Analysis of Anti-windup Techniques in PID Control of Processes with Measurement Noise *

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Abstract: This work presents an analysis of the effect of measurement noise on the closed-loop performance for three anti-windup strategies, used together with a PID controller. The study is done both analytically and experimentally and considered stable, integrating and unstable processes with dead time subjected to saturation of the actuator. The PID tuning rule, used for all the presented case studies, is based in a low-order approximation of the filtered Smith predictor. The analysis shows that the error recalculation anti-windup technique gives better reference tracking performance when compared to the incremental algorithm and back-calculation techniques, being able to reduce the effects of noisy measurements on the calculation of the control action, thus resulting in lower control and process variable variability. In addition, it is shown that when the process operating point is near a saturation limit, noise can cause an offset between the process variable and the reference and it is also proven that the error recalculation anti-windup strategy can significantly attenuate this behavior.

Keywords: Windup, PID, saturation, actuator, transport delay, measurement noise.

1. INTRODUCTION

It is estimated that approximately 90% of the current industrial controllers are of the PID type due to its simplicity, low cost and robustness (de Castro et al., 2016; Oviedo et al., 2006; Abdel-Geliel et al., 2014). However, for practical applications with nonlinearities or dead time, which are very common characteristics in industrial processes, the performance of regular PID controllers is not satisfactory. In some specific cases, it is necessary to use more complex control strategies, but generally it is possible to use regular PID controllers with special tuning or modified versions of PID controllers even in these cases. A typical example of nonlinearity which can compromise the performance (or even stability) of a PID controller and is common in industrial processes is the saturation of the actuator. In fact, any nonlinearity which can make the plant input different from the controller output causes a wrong update of the controller states if the output of the controller at past time instants is used to determine the current output, what is known as controller windup (Doyle et al., 1987; Hippe, 2006).

The literature presents a variety of techniques, known as anti-windup (AW) strategies, to avoid the undesired effect in the closed-loop performance caused by windup (Fertik and Ross, 1967; Hansu et al., 1987; Åström and Wittenmark, 1984). Such methods are based on a two-step approach: (i) tune the controller parameters ignoring the saturation limits of the actuator; (ii) design an auxiliary scheme to reduce the effects of the actuator constraints. It is also possible to design a controller which explicitly considers the nonlinearities during its design to avoid windup, such as model-based predictive controllers (Goodwin et al., 2006). However anti-windup strategies allow the use of the well-known PID controllers and are common in industry.

In Åström and Wittenmark (1984), an incremental algorithm is presented. This technique is widely used in industry for its simplicity of implementation in digital controllers. Furthermore, the other two main advantages of this technique are: (i) no integration of the error when the control signal reaches a saturation limit; (ii) bumpless transfer when the operator switches the controller operation mode from manual to automatic control, or vice-versa (Kothare et al., 1994). Fertik and Ross (1967) have proposed a strategy known as back-calculation, which is still widely used in industry, due to its good performance. This technique adds an extra feedback signal to the input of the integrator, which is composed of the error between the output signal of the controller and the signal that is applied to the plant multiplied by a constant gain, known as tracking time parameter. The tuning of this gain was object of study of several works (Åström and Wittenmark, 1984; Jalil et al., 2014; Kothare et al., 1994). In Hansson
et al. (1994) a conditional integration method is proposed. This method consists in updating the integral term of the controller only if certain conditions are met. Another AW strategy, originally proposed in Bruciapaglia and Apolônio (1986) and recently used in a Smith predictor approach (Flesch et al., 2017), uses a recalculation of the error signal to maintain the consistency between the calculated control signal and the real value applied to the process. Several other strategies to avoid the problems related to windup can be found in (Tarbouriech and Turner, 2009; Zaccarian and Teel, 2011).

Although anti-windup techniques have been used in many studies, there are not much works focusing on a comparative analysis of their performances in the presence of measurement noise, which is a common characteristic of many practical applications. In this work, the effect of measurement noise on the closed-loop performance is analyzed for some AW strategies used together with a PID controller applied in processes with saturation of the actuator and dead time. The AW strategies used are: (i) incremental algorithm; (ii) back-calculation; (iii) error recalculation. For the PID tuning, the technique presented in Normey-Rico and Guzmán (2013), which consists in a low-order approximation of a filtered Smith predictor, is used.

The paper is organized as follows. Section 2 presents the formulation of some AW techniques. Section 3 compares the closed-loop performance for some AW techniques, for a stable process, and also analyzes the effect of the noise on the calculation of the control action. Two case studies are presented in section 4 with the goal to compare the performance of the AW techniques in processes subjected to measurement noise. Finally, the conclusions of the paper are presented in section 5.

2. REVIEW OF SOME ANTI-WINDUP TECHNIQUES

According to Doyle et al. (1987), any controller with slow or unstable modes may present windup problems if there are actuator constraints. The windup phenomenon appears when there is an inconsistency between the controller output and the plant input. When this happens, the feedback loop is broken, so any further increase of the control signal does not lead to faster response of the system. In case of PID controllers, the integral term can become very large, which can cause the control signal to be outside the operating region for a long time, causing large overshoots and high settling times (Hippe, 2006).

In this work, a discrete-time PID controller in parallel form with filtered derivative action is used for the analysis of the effect of measurement noise on the closed-loop behavior for three different AW techniques. Even though the analysis presented here is valid for any form of PID controller, this specific form was chosen to facilitate the understanding. This form can be obtained by the discretization of its equivalent continuous-time counterpart, presented in (1), using the backward difference technique, resulting in (2), where $K_p$ is the proportional gain, $K_i$ is the integral gain, $K_d$ is the derivative gain, $T_s$ is the sampling period, and $\alpha$ is the parameter of the filter of the derivative part.

$$C(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{\alpha s + 1} \quad (1)$$

Using the PID form presented in (2), the control signal $u(k)$ can be calculated as

$$u(k) = u_p(k) + u_i(k) + u_d(k), \quad (3)$$

where $u_p(k)$ is the proportional control term, $u_i(k)$ is the integral control term, and $u_d(k)$ is the derivative control term. These three terms can be calculated as

$$u_p(k) = K_p e(k), \quad (4)$$
$$u_i(k) = u_i(k-1) + K_i T_s e(k), \quad (5)$$
$$u_d(k) = \frac{K_d [e(k) - e(k-1)] + \alpha u_d(k-1)}{\alpha + T_s}. \quad (6)$$

The structure of a system with saturation of the actuator, considered for the analysis of the AW techniques, is presented in Fig. 1 in discrete-time domain, where $r(k)$ is the reference, $e(k)$ is the error, $u(k)$ is the calculated control signal, $u_r(k)$ is the control signal applied to the plant, $y^*(k)$ is the plant output, $\eta(k)$ is the noise signal, $y(k)$ is the measured system output, $C(z)$ is the controller, and $P(z)$ is the plant.

![Fig. 1. Block diagram of a system with saturation of the actuator and anti-windup](image)

2.1 Back-calculation

The back-calculation approach, proposed in Fertik and Ross (1967), aims to prevent the integral term to accumulate a large value, when the controller output $u(k)$ is saturated, i.e. $u(k) \neq u_r(k)$, by using an extra compensation which feeds back, to the integral term, the difference between the controller output and the system input multiplied by a gain $1/T_i$, where $T_i$ is a parameter called tracking time constant. This gain determines how fast the integral term will be reset. The back-calculation procedure is presented in (7), where $u^*_i(k)$ is the new integral control term value and $e_p(k) = u_r(k) - u(k)$.

$$u^*_i(k) = u_i(k-1) + \left[ K_i e(k) + \frac{1}{T_i} e_p(k) \right] T_s \quad (7)$$

Substituting (7) in (3) it is possible to calculate the new control signal $u^*(k)$ as

$$u^*(k) = u_p(k) + u^*_i(k) + u_d(k). \quad (8)$$

2.2 Incremental algorithm

The incremental algorithm technique consists in calculating a control increment, $\Delta u(k)$, at each sampling period and adding to the previous control signal, $u(k-1)$, only the amount that does not saturate the actuator. According to Visioli (2006), this strategy can avoid the windup effect due to the fact that the integral action is outside the control law.
To find the equation that calculates the control increment \( \Delta u(k) \), it is necessary to rewrite (2) using the backshift operator \( z^{-1} \) as
\[
C(z^{-1}) = \frac{b_{c_0} + b_{c_1} z^{-1} + b_{c_2} z^{-2}}{(1 - \beta z^{-1})(1 - \beta z^{-1})},
\]
where
\[
b_{c_0} = K_p (T_s + \alpha) + K_i (T_s^2 + T_s \alpha) + K_d,
\]
\[
b_{c_1} = \frac{K_p (-2\alpha - T_s) - K_i T_s \alpha - 2K_d}{T_s + \alpha},
\]
\[
b_{c_2} = \frac{K_i \alpha + K_d}{T_s + \alpha},
\]
\[
\beta = \frac{\alpha}{T_s + \alpha}.
\]
Then, using the PID form presented in (9), it is possible to calculate the control increment as
\[
\Delta u(k) = \beta \Delta u(k-1) + b_{c_0} e(k) + b_{c_1} e(k-1) + b_{c_2} e(k-2),
\]
and the control signal can be calculated as
\[
u(k) = \Delta u(k) + u(k-1).
\]
Thus, if the condition \( \Delta u(k) = u_r(k) \) is satisfied, the control increment is recalculated as
\[
\Delta u(k) = u_r(k) - u(k-1),
\]
and the current control signal is modified to \( u(k) = u_r(k) \).

### 2.3 Error recalculation

The AW strategy proposed in Bruciapaglia and Apolónio (1986) consists in modifying the current control signal and the current error signal to maintain the consistency between the control signal calculated by the controller and the input signal that is effectively applied to the plant.

The denominator of the controller \( C(z^{-1}) \), presented in (9), can be written in a polynomial form as
\[
C(z^{-1}) = \frac{B_c(z^{-1})}{A_c(z^{-1})} = \frac{b_{c_0} + b_{c_1} z^{-1} + b_{c_2} z^{-2}}{1 + a_{c_1} z^{-1} + a_{c_2} z^{-2}},
\]
where
\[
a_{c_1} = -\beta - 1,
a_{c_2} = \beta.
\]
Using this representation, the control signal can be calculated from the two previous control signals, two previous errors, and the current error as
\[
u(k) = b_{c_0} e(k) + b_{c_1} e(k-1) + b_{c_2} e(k-2) - a_{c_1} u(k-1) - a_{c_2} u(k-2).
\]
If the control signals \( u(k-1) \) and \( u(k-2) \) are within the saturation limits and the error signals \( e(k-1) \) and \( e(k-2) \) are consistent with those that would be expected to occur, (17) is valid, where \( e^*(k-1) \) and \( e^*(k-2) \) represent the previous error signal values expected to verify the equality \( u(k) = u_r(k) \).
\[
u(k) = b_{c_0} e(k) + b_{c_1} e^*(k-1) + b_{c_2} e^*(k-2) - a_{c_1} u_r(k-1) - a_{c_2} u_r(k-2).
\]
Thus, if the current control signal \( u(k) \) is saturated, the value of the current error signal \( e(k) \) must be modified to ensure that \( u(k) = u_r(k) \). This can be expressed as
\[
u_r(k) = \frac{[1 - \alpha_i(z^{-1})]}{[B_c(z^{-1}) - b_{c_0}]} e^*(k),
\]
where \( e^*(k) \) represents the actual error that would be expected for \( u(k) = u_r(k) \).

Subtracting (18) from (17), it is possible to find an expression for \( e^*(k) \) as
\[
e^*(k) = e(k) - \left[ \frac{u(k) - u_r(k)}{b_{c_0}} \right].
\]

Using this strategy, the controller does not windup because the control signal is modified to always be equal to the plant input and the error signal is modified so that all previous error signals are also consistent with the fact that the controller output at an instant of time lies exactly at the saturation limit (Flesch et al., 2017).

### 3. PERFORMANCE WITH NOISY MEASUREMENTS

This section analyzes the closed-loop performance of the three AW strategies detailed in section 2, for a particular case, to motivate the discussion of the performances in the presence of measurement noise. The section also includes an analysis of the influence of measurement noise on the calculation of the control action and the noise rejection capability of the AW techniques.

#### 3.1 A motivating example

The motivating example considers the first-order plus dead-time (FOPDT) model
\[
P(s) = \frac{1}{s + 1} e^{-\tau s},
\]
with time given in seconds. The PID tuning method used in this example, and in the simulation case studies, was proposed in Normey-Rico and Guzmán (2013). This method can be used to tune PID controllers for stable, integrating and unstable dead-time processes, and it is based on a modification of the filtered Smith predictor dead-time compensator. The resulting PID parameters, \( K_p, K_i, K_d \) and \( \alpha \), are obtained as a function of the plant model parameters and by the tuning of only one parameter, \( T_p \), which represents a trade-off between robustness and performance.

For this example, the PID tuning parameters are \( K_p = 1.2, K_i = 0.923, K_d = 0.284, \alpha = 0.115 \), which are obtained with \( T_p = 0.5 \) s. For the choice of parameter \( T_i \), the rule proposed in Aström and Wittenmark (1984) was used, resulting in \( T_i = 0.677 \) s. The plant and the controller were discretized, respectively, using a zero-order holder (ZOH) and the backward difference technique with a sampling period of \( T_s = 0.1 \) s, resulting in
\[
P(z) = \frac{0.904}{z - 0.904},
\]
\[
C(z) = \frac{2.967 z^2 - 5.088 z + 2.177}{z^2 - 1.395 z + 0.395}.
\]

Fig. 2 shows a simulation of the system and compares the performances of three AW techniques, without considering measurement noise, for a step reference of amplitude 0.9 applied at time \( t = 12 \) s. The minimum and maximum saturation limitations of the control signal are \( u_{\text{min}} = 0 \) and \( u_{\text{max}} = 1 \). As can be seen in Fig. 2, the error recalculation technique has a considerably better performance when compared to incremental algorithm and back-calculation techniques.
When the control signal is near to a saturation limit, noisy measurements can cause saturation, that is
\[ u(k) = b_c r(k) + b_c (y^*(k) - \eta(k)) + b_c e(k - 1) + b_c e(k - 2) - a_c u(k - 1) - a_c u(k - 2) > u_{\text{max}}. \] (24)

If the condition presented in (24) is satisfied, \( e(k) \) is recalculated such that the new control signal \( u^*(k) = u_{\text{max}} \), which is equivalent to
\[ u^*(k) = b_c r(k) + b_c (y^*(k) - \eta^*(k)) + b_c e(k - 1) + b_c e(k - 2) - a_c u(k - 1) - a_c u(k - 2), \] (25)
with \( \eta^*(k) < \eta(k) \). Thus, the equivalent noise is reduced and this implies in a control action less influenced by the noise, resulting in a process output very close to the reference signal. This is not the case for the other two AW strategies, for which only the current control signal is modified and the current error signal is not recalculated. The consequence of this is that, when there is saturation of the control signal, in the next sampling periods, the PID controller will calculate the control signal using previous error signals inconsistent with the previous control signals that were applied to the plant. In the case of processes with noisy measurements, this may increase the variability of the control action, resulting in an average that is not enough to take the process output next to its optimal operating point. This effect may also appear in other control strategies, such as model predictive control (Camacho and Bordons, 2013).

4. SIMULATION CASE STUDIES

In this section two simulation case studies are presented: one for an integrating process and another for an unstable process. Noisy measurements are considered for the analysis of the performances of the techniques. The same PID tuning method presented in section 3 is used in this section for a comparative analysis. Parameter \( T_1 \) of the back-calculation AW technique was tuned using the method proposed in (Åström and Wittenmark, 1984), but other tuning methods could be used, resulting in different noise rejection performances. For all the case studies, the PID tuning parameter \( T_a \) was chosen to obtain a controller with fast response. Although the three AW techniques presented in section 3 are able to stabilize the two processes for the cases without noise, the incremental algorithm technique was not able to stabilize both processes considering noise, therefore its performance results are not presented. It is also important to point out that the performance of the conditional integration AW technique was also analyzed, however its results were conceptually similar to the ones of back-calculation. Therefore they are not presented in this work due to lack of space.

4.1 Integrating case

The following transfer function, presented in Skogestad (2003), will be considered for the integrating case study
\[ P(s) = \frac{e^{-s}}{s}. \] (26)

with time given in seconds. For this case, the PID tuning parameters are \( K_p = 0.959, K_i = 0.285, K_d = 0.291, \alpha = 0.142 \), which are obtained with \( T_a = 1 \text{ s}, \) and the tracking time constant for the back-calculation AW technique is
$T_i = 1.224 \text{ s}$. The plant and the controller were discretized, respectively, using a ZOH and the backward difference technique with a sampling period of $T_s = 0.1 \text{ s}$, resulting in

$$P(z) = \frac{0.1}{z - 1} z^{-10}, \quad (27)$$

$$C(z) = \frac{2.485 z^2 - 4.437 z + 1.967}{z^2 - 1.481 z + 0.481}. \quad (28)$$

The minimum and maximum saturation limits of the control signal are $u_{\text{min}} = -0.06$ and $u_{\text{max}} = +0.1$. In this case, measurement noise with normal distribution, SNR of 10 dB and variance of 0.03 is considered. Fig. 4 shows the simulation of the system comparing the back-calculation and the error recalculation AW techniques for a step reference of amplitude 0.3 applied at time $t = 4 \text{ s}$.

![Fig. 4. Comparative analysis between the performance of the AW techniques for the integrating case.](image)

The results show that the back-calculation technique presents a considerable error in steady state due to the fact that the average of the control action applied is not enough to take the system output to the reference. On the other hand, the error recalculation technique presents good reference tracking performance, being able to take the system output to a value closer to the reference when compared to the back-calculation technique. Table 1 shows three performance indices for reference tracking of the integrating case: steady-state average error (SSAE), integral absolute error (IAE), and total variation of the control signal (TV). As can be seen in Table 1, the error recalculation technique presents a slightly lower TV value, which represents a lower variability in the control action, when compared to the back-calculation technique. Due to this lower variability, the average of the output of the system was able to operate in a region very close to the reference signal, which resulted in lower SSAE and IAE indices.

<table>
<thead>
<tr>
<th>Technique</th>
<th>SSAE (%)</th>
<th>IAE</th>
<th>TV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Back-calculation</td>
<td>13.6</td>
<td>21.2</td>
<td>30.7</td>
</tr>
<tr>
<td>Error recalculation</td>
<td>3.3</td>
<td>14.9</td>
<td>28.5</td>
</tr>
</tbody>
</table>

4.2 Unstable case

For the unstable case, the example presented in Sree and Chidambaram (2003) will be considered, in which a chemical reactor is described by the Chollete model. The nonlinear model of the process is given by

$$\frac{dW}{dt} = \frac{F(t)}{V} [W_i(t) - W(t)] - \frac{k_i C(t)}{(k_2 W(t) + 1)^2}, \quad (29)$$

where $F(t)$ is the input flow rate in l/s, $W_i(t)$ is the input concentration in mol/l (manipulated variable) and $W(t)$ is the output concentration in mol/l (process variable). The values for the operating point of the process that will be considered are $F = 0.033331 \text{ l/s}$, $V = 11$, $k_1 = 101 \text{ l/s}$, $k_2 = 101 \text{ mol}$, $W_i = 3.2881 \text{ mol/l}$, and $W = 1.316 \text{ mol/l}$. The nonlinear model is used for the simulation. A linearized model obtained for the operating point and represented in (30) is used for the PID tuning.

$$\frac{W(s)}{W_i(s)} = \frac{P(s)}{T(s)} = \frac{3.433 e^{-20s}}{103.1s + 1}. \quad (30)$$

The time constant $T = 103.1 \text{ s}$ and the dead time $D = 20 \text{ s}$ are in seconds. For this case, the PID tuning parameters are $K_p = 2.359$, $K_i = 0.045$, $K_d = 17.143$ and $\alpha = 0.979$, which are obtained with $T_s = 8 \text{ s}$, and the tracking time constant, for the back-calculation technique, is $T_i = 20.57 \text{ s}$. The plant and the controller were discretized, respectively, using a ZOH and the backward difference technique with a sampling period of $T_s = 5 \text{ s}$, resulting in

$$P(z) = \frac{0.17}{z - 1.05} z^{-4}, \quad (31)$$

$$C(z) = \frac{7.402 z^2 - 11.021 z + 3.946}{z^3 - 0.562 z - 0.437}. \quad (32)$$

In this case, a symmetrical saturation limit of the control signal, with magnitude of 0.06 mol/l, is considered.

Fig. 5 shows the simulation comparing the performances of the back-calculation and the error recalculation techniques, for a step reference of 0.3 mol/l at $t = 20 \text{ s}$, and considering measurement noise with normal distribution, SNR of 10 dB and variance of 0.03 mol/l.

![Fig. 5. Comparative analysis between the performance of the AW techniques for the unstable case.](image)

Again, the results show a superior reference tracking performance of the error recalculation technique when compared to the back-calculation technique, which presented...
Table 2. Performance comparison for the unstable case study

<table>
<thead>
<tr>
<th>Technique</th>
<th>SSAE (%)</th>
<th>IAE</th>
<th>TV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Back-calculation</td>
<td>20.6</td>
<td>20.3</td>
<td>21.0</td>
</tr>
<tr>
<td>Error recalculation</td>
<td>4.3</td>
<td>8.6</td>
<td>16.6</td>
</tr>
</tbody>
</table>

larger error variance. Table 2 shows three performance indices for reference tracking of the unstable case.

The performance indices presented in Table 2 show the advantage of the error recalculation technique, which presented considerably lower values, when compared to the back-calculation, due to its capability of better noise rejection. This capability makes it possible to reduce the variability of the control action and improve the reference tracking performance.

5. CONCLUSIONS

This paper has focused on presenting an analysis of the performance of some AW techniques applied to processes subjected to measurement noise, saturation of the actuator and dead time. First, a stable case example was presented to motivate the discussion of the behavior of the AW techniques and their noise rejection capabilities in processes subjected to measurement noise. The back-calculation and the error recalculation closed-loop performances were compared in two case studies, presented in section 4. The closed-loop performance of the incremental algorithm was not presented for the integrating and unstable cases because this technique was not able to stabilize these two systems when noise was considered.

The presented results show that for the cases in which the model of the process is well known, but the measurement noise is large, the error recalculation technique has a better closed-loop performance when compared to the incremental and back-calculation techniques. The advantage of this technique is due to its ability to reduce the influence of noise on the calculation of the control signal by updating the error signal in order to maintain the consistency between the controller output and the plant input. In this way, the system presents a control action with reduced variability and a better reference tracking performance. Therefore, in cases for which the model is a good approximation of the plant but there is considerable noise, the use of the error recalculation technique is quite interesting, due to its easy implementation, good noise rejection capability and good reference tracking performance.

REFERENCES


