive real, where \( W_i(s) \) is the transfer function of the system (15)–(16).

Choose \( \varpi = 0 \), then to prove the absolute stability of the system it is sufficient to show strictly positive realness of the transfer function \( W(s) = 1 + W_i(s) \).

Calculate the transfer function \( W_i(s) \)

\[
W_i(s) = c^T (s I - A)^{-1} b = \begin{bmatrix}
-\kappa c^T - \kappa c_q^T \\
\gamma b
\end{bmatrix} A = \begin{bmatrix}
A_q & 0 \\
A_{12} & -\gamma
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}^{-1} A,
\]

where

\[
A_{11} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}^{-1} A = \begin{bmatrix}
A_q & 0 \\
A_{12} & -\gamma
\end{bmatrix} = \begin{bmatrix}
A_q & 0 \\
A_{12} & -\gamma
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}^{-1} A,
\]

and use the Frobenius formula for block matrices inversion.

Calculate the inverse matrix of the block \( A_{22} \)

\[
A_{22}^{-1} = \begin{bmatrix}
A_{21} & A_{22}
\end{bmatrix}^{-1} = \begin{bmatrix}
A_{21} & A_{22}
\end{bmatrix} = \begin{bmatrix}
A_q & 0 \\
A_{12} & -\gamma
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}^{-1} A
\]

in the system (15)–(16) and static nonlinearity \( \zeta(v) \) is absolutely stable with all initial conditions.
where $A_q = (sI - A_q)^{-1}$.
Calculate all the blocks of the matrix $A$

\[
A_{11} = \begin{pmatrix}
\bar{A}_{11} & \bar{A}_{12} \\
A_{22} & A_{11}
\end{pmatrix}
\]

\[
A_{21} = \begin{pmatrix}
A_q(s)b_s^T A_{11} \\
\frac{rb_q(s)}{s} A_q(s)
\end{pmatrix}
\]

where

\[
\begin{aligned}
b_{q}(s) &= c_{q}^{T} A_{q}(s) b_{q} + 1, \\
\beta(s) &= c_{q}^{T} A_{q} b_{q} \\
\frac{\beta(s)}{\alpha(s)} &= c_{q}^{T} A_{q} b_{q} + \frac{b(q) s a_{q}(s)}{s a_{q}(s) + r b_{q}(s)(s + \gamma) b(s)}.
\end{aligned}
\]

Remark 7. Note that Hurwitzness of the numerator and denominator of the transfer function (18) can be achieved by choosing matrix $A_q$ and vectors $b_q$ and $c_q$.

Substitute derived expressions of the blocks $A_{11}$, $A_{12}$, $A_{21}$ and $A_{22}$ into the matrix $A$ (for the sake of brevity the middle column is omitted)

\[
A = \begin{pmatrix}
A_{11} & \ast & \ast & \ast & \ast & \ast \\
A_q(s)b_s T A_{11} & \ast & \ast & \ast & \ast & \ast \\
\frac{rb_q(s)}{s} & T A_{11} & \ast & \ast & \ast & \ast \\
\end{pmatrix}
\]

Calculate the transfer function $W_f(s)$

\[
W_f(s) = \begin{pmatrix}
-b \\
0
\end{pmatrix} \begin{bmatrix}
\beta_{q}(s) = \frac{k b_q(s)(s + \gamma) b(s)}{s a_q(s)a(s)} + \frac{\beta_{a}(s)}{\alpha(s)}, \\
\frac{\beta_{a}(s)}{\alpha(s)} = \frac{k b_q(s)(s + \gamma) b(s)}{s a_q(s)a(s)} + \frac{\beta_{a}(s)}{\alpha(s)}
\end{bmatrix}
\]

Next, calculate the transfer function $W(s)$ and show its strictly positive realness

\[
W(s) = W_f(s) + 1 = \frac{2 b_q(\gamma(\gamma) b(s)(s + \gamma + (s - \gamma) a_q(s)) a(s)}{s a_q(s)a(s) + k b_q(s)(s + \gamma) b(s)}
\]

Indeed, together with Remark 7 and Assumption 1 it easy to show, that there exists a number $\kappa_0$ such that for $\kappa \geq \kappa_0$ both the numerator and denominator of the transfer function $W(s)$ are Hurwitz. The relative degree of $W(s)$ is zero. As a consequence, strictly positive realness follows, i.e. Re$\{W(j\omega)\} > 0$, $\forall \omega \in [-\infty, \infty]$ or equivalently Re$\{W_f(j\omega)\} > 0, \forall \omega \in [-\infty, \infty]$ and the absolute stability of the system (15), (16) follows in accordance to the Popov criterion Khalil (2002).

Return to the case of external disturbance presence ($w \neq 0$) and analyze the steady error. Obviously, the following relation holds at steady state $v = \text{sat}(v)$, consequently $x(v) = v - \text{sat}(v) = 0$. In order to determine steady state behavior use the Sylvester equation applied to the model (12). In particular focus on the fourth row

\[
\Sigma S = \kappa(c_q^T \Pi_2 + c_q^T \Pi_\xi),
\]

which shows that at steady state

\[
c_q^T \Pi_2 + c_q^T \Pi_\xi = 0. \tag{21}
\]

Find the relation between $c_q^T \Pi_2$ and $c_q^T \Pi_\xi$. Consider the auxiliary variable $z_0$

\[
z_0 = y + c_q^T \xi, \tag{22}
\]

taking into account (21), the steady state value of which is zero.

From (9) follows

\[
\xi(s) = (s I - A_q)^{-1}(b_q y(s) + \xi(0)),
\]

where $\xi(0)$ is a vector of initial conditions.

Then, rewrite (22)

\[
z_0(s) = (c_q^T (s I - A_q)^{-1} b_q + 1) y(s) + \epsilon(s),
\]

where $\epsilon(s) = c_q^T (s I - A_q)^{-1} \xi(0) + \xi(0)$ corresponds to the exponentially decaying function $\epsilon(t)$.

If $c_q^T$ is chosen so that the numerator of the transfer function $(c_q^T (s I - A_q)^{-1} b_q + 1)$ is Hurwitz and the relative degree is zero, then from

\[
y(s) = (c_q^T (s I - A_q)^{-1} b_q + 1)^{-1}(z_0(s) - \epsilon(s))
\]

find that the steady-state error of $y$ as well as $c_q^T \xi$ converges to zero.

5. QUADROPTER CONTROL

Consider the quadcopter mathematical model assuming the drag coefficient equal to zero at low speed Altuğ et al. (2002):

\[
\begin{align*}
\dot{x} &= \sum_{i=1}^{4} u_i(c_{\phi} s_{\theta} c_{\psi} + s_{\phi} s_{\psi}), \\
J_\theta \ddot{\theta} &= \ell(-u_1 - u_2 + u_3 + u_4), \\
J_y \ddot{\psi} &= \ell(-u_1 + u_2 + u_3 - u_4), \\
J_z \ddot{\psi} &= \sum_{i=1}^{4} u_i(c_{\phi} s_{\psi} c_{\theta} - s_{\phi} s_{\psi}), \\
J_{\phi} \ddot{\phi} &= C(u_1 - u_2 + u_3 - u_4),
\end{align*}
\]

where $x, y, z$ are the Cartesian coordinates, $\theta, \psi, \phi$ are the Euler angles, which correspond to the pitch, roll and yaw angles, $u_i, i = \{1, 2, 3, 4\}$ are the control signals representing the lift force of each motor, $m$ is the mass of the quadcopter, $g$ is the gravitational acceleration, $\ell$ is the distance between the quadcopter geometric center and the plane passing through axes of rotation of two adjacent motors, $J_\theta, J_y, J_\phi$ are the moments of inertia, $C$ is the conversion factor from force to torque, $c_{\phi} \equiv \cos \phi, s_{\phi} \equiv \sin \phi$.

Decompose the quadcopter model choosing the set of quasi-control signals

\[
\begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 1 & 1 \\
-1 & -1 & 1 & 1 \\
1 & 1 & 1 & -1 \\
1 & -1 & 1 & -1
\end{bmatrix} \begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
u_4
\end{bmatrix}.
\]

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where the quasi-control signals $U_i, i = \{1,2,3,4\}$ satisfy the saturation condition

$$U_i = \text{sat}(v_i) = \begin{cases} U_{i,max}, & v_i \geq u_{i,max} \\ U_i, & u_{i,min} < v_i < u_{i,max} \\ U_{i,min}, & v_i \leq u_{i,min}. \end{cases}$$  \tag{24}$$

$U_{i,min}$ and $U_{i,max}$ are the input saturation limits satisfying Assumption 4, $v_i$ are the nominal control signals generated by the independent SISO linear regulators.

Following transformations similar to Borisov et al. (2016, 2017) use the linearized quadcopter model

$$m\ddot{x} = -U_5, \quad J_\theta \ddot{\theta} = \ell U_2, \quad (25)$$
$$m\ddot{y} = -U_6, \quad J_\psi \ddot{\psi} = \ell U_3, \quad (26)$$
$$m\ddot{z} = U_1 - mg, \quad J_\phi \ddot{\phi} = CU_4, \quad (27)$$

where the desired values of the roll and pitch angles are calculated as

$$\theta^* = \frac{U_5}{U_1}, \quad \psi^* = -\frac{U_6}{U_1}, \quad U_5 = U_1 \theta, \quad U_6 = -U_1 \psi. \quad (28)$$

**Remark 8.** Note that in the final engineering implementation the calculation algorithm should be modified in order to avoid singularities in (28). Zero values of $\theta^*$ and $\psi^*$ should be assigned when $U_1$ is approaching to zero. Values of $\theta^*$ and $\psi^*$ should be bounded to avoid critical reference peaks within the transient processes.

The relative degrees of all the subsystems (25)–(27) equal $\rho_i = 2$. In view of that the adapted regulator has the form

$$v_i = -\kappa_i (q_i, \ddot{q}_i, \dot{q}_i + y_i) - \gamma_i \eta_i, \quad (29)$$
$$\dot{q}_i = A_{q,i} \dot{q}_i + b_{q,i} y_i, \quad (30)$$
$$\dot{\eta}_i = \kappa_i (q_i, \ddot{q}_i, \dot{q}_i + y_i) + \nu_i \kappa_i(v_i), \quad (31)$$
$$\kappa_i(v_i) = v_i - \text{sat}(v_i), \quad (32)$$

where $\kappa_i(v_i)$ are the nonlinear anti-windup signals, $\kappa_i > 0$, $\gamma_i > 0$, $\nu_i > 0$, the matrices $A_{q,i}$ and vectors $b_{q,i}$ are of the form

$$A_{q,i} = \begin{bmatrix} -q_i^2 \sigma_i & 1 \\ -q_i \dot{q}_i \sigma_i & \sigma_i^2 \end{bmatrix}, \quad b_{q,i} = \begin{bmatrix} q_i \sigma_i, q_i^2 \sigma_i \\ q_i \dot{q}_i \sigma_i^2 \end{bmatrix}, \quad c_{q,i} = \begin{bmatrix} q_i^2 \sigma_i \\ q_i \dot{q}_i \sigma_i \end{bmatrix},$$

where $\sigma_i > 0$.

As a last step, we need to do inverse transformation of (23) to calculate the input signals to be sent to the actuators

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = 0.25 \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}. \quad (23)$$

**6. EXPERIMENTAL APPROVAL**

Experimental study of the proposed control algorithm has been carried out using the 2-DOF indoor quadcopter testbed “KOMEX-1” (see Fig. 1). This testbed represents a wooden stationary structure with two moving elements. A bar, at the middle of which a quadcopter is rigidly mounted, is connected to an outer frame by means of hinges. The outer frame is attached to the vertical pillars. Such construction provides two degrees of freedom of the quadcopter in the directions of roll and pitch angles. Thus, the testbed allows to test dual-channel stabilization of the quadcopter under laboratory conditions without any risks of damage. Hardware and software parts of the quadcopter testbed are described in the works Tomashevich and Belyavskiy (2016); Tomashevich et al. (2017a).

The experimental results of the stabilization of the both pitch and roll angles are shown in Fig. 2. The control law (29)–(32) has been chosen with the parameters $\kappa_2 = \kappa_3 = 5$, $\gamma_2 = \gamma_3 = 1$, $\nu_2 = \nu_3 = 1$, $A_{q,2} = A_{q,3} = \begin{bmatrix} -10 & 1 \\ -100 & 0 \end{bmatrix}$,

$$b_{q,2} = b_{q,3} = \begin{bmatrix} 10 \\ 100 \end{bmatrix}, \quad c_{q,2} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad c_{q,3} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \sigma_2 = \sigma_3 = 10.$$

The both control signals were saturated (see Fig. 2a and 2c). The input saturation limits chosen for each channel are $U_{2,lim} = 800$ RU, $U_{3,lim} = 400$ RU.

Note that the quadcopter is affected by the external disturbance caused by the air flow generated by the quadcopter propellers and reflected from the floor. Despite this, the transient processes of the both angles are stable under condition of the bounded input signals.

**7. CONCLUSION**

The well-known structure of PID controllers can be generalized using the consecutive compensator approach together with integral loop and anti-windup compensation scheme. The particular form of this controller explicitly related with a classical PID structure is presented in Tomashevich et al. (2017b). In this research the consecutive compensator approach is rewritten in the state-space representation, that allows to analyze the absolute stability of the closed-loop system with the input saturation nonlinearity using the Popov criterion. Practical value of the designed control law is confirmed by the experimental study of simultaneous stabilization of pitch and roll angles of the quadcopter.

There are several directions of the further research on the addressed topic. As one can see from (3), this study focuses on the regular component of the disturbance. However, the irregular component of the disturbance does not violate the stability proof in general. Using the Lyapunov function approach it is easy to estimate the steady-state error with respect to the noise magnitude. Besides that, the proposed approach might be improved using the internal model in general form to compensate sinusoidal disturbances. Adaptation laws can be implemented in case if the disturbance frequencies are unknown, which is reasonable for real applications.
Fig. 2. Experimental results (RU stands for “Relative Units”)

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