Abstract: The present study proposes a new approach to a data-driven PID control design method based on one-shot closed-loop input and output data. Even if the proposed controller is designed using only one-shot data, both the prescribed robust stability and tracking performance optimization are attained. The proposed control law is designed by solving a constrained optimization problem, in which the robust stability as a constraint condition is designed by a sensitivity function estimated using the discrete-time Fourier transform, and the performance function defined using a fictitious reference is minimized. As a result, the proposed method provides trade-off design between the tracking performance and robust stability, where the robust stability is arbitrarily selected depending on the plant perturbation. The effectiveness of the proposed method is demonstrated through numerical examples.

Keywords: PID control, iterative method, transient response, robust stability, sensitivity function, frequency estimation, multiobjective optimization, sampled-data system

1. INTRODUCTION

Proportional-integral-derivative (PID) control (Åström and Hägglund, 2006; Vilanova and Visioli, 2012; Vilanova et al., 2017) is well known and widely used because of its simple structure and useful properties. Most of the tuning approaches for PID controllers are model-based: a mathematical model (usually of low order) of the controlled plant is needed and a tuning relation relates the process model parameters with the controller parameters. One of the drawbacks of this approach is the need to encapsulate the process data into a low order model (O’Dwyer, 2003). In contrast to this approach, the data-driven approach for tuning the PID control has also been approached from different view. In unfalsified control (Safonov and Tsao, 1997), iterative feedback tuning (Hjalmarsson et al., 1998; Hjalmarsson, 2002) and correlation-based tuning (CbT) (Karimi et al., 2004; Mišković et al., 2005), although the controller parameters are decided directly by the controlled process data, iteration experiments are needed for optimization. In addition to these ones, noniterative tuning methods, such as virtual reference feedback tuning (VRFT) (Campi et al., 2002; Masuda et al., 2017), fictitious reference iterative tuning (FRIT) (Souma et al., 2004; Jeng et al., 2017), and noniterative correlation-based tuning (NCbT) (Karimi et al., 2007), have been proposed. Furthermore, VRFT and FRIT have been extended such that the reference model is also tuned in order to improve the reference model tracking (Kano et al., 2010; Saeki et al., 2013).

Conventional direct tuning methods have focused on reference model tracking, whereas actual systems also require robust stability (Saeki and Sugitani, 2011; Parastvand and Khosrowjerdi, 2014; Koenings et al., 2017) because the stability of the control system is critical. The relationship between the tracking performance and the robust stability experiment a trade-off relationship, so that both the tracking performance and the robust stability cannot be optimized simultaneously. Therefore, a trade-off design based on the required modeling accuracy represents a feasible alternative design approach. To this end, by using the sensitivity function, the stability margin is selected to be appropriate for the model uncertainty (Arrieta and Vilanova, 2011, 2012; Kurokawa et al., 2017). In trade-off design, the higher the tracking performance, the smaller the stability margin, and vice versa. Since conventional methods require a plant model, data-driven trade-off design is the motivation of the present study.

As an open loop data-driven robust tuning, van Heusden et al. (2011) discussed closed-loop stability, and Rojas and Vilanova (2011, 2012) studied robust stability using the empirical transfer-function estimate (Keesman, 2011). On the other hand, the present study proposes a closed-loop data-driven robust tuning method subject to prescribed robust stability. To this end, the maximum sensitivity is obtained based on frequency characteristic estimation.
Fig. 1. Sampled-data control system (Matsui et al., 2010). Furthermore, the tracking performance is optimized using extended FRIT (E-FRIT) (Kano et al., 2010). As a result, trade-off design between tracking performance and robust stability is achieved. Generally, the higher the robust stability, the worse the tracking performance. Therefore, in the proposed method, the robust stability is selected depending on the plant perturbation, and the tracking performance is then optimized subject to the prescribed robust stability. Consequently, the tracking performance is better when the plant perturbation is small enough.

The remainder of the present paper is organized as follows. Section 2 presents the control system to be designed and the design objective of the present study. In Section 3, a data-driven constrained optimization problem is designed, and the optimal PID parameters are decided. In Section 4, the effectiveness of the proposed method is evaluated through numerical examples. Finally, concluding remarks are presented in Section 5.

2. DESIGN OBJECTIVE

2.1 PID Control Law

Consider the control system illustrated in Fig. 1, where \( P(s) \) is a controlled continuous-time plant, and \( H \) and \( S \) denote the zero-th order holder and sampler, respectively. A discrete-time control input \( u_d(k) \) is decided by a discrete-time controller and is converted by the holder to a continuous-time signal \( u(t) \), which is to be sent to the plant. A continuous-time plant output \( y(t) \) is converted to a discrete-time signal \( y_d(k) \) by the sampler, which is used in a discrete-time controller to decide the discrete-time control input. The discrete-time control input is calculated by a PID control law given as follows:

\[
\begin{align*}
    u_d(k) &= C_e(z^{-1})e_d(k) - C_y(z^{-1})y_d(k) \\
    e_d(k) &= r_d(k) - y_d(k) \\
    C_e(z^{-1}) &= K_p + K_i \frac{T_s}{\Delta} \\
    C_y(z^{-1}) &= K_d \frac{\Delta}{T_d} \\
    \Delta &= 1 - z^{-1}
\end{align*}
\]

where \( r_d(k) \) is a reference input, \( T_s \) denotes the sampling interval, and \( z \) denotes the forward shift operator. Moreover, \( K_p \), \( K_i \), and \( K_d \) are the proportional gain, the integral gain, and the derivative gain, respectively, and are referred to collectively as PID gains. The PID gains are tuned according to the design objective described in 2.2.

Fig. 2. Error system

2.2 Constrained Optimization Problem

The design objective of the present study is to have the closed-loop plant output follow the reference model output subject to an assigned stability margin.

A block-diagram of the error system is illustrated in Fig. 2. In this figure, \( M_d(z^{-1}) \) is the reference model which is the discrete-time model of a continuous-time reference model \( M(s) \), and \( G_d(z^{-1}) \) is a closed-loop system from \( r_d(k) \) to \( y_d(k) \) in Fig. 1. Hence, \( \varepsilon_d(k) \) is defined as follows:

\[
\varepsilon_d(k) = (M_d(z^{-1}) - G_d(z^{-1}))r_d(k)
\]

In the present study, the reference model \( M_d(z^{-1}) \) is defined by the following second-order plus dead-time transfer function:

\[
M(s) = \frac{\omega_0^2}{s^2 + 2\omega_0s + \omega_0^2}e^{-L_0s}
\]

where \( \omega_0 \) and \( L_0 \) are the natural angular frequency and dead-time, respectively.

Using Eq. (2), a performance function is defined as follows:

\[
J = \frac{1}{N} \sum_{k=1}^{N} \varepsilon_d(k)^2
\]

where \( N \) denotes the control horizon.

The design objective is not only to minimize the performance function but also to guarantee robust stability. To this end, the following sensitivity function is introduced:

\[
S_f(e^{-j\omega}) = \frac{1}{1 + C_d(e^{-j\omega})P_d(e^{-j\omega})}
\]

\[
C_d(e^{-j\omega}) = C_e(e^{-j\omega}) + C_y(e^{-j\omega})
\]

where \( P_d(z^{-1}) \) denotes a plant model in discrete time. Using the sensitivity function, the maximum sensitivity is also obtained as follows:

\[
M_s = \max_{\omega} |S_f(e^{-j\omega})|
\]

In the present study, in order to obtain the assigned stability margin, the following constraint condition is defined:

\[
|M_s - M^d_s| = 0
\]

\( M^d_s \) is the desired maximum sensitivity and is assigned by the designer. The recommended range of \( M^d_s \) is known to be between 1.4 and 2.0 (Åström and Hägglund, 2006). Here, \( M_s \) is defined such that as the stability margin increases, \( M_s \) decreases. In contrast, as the tracking performance decreases, \( M_s \) increases.

As a result, the constrained optimization problem is defined as follows:

\[
\min_{K_p, K_i, K_d} J
\]

subject to \( |M_s - M^d_s| = 0 \)
In the present study, this problem is solved without conducting an iterative experiment and without previous knowledge of a plant model. However, the plant model is included in Eq. (5), and Eq. (4) must be minimized non-iteratively. In the next section, constraint condition Eq. (7) is first estimated using one-shot data in 3.1, and the performance function Eq. (4), which can be minimized based on the one-shot data in 3.2, is then rewritten.

3. PROPOSED DESIGN USING ONE-SHOT DATA

3.1 Estimation of the maximum sensitivity

In order to obtain \( M_s \) without the plant model, the gain and phase characteristic curves are estimated using the available data as in Matsui et al. (2010). In this estimation method, a bandpass filter is introduced so that a Discrete Fourier Transform (DFT) can be used to estimate the frequency characteristics. As a result, the data is absolutely integrable, and thus the discontinuity between the start and end data is resolved. The bandpass filter is used as follows:

\[
B(s) = \frac{T_1 s}{(T_1 s + 1)(T_2 s + 1)}
\]

where \( \omega_l \) and \( \omega_h \) are design parameters, and the estimated frequency range is decided by selecting them. The bandpass filter is converted to a discrete-time filter \( B_d(z^{-1}) \), and initial closed-loop data \( u_d^0(k) \) and \( y_d^0(k) \) are transformed as follows:

\[
U_f(\omega) = \mathcal{F}[B_d(z^{-1})u_d^0(k)]
\]

\[
Y_f(\omega) = \mathcal{F}[B_d(z^{-1})y_d^0(k)]
\]

where \( \mathcal{F}[\cdot] \) denotes the DFT. Using Eq. (11) and Eq. (12), the frequency characteristic is estimated as follows:

\[
\hat{P}_d(e^{-j\omega}) = \frac{Y_f(\omega)}{U_f(\omega)}
\]

The estimated frequency characteristics are used in Eq. (5), and hence the sensitivity function can be estimated as follows, although the high-frequency area is removed by the bandpass filter:

\[
\hat{S}_f(e^{-j\omega}) = \frac{1}{1 + C_d(e^{-j\omega})\hat{P}_d(e^{-j\omega})}
\]

Hence, in the present study, the maximum sensitivity is also estimated as follows:

\[
M_s = \max_\omega |\hat{S}_f(e^{-j\omega})|
\]

3.2 Tracking Performance Optimization Using E-FRIT

A closed-loop data-based noniterative tuning is designed using E-FRIT (Kano et al., 2010). This method evaluates the error between the initial output \( y_d^0(k) \) and its fictitious reference output \( \tilde{y}_d(k) \), and hence the performance function is defined as follows:

\[
J = \frac{1}{N} \sum_{k=1}^{N} (\hat{e}_d(k)^2 + \lambda \Delta \tilde{u}_d(k)^2)
\]

\[
\hat{e}_d(k) = y_d^0(k) - \tilde{y}_d(k)
\]

\[
\tilde{y}_d(k) = M_d(z^{-1})\hat{r}_d(k)
\]

\[
\hat{r}_d(k) = C_e(z^{-1})^{-1}(u_d^0(k) + C_d(z^{-1})y_d^0(k))
\]

\[
\tilde{u}_d(k) = C_e(z^{-1})\hat{r}_d(k) - C_d(z^{-1})M_d(z^{-1})r_d^0(k)
\]

where \( \lambda \) is a weighting factor for the input deviation, and \( r_d^0(k) \) denotes the initial reference input. The minimization of Eq. (15) is performed over that of Eq. (4) when \( \lambda = 0 \). Hence, the controller parameters that minimize Eq. (4) are obtained by minimizing Eq. (15) using the one-shot closed-loop data. In the design of E-FRIT, \( \omega_0 \) and \( L_0 \) are also estimated for minimizing Eq. (15). In other words, \( \omega_0 \) and \( L_0 \) are selected so that the best reference model is selected. Consequently, the optimization problem to be solved is redefined as follows:

\[
\min_{K_p, K_i, K_d, \omega_0, L_0} J
\]

subject to \( \|M_s - M_s^d\| = 0 \)

Solving this problem, the PID and reference model parameters are decided such that the tracking performance is optimized subject to the prescribed robust stability. In the original E-FRIT, the control system stability is achieved by weighting the input deviation, although the stability condition is not used. On the other hand, in the proposed method, since the PID parameters are decided such that the prescribed stability margin is achieved, the weight for the input deviation is not used, namely, \( \lambda = 0 \). Therefore, the coordination of the weighting factor is not needed.

4. NUMERICAL EXAMPLE

Consider the following transfer function:

\[
P(s) = \frac{1}{s + 1} e^{-0.6s}
\]

As this model is assumed to be unknown, a control system is designed. The sampling interval \( T_s \) is set to 0.01 s, and the reference input is defined as a unit step function. The initial controller parameters are set as \( K_p = 0.05 \), \( K_i = 0.1 \), and \( K_d = 0.01 \) by trial and error such that the closed-loop system is stabilized. The obtained initial control input and output responses are plotted by dashed lines in the upper and lower panels, respectively, of Fig. 3. The parameters are decided based on this data.

In order to realize robust stability as the constraint condition, the frequency characteristic is identified using the estimation method given in Section 3.1. The filtered input and output data are plotted by solid lines in the upper and lower panels, respectively, of Fig. 3. In the present study, in order to estimate the frequency characteristic in the range from \( 10^{-1} \) rad/s to \( 10^2 \) rad/s, \( \omega_l = 100 \) and \( \omega_h = 10 \). The identified gain and phase characteristics are then shown in Fig. 4. From Fig. 4, the frequency characteristics are identified approximately correctly without the high-frequency area. Using the identified frequency characteristics, the constraint condition is implemented using the selected \( M_s^d \).

The optimization problem stated in Eq. (16) is solved using the initial data and estimated frequency characteristics. In
In the present study, the problem is solved using “fmincon” with option “interior point” \(^1\). In the present study, the control system is designed with \(M_d^2\) varied as 1.4, 1.6, 1.8, and 2.0, respectively. For comparison with the proposed method, E-FRIT is also designed. Since E-FRIT can guarantee closed-loop stability by weighing the control input deviation, the control system is designed using \(\lambda\) varied as \(10^{-2}\) and \(10^{-3}\), respectively, where the robust stability constraint is not considered in E-FRIT. The parameters obtained using the proposed method are shown in Table 1. Note that as \(M_d^2\) increases, the natural angular frequency of the reference model increases, and hence the trade-off design is achieved because the stability margin is inversely proportional to \(M_s\). The parameters obtained using E-FRIT are shown in Table 2, in which \(\lambda\) decreases as the natural angular frequency increases.

The tracking performance and robust stability are illustrated in Section 4.1 and Section 4.2, respectively.

### 4.1 Tracking Performance

The closed-loop control results obtained using the obtained parameters are shown in Fig. 5. In the figure, the red, green, blue, and black solid lines indicate the output responses obtained using the proposed method with \(M_d^2\) varied as 1.4, 1.6, 1.8, and 2.0, respectively, and the cyan and orange solid lines indicate the output responses obtained using E-FRIT with \(\lambda = 10^{-2}\) and \(10^{-3}\), respectively. The magenta dashed-dotted line indicate the initial output response using the initial PID parameters. In the proposed method, the tracking performance depends on \(M_d^2\) because \(M_d^2\) increases as the reference model response becomes faster. On the other hand, the output response obtained using E-FRIT with a small value of \(\lambda\) is faster than that obtained using E-FRIT with a large value of \(\lambda\), where the output result obtained using \(\lambda = 0\) is not shown because it is unstable. Note that the tracking performance of E-FRIT depends on the selection of \(\lambda\).

The tracking performance for \(M_d^2\) is evaluated quantitatively. The control performance of the plant output for the reference input is evaluated by Eq. (18).

\[
J_{\text{control}} = \frac{1}{N} \sum_{k=1}^{N} (r_d(k) - y_d(k))^2
\]  

(18)

The evaluated values of the proposed method are shown in Table 3, and trade-off design is achieved by selecting \(M_d^2\) because the tracking performance depends on \(M_d^2\). The evaluated values of the conventional E-FRIT are also shown in Table 3. As shown in this table, the convergence to the reference input is worse than that of the proposed method.

### Table 1. PID and reference model parameters of the proposed method for \(M_d^2\) varied as 1.4, 1.6, 1.8, and 2.0

<table>
<thead>
<tr>
<th>(M_d^2)</th>
<th>(\omega_0)</th>
<th>(L_0)</th>
<th>(K_p)</th>
<th>(K_s)</th>
<th>(K_d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>2.3806</td>
<td>0.4872</td>
<td>0.8899</td>
<td>0.7508</td>
<td>0.1253</td>
</tr>
<tr>
<td>1.6</td>
<td>3.7213</td>
<td>0.5499</td>
<td>1.2406</td>
<td>0.9157</td>
<td>0.2396</td>
</tr>
<tr>
<td>1.8</td>
<td>4.9016</td>
<td>0.5830</td>
<td>1.4559</td>
<td>1.0041</td>
<td>0.3112</td>
</tr>
<tr>
<td>2.0</td>
<td>5.7176</td>
<td>0.5671</td>
<td>1.6457</td>
<td>1.0830</td>
<td>0.5437</td>
</tr>
</tbody>
</table>

### Table 2. PID and reference model parameters of E-FRIT with control weight

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>(\omega_0)</th>
<th>(L_0)</th>
<th>(K_p)</th>
<th>(K_s)</th>
<th>(K_d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^{-2})</td>
<td>0.4277</td>
<td>0.5819</td>
<td>0.0013</td>
<td>0.1900</td>
<td>9.7584\times10^{-4}</td>
</tr>
<tr>
<td>(10^{-3})</td>
<td>1.6619</td>
<td>0.4484</td>
<td>0.5951</td>
<td>0.6036</td>
<td>2.0164\times10^{-4}</td>
</tr>
</tbody>
</table>

\(^1\) Mathworks, Inc.
Table 3. Tracking performance evaluation in Fig. 5 and obtained $M_s$

<table>
<thead>
<tr>
<th>Design parameter</th>
<th>$J_{control}$</th>
<th>$M_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed method</td>
<td>$M_s^d=1.4$</td>
<td>0.00102</td>
</tr>
<tr>
<td></td>
<td>$M_s^d=1.6$</td>
<td>0.00089</td>
</tr>
<tr>
<td></td>
<td>$M_s^d=1.8$</td>
<td>0.00083</td>
</tr>
<tr>
<td></td>
<td>$M_s^d=2.0$</td>
<td>0.00082</td>
</tr>
<tr>
<td>Conventional method</td>
<td>$\lambda = 10^{-2}$</td>
<td>0.00065</td>
</tr>
<tr>
<td></td>
<td>$\lambda = 10^{-3}$</td>
<td>0.00121</td>
</tr>
</tbody>
</table>

Fig. 6. Output responses with plant perturbation

4.2 Robust Stability

The values of $M_s$ obtained using the proposed method for $M_s^d$ are shown in Table 3. The prescribed robust stability is attained. Furthermore, as $M_s^d$ decreases, the robust stability increases because the robust stability has a trade-off relationship with respect to the tracking performance. In order to clarify this trade-off relationship, the robust stability corresponding to $M_s^d$ is confirmed. The simulation results for selected $M_s^d$ are shown in Fig. 6. In this figure, the plant is not perturbed and is Eq. (17) from the start until 100 s. Then, after 100 s, the plant is perturbed as follows:

$$P'(s) = \frac{1.5}{0.73s + 1} e^{-0.7s}$$

The output response designed using $M_s^d = 2.0$ diverges after 100 s. An enlarged view of Fig. 6 is shown in Fig. 7, where, for clarity, the output response using $M_s^d = 2.0$ and the initial response are not shown. This figure shows that the output responses using a small $M_s^d$ are stabilized. Therefore, the trade-off relationship between the tracking performance and the robust stability is confirmed.

The values of $M_s$ obtained using the conventional E-FRIT are also shown in Table 3. Since $M_s$ obtained using the conventional E-FRIT is smaller than that obtained using the proposed method, the system designed using the conventional E-FRIT is sufficiently robust. Thus, the output responses obtained using the conventional E-FRIT are also stabilized with non-zero $\lambda$. Although trade-off design using the conventional E-FRIT is possible by selecting $\lambda$, the robust stability is not explicitly assigned.

Fig. 7. Enlarged view of Fig. 6

5. CONCLUSION

In the present study, we proposed a new data-driven approach. In the proposed method, a constrained optimization problem is solved, in which the performance function was designed by E-FRIT, and robust stability was achieved using the sensitivity function as a constraint condition. As a result, using one-shot closed-loop data, controller parameters were decided such that the tracking performance was optimized subject to the prescribed robust stability. Since the robust stability was selected, a trade-off design between tracking performance and robust stability was achieved. Hence, the tracking performance was improved by selecting a small stability margin when the plant perturbation is small enough. Finally, the effectiveness of the proposed design method was demonstrated through numerical examples.

The proposed method focuses on servo tracking, although regulation control for a disturbance has been also investigated (Ishii et al., 2015). Therefore, in the future, we intend to investigate regulation control for disturbance attenuation. Furthermore, in the present study, a non-convex problem was solved in order to obtain the trade-off design, and so the obtained control system was influenced by the initial conditions. As such, in the future, this influence must be reduced.

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