PI and Adaptive Model Matching Control System
that Satisfies the Setting Settling Time
application to engine speed control

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Abstract: A new design method that satisfies the setting settling time with small maximum overshoot in a servo controller was developed using a PI controller and an internal feedback system. The internal feedback system consists of the model parameter of the controlled object that was approximated as a second-order lag-time system. Therefore, an adaptive ability that counteracts the changing model parameters was required. In this paper, the variable parameters of the controlled object were obtained by on-line execution of a non-linear least-squares method. Suitable adaptation by the developed method was confirmed in simulation and experimental tests.

Keywords: Engine Control, Adaptive Control, Setting Time, PI Control, Model Matching, Non-linear Least-Squares Sequential Quadratic Programming

1. INTRODUCTION

In order to reduce the CO2 emission of an automobile, an optimal control is necessary to operate the power train with high efficiency. Then, since the parameters of a dynamic characteristic of the power train vary according to the driving condition, non-linear compensation is required for the control system [Takiyama (2014)]. Besides, since the number of power train component is increased for high efficient operation, it is desirable that the control system can be easily and conveniently constructed for an efficient development.

PID controllers are widely used for closed-loop systems from the viewpoints of simplicity and control performance. Though various methods are considered to determine the control parameters [Ålström and Hägglund (1995)], those are obtained by evaluating the settling time or overshoot of the time response in numerous trial-and-error cases. A coefficient diagram method that consider both the dynamics of the target system and the time response were proposed [Manabe (1998)], however, it is complicated. And, the dead-beat controller is a well-known controller that satisfies the setting settling time, but lacks robustness. Recently, a method for optimizing the control parameters using a non-linear programming method has been proposed based on the measured input/output data from the controlled object [Kaneko (2013)]. Therefore, the non-linear programming method is expected to obtain a suitable parameter of the dynamic characteristics of the controlled object.

In a normal second-order lag-time system, the time response is known to depend on the damping coefficient or frequency parameter. Based on this background, we investigate a new design method of a servo system that satisfies the setting settling time with small overshoot amount without trial-and-error. The controller consists of a proportional integral (PI) controller and an internal controller for a second-order lag-time system using the model parameters of the control objects [Ohta, et al. (2014)]. Since the control system can be simply constructed by using the desired settling time, it is very useful to improve the efficient development of control system. Furthermore, changes in the parameters of the controlled object are handled by an adaptation method implemented in the control system by an online parameter search using a non-linear programming method.

This paper investigated about PI and adaptive model matching control system that satisfies the setting settling time. Then, it was applied to the speed controller of a gasoline engine and the experimental examination were carried out.

2. SETTING SETTLING TIME SERVO SYSTEM

2.1 PI Controller and Setting Time in the Second-order Lag-time System

Figure 1 shows the unity feedback system. In (1) and (2) respectively, $P(s)$ denotes a virtual control object in the second-order lag-time system and $C_{PI}(s)$ denotes the PI controller. Equation (3) defines the $M\%$ overshoot. The smallest $T_s$, that satisfies (3), is called the $M\%$ settling time, denoted as $T_s(M)$.

\[
P(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_ns + \omega_n^2}
\]

\[
C_{PI}(s) = K_P + K_I \frac{1}{s}
\]

\[
\frac{|y(t) - y(\infty)|}{|y(\infty)|} \times 100 \leq M, \quad t \geq T_s
\]

Fig. 1. Unity feedback system
The step-response characteristics of this closed loop system were investigated in numerical experiments. In these experiments, we select \( \omega_n=10, M \in \{1, 2, 5, 10\}, \zeta \in \{0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1\} \) and \( K_p \in \{0.01, 0.02, \ldots, 10.00\} \). Then, we investigated the \( K_t \) value that stabilized the closed loop and minimized \( T_s(M) \) for different combinations of \( (M, \zeta, K_p) \) tuples. Then, in order to diminish the maximum overshoot less than \( M\% \), \( \zeta \) was investigated that achieve a small \( T_s \) at a relatively low gain crossover frequency \( \omega_c \). Ultimately, we selected \( \zeta=0.7 \) for \( M=2 \). The experimental results as \( M \) was varied from 1 to 10 are presented in Table 1 [Ohta et al. (2014)].

<table>
<thead>
<tr>
<th>( M )</th>
<th>( \zeta )</th>
<th>( K_p )</th>
<th>( K_t )</th>
<th>( T_s(M) )</th>
<th>( \phi_n )</th>
<th>( \omega_c )</th>
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<td>3.44</td>
<td>0.597</td>
<td>67.6</td>
<td>3.44</td>
</tr>
<tr>
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<td>0.7</td>
<td>0.28</td>
<td>4.39</td>
<td>0.442</td>
<td>67.7</td>
<td>4.49</td>
</tr>
<tr>
<td>5</td>
<td>0.6</td>
<td>0.27</td>
<td>4.77</td>
<td>0.356</td>
<td>66.0</td>
<td>5.18</td>
</tr>
<tr>
<td>10</td>
<td>0.6</td>
<td>0.71</td>
<td>6.88</td>
<td>0.223</td>
<td>55.5</td>
<td>8.64</td>
</tr>
</tbody>
</table>

2.2 Control System that Satisfies the Setting Time Condition

For example, when \( \omega_n=10 \) and a 2\% settling time \( \bar{T}_s \) is required, the controller parameters can be read from Table 1: \( \zeta=0.7, K_p=0.28, K_t=4.39 \) and \( T_s(2)=0.442 \) for \( M=2 \). According to the similarity theorem of the Laplace transform, the M\% settling time \( T_s(M) \) and the setting time \( T_s \) are related through \( \alpha \) (see (4)) [Ohta et al. (2014)]. Applying the PI controller of (6) to the system governed by (5), the 2\% settling time of the step response of the closed-loop transfer function (\( G_r(s) \) in (7)) satisfies the \( \bar{T}_s \). Hereafter, we refer to \( G_r(s) \) as the reference model (where the subscript \( r \) denotes reference).

\[
\alpha = \frac{T_s(M)}{T_s} = \frac{T_s(2)}{T_s} = 0.442 = \omega_n = 10 \tag{4}
\]

\[
P_r(s) = \frac{(s \zeta \omega_n^2)}{s^2 + 2s \zeta \omega_n + (\omega_n^2)} \tag{5}
\]

\[
C_p(s) = K_p + \frac{\alpha K_t}{s} = 0.28 + \frac{4.39}{s} \tag{6}
\]

\[
G_r(s) = \frac{C_p(s)P_r(s)}{1 + C_p(s)P_r(s)} \tag{7}
\]

2.3 Servo System using Internal Feedback

Equation (8) describes a general second-order lag-time system \( P_f(s) \). A servo system that satisfies the setting \( M\% \) settling time \( \bar{T}_s(M) \) was constructed using an internal feedback controller. The internal feedback controller was constructed following the pole-assignment method with a dynamic controller using a minimum-order observer [Ichikawa (1985)]. Figure 2 shows the structure of the system. The transfer function \( \tilde{P}_f(s) \) from \( u_b \) to \( y \), given by (9), depends on the gain \( K_c \) and the internal feedbacks \( C_{bu}(s) \) and \( C_{by}(s) \).

\[
P_f(s) = \frac{b_0}{s^2 + a_1 s + a_0} \tag{8}
\]

\[
\tilde{P}_f(s) = \frac{\bar{U}_d(s)}{\bar{U}_c(s)} = \frac{K_c P_f(s)}{1 + C_{bu}(s) + P_f(s) C_{by}(s)} \tag{9}
\]

\[
K_c = \frac{(a_2 \omega_n^2)}{b_0}, C_{bu}(s) = \frac{q \gamma}{s \gamma + q}, C_{by}(s) = \frac{r_1 + s \gamma}{b_0 (s \gamma + q)} \tag{10}
\]

\[
r_1 = (a_2 \omega_n)^2 + 2q \gamma (a_2 \omega_n) - a_1 q - a_0 \tag{11}
\]

\[
r_0 = \gamma (a_2 \omega_n)^2 - a_0 q \tag{12}
\]

\[
\tilde{P}_f(s) = \frac{K_c P_f(s)}{1 + C_{bu}(s) + P_f(s) C_{by}(s)} = \frac{(s + \gamma) K_c b_0}{(s + \gamma) D(s)} \tag{13}
\]

\[
G_f(s) = \frac{C_p(s) \tilde{P}_f(s)}{1 + C_p(s) \tilde{P}_f(s)} = \frac{(s + \gamma) C_p(s) P_f(s)}{(s + \gamma) [1 + C_p(s) P_f(s)]} = G_r(s) \tag{14}
\]

Using the parameters described in (10)-(13), \( \tilde{P}_f(s) \) equals \( P_f(s) \) and the closed-loop transfer function \( G_r(s) \) equals \( G_f(s) \). These relationships are described by (14) and (15), respectively. Therefore, the general second-order lag-time system with the internal feedback match the reference model and the system can satisfy the setting settling time. The control system that satisfies the \( M\% \) settling time \( \bar{T}_s \) is called the setting settling time controller (SSTC) hereafter. Note that the denominator and numerator of (14) and (15) are offset by \( (s + \gamma) \), where \( \gamma \) denotes the pole of the minimum-order observer. To preserve the responsiveness of the system, the value of \( \gamma \) should be sufficiently larger than the other poles and zeros of \( G_f(s) \) (for example, five times or higher).

3. Non-linear Programming

The model parameters of the controlled object vary under several influences. This variation must be corrected by some kind of countermeasure for the SSTC. In this paper, a changing parameter is detected from variations in the dynamic characteristics of the transient response. The calculation is performed by non-linear least squares sequential quadratic programming (NLSSQP). The NLSSQP method combines the quasi-Newton method for the unconstrained non-linear least-squares problem with the SQP method for general non-linear minimization problems [Takahashi (1987)]. The objective is to determine the \( x \) that minimizes the objective function (18) under the constraints described by (16) and (17). In (18), \( r(x) \) stands for the residual vector.

\[
g(x) \leq 0, g(x) = [g_1(x), \ldots, g_m(x)]^T \tag{16}
\]

\[
h(x) = 0, h(x) = [h_1(x), \ldots, h_l(x)]^T \tag{17}
\]

\[
f(x) = \frac{1}{2} r(x)^T r(x) = \frac{1}{2} \sum_{j=1}^{q} [r_j(x)]^2 \tag{18}
\]

This problem was solved by a sequential quadratic-programming algorithm. Given \( x_0 \), this partial QP problem seeks the \( d \in \mathbb{R}^n \) that minimizes (21) under the linear constraint conditions described by (19) and (20). Figure 3 is a flow-chart of the NLSSQP method. The direction \( d \) was searched by the Goldfarb-Idnani method, and the minimum step size \( \beta \) was determined by a golden sectioning method.
4. CONTROL SYSTEM

4.1 Experimental Apparatus and Objective System

The experiments were performed on a gasoline engine installed on a test-bench without an equivalent inertial mass. The engine has three cylinders and a displacement of 660cc. It was controlled by a digital signal processing (mbiBS/DSP71011). The measurement and control were executed at every suction action of the top dead center of cylinder #1. Therefore, the sampling-and-control time \( \Delta t \) was set to \( \Delta t = 2 / n_c \), where \( n_c \) represents the engine speed (s\(^{-1}\)).

The target was the engine speed \( n_c \), controlled by commanding the ignition timing \( I_c \). The objective model (see Fig. 4) was based on our previous investigation and the step response shown in Fig. 6. The intake valve opening (IVO) angle was set to 0 or 20 CA (where CA denotes the crank angle). Figure 6 shows a typical identification result.

The upper and lower panels display the time responses of the ignition timing \( I_c \) and the engine speed \( n_c \), respectively. During the step-down of the \( I_c \), the time responses of the objective model (sim.) and experimental results (exp.) slightly deviated because parameters such as the time constant of the intake system differ at deceleration and acceleration. Otherwise, the model output well agreed with the engine output, verifying the suitability of this objective model as a controller design.

Table 2 provides the identified model parameters under each operating condition. Although the engine speed varied within 10 s\(^{-1}\), the time constant \( T_1 \) and gain \( K_1 \) varied by 20-30%. These parameters were then targeted for adaptation.

![Fig. 7. Structure of the engine control system](image)

4.2 Construction of SSTC for Speed Controller

The relative order of the transfer function of the controlled objective \( P_{p0}(s) \) (22) is a first-order. To translate the second-order system, a compensator \( C_p(s) \) was connected in series with (22). Equation (23) denotes the second-order lag-time system after compensation. Substituting the parameters \( a_1, a_0, \) and \( b_0 \) (defined by (24)) into (10)-(13), we construct the control system shown in Fig. 7. In this way, we can automatically construct the SSTC based on the correct model parameters and the setting settling time.

\[
P_{p0}(s) = \frac{K_1}{T_2s + 1} \frac{T_2s + 1}{s^2 + \frac{T_1}{T_2}s + \frac{1}{T_2}K_2} = \frac{1}{T_2s + 1} \tag{22}
\]

![Fig. 8. Schematic of the adaptive control system](image)

4.3 Construction of Real-Time Adaptive Control System

Figure 8 schematizes the adaptive control system with SSTC. The \( P_p(s) \) in Fig. 8 represents a virtual model with the same structure as the engine model \( P_{p0}(s) \). The \( C(s) \) represents the overall control system comprising the SSTC’s internal feedback control system \( (K_{ai}, C_{ai}(s), C_{by}(s)) \) described by (10)) and the compensator \( C_p(s) \). These compartments are defined by the dashed line in Fig. 7. The parameter value of \( C(s) \) was set equal to that of \( P_p(s) \).
The feedback control to the controlled object $P_p(s)$ was performed through the output $y_{po}$. To obtain the open-loop response $y_p$, the control signal was commanded to the $P_p(s)$ at the same time. As the virtual model $P_p(s)$ and the controlled object $P_{po}(s)$ are structurally identical and the same input is commanded, the parameter difference is expected to be detectable from the output difference.

4.4 Procedure of On-line Real-time NLSSQP Operation

Figure 9 displays the procedure of the on-line real-time NLSSQP. Note that the model parameters $[T_1, K_1, T_2, K_2]$ in the controlled object $P_{po}(s)$ are expressed in vector form as $x_E$ (where the subscript $E$ denotes engine), and those in the virtual model $P_p(s)$ and the SSSC $C(s)$ are expressed as $x_C$ (where the subscript $C$ denotes controller).

While $1 \leq k \leq 150$, the first procedure (indicated by (s)) stores the input $u_p$ and the output $y_{po}$ of the controlled object. The following procedure (a) calculates the open response of $P_p(s)$ using $y_{po}$, (b) sets the residual vector, and (c) executes NLSSQP to obtain $x_C$. Finally, it (d) updates $P_p(s)$ and $C(s)$ using $x_C$. Routines (a)-(d) are iterated for $151 \leq k \leq 250$. When $n_e$ changes, the procedure returns to (s) and the adaptation repeats.

4.5 Constraint Conditions

Equations (26)-(27) are assigned as constraint conditions to accelerate convergence of the parameter exploration. Equation (26) restricts the range of each parameter. Based on the identification results in Table 2, these ranges were determined as 20% above the maximum and 20% below the minimum value.

$$0.45T_1 \leq 1.1, \ 0.21 \leq K_1 \leq 0.51, \ 4 \leq T_2 \leq 6, \ 0.05 \leq K_2 \leq 0.29 \quad (26)$$
$$g_1 = j_1 T_1 + j_2 K_1 + j_3 T_2 + j_4 K_2 \leq 0 \quad (27)$$

For every change in $T_1$ and $K_1$, $T_2$ and $K_2$ were changed through (27). Investigation showed that the convergence speed and accuracy of matching $x_C$ and $x_E$ depend on the values of the coefficients $j_1$-$j_4$ in (27). Figure 10 plots the concordance rate $x_C/x_E$ for each combination $p_1$-$p_5$ of $j_1$-$j_4$ listed in Table 3. When the concordance rate approximates 1, $x_C$ and $x_E$ are well-matched. Based on these results, we selected the $p_2$ combination with $j_1=-1$, $j_2=-1$, $j_3=0.2$ and $j_4=0.1$.

5. SIMULATION AND EXPERIMENTATION

To clarify the adaptation process, the dynamic behavior of the controller was set slower than that of the controlled object. To this end, the parameter at $\theta_V$ with the largest time constant $T_1$ was applied to $x_C$, while the parameter at $\theta_V$ of $V_1$ with the smallest $T_1$ was applied to $x_E$. The $C(s)$ was designed with a 2% settling time $T_s(2)=3s$, and $\alpha$ was multiplied by 1.01 to provide a margin. As mentioned at 4.1, $\Delta t=2/n_e=0.06(s)$.

Figure 11 illustrates the simulation results. The target engine speed ($\dot{n}_e$) was periodically increased and decreased. Figure 11(A) illustrates the time responses of the input and output signals of the controlled object. The upper panel presents the input $I_2(u_p)$, and the lower panel plots the $\dot{n}_e$ and output $n_e$. The $n_e$ and $\dot{n}_e$ values are consistent, confirming a well-controlled output. Figure 11(A) also shows the input and output behavior of the reference model $G_s(s)$, represented by $u_i$ and $y_i$, respectively.

(A). Time responses of inputs and outputs

(B). Inequality constraints

(C). Integrated square errors

(D). Coincidence ratios of model parameters

Fig. 11. Results of the simulated adaptive control
As \( n_e(y_p) \) and \( y_r \) almost overlap after the second period of the \( \tilde{n}_e \) change, the NLSSQP is deemed effective.

Figure 11(B) illustrates the time response of each parameter of \( x_C \). The same time responses are elicited by the inequality constraints and by \( g_1 \) in (27). The range of the vertical axis covers the range of inequality constraints, confirming that each inequality constraint is satisfied. The dotted line in each panel plots the \( x_C \). At every \( \tilde{n}_e \) change, the parameter \( x_C \) at the controller side gradually changed over steps 151 \( \leq k \leq 250 \) in one period, as mentioned above. In the third period of \( \tilde{n}_e \) change, the \( x_C \) almost coincided with the parameter \( x_C \) of the controlled object. This implies successful adaptive operation by NLSSQP. The integrated square error (ISE) (Fig. 11(C)) behaved similarly to the objective function of NLSSQP. During one period, ISE decreased throughout 151 \( \leq k \leq 250 \) with every iteration of the \( \tilde{n}_e \) change. Almost the same behavior occurred during an \( x_C \) change. Figure 11(D) illustrates the error ratio between the obtained parameter \( x_C \) and the controlled object parameter \( x_C \). The small residual was attributed to the limit of the parameter search when ISE became very small.

5.1 Consideration of Adaptive Behavior

To clarify the adaptive action during one period of the \( \tilde{n}_e \) change, Fig. 12 superposes the adaptive behaviors at the first (\( k=151 \)), fifty-fifth (\( k=200 \)) and hundredth (\( k=250 \)) instances in the first period of the \( \tilde{n}_e \) change. The upper panels of Fig. 12 depict \( y_p \) and \( y_p(x_C(k), l) \). The \( y_p \) is the stored data used in the NLSSQP processing, so remains constant throughout the period. The \( y_p(x_C(k), l) \) was obtained from the open response of \( P_p(s) \) and supplied to the NLSSQP calculation at every \( k \). During the \( \tilde{n}_e \) step-up in the first period of a \( \tilde{n}_e \) change, the virtual model output \( y_p(1) \) and the controlled object output \( y_p \) were mismatched because the value of \( x_C \) at the controller side deviated from that of \( x_C \) at the controlled object side. Furthermore, \( y_p \) did not satisfy the setting \( M\% \) settling time because the SSTC’s parameters were designed using \( x_C \). However, between 50 and 100 steps, the \( y_p \) gradually edged closer to \( y_p \). The bottom panels of Fig. 12(A) plot the the squared error

\[
er^2(l) = \left| y_p(l) - y_p(l) \right|^2
\]

at first, fiftieth and hundredth. During the \( \tilde{n}_e \) step-up, the error \( er^2 \) gradually decreased, demonstrating strong convergence of \( y_p \) to \( y_p \). During a \( \tilde{n}_e \) step-down in the first period of a \( \tilde{n}_e \) change, \( y_p \) appears to reasonably agree with \( y_p \) (Fig. 12(B), top), but \( er^2 \) was not complementary attenuated (Fig. 12(B), bottom). The \( er^2 \) shown in Fig. 12(B) bottom had decreased to \( 1/10 \) of those in Fig. 12(A) bottom. Then, the difference of \( er^2 \) at 50 steps and at 100 steps is small. Therefore, the \( er^2 \) was to be sufficiently decreased and the adaptation was considered to be almost accomplished.

5.2 Consideration of Settling Time

Figure 13 illustrates the amplified \( y_p \) (\( n_e \)) behavior of the transient response in five periods of the \( \tilde{n}_e \) change for \( T_i(2)=3 \) and 4s. The upper and lower panels present the responses during a \( \tilde{n}_e \) step-up and \( \tilde{n}_e \) step-down, respectively. The time response of \( y_r \) of reference model \( G_r(s) \) is clarified by the \( \times \) symbols in Fig. 13. Since \( y_r \) satisfies the setting settling time, the response through the \( \times \) symbols also demonstrates satisfying the setting settling time. The indices s1-s10 indicate when the \( \tilde{n}_e \) changed. Odd and even indices denote a \( \tilde{n}_e \) step-up and a \( \tilde{n}_e \) step-down period, respectively. During the first period, the \( y_p(s1) \) (blue line) did not pass over the \( \times \). However, the \( y_p(s3-s9) \) passed over and almost overlapped the \( \times \) points. This confirms that the time responses of \( y_p \) satisfies the setting \( M\% \) settling time \( \tilde{T}_i(M) \). In the \( \tilde{n}_e \) step-down period (Fig. 13, lower), all behaviors almost passed over the \( \times \) points, demonstrating near coincided of the time responses of \( y_p \) and \( y_r \). Therefore, the setting \( M\% \) settling time \( \tilde{T}_i(M) \) was well satisfied in this case. The adaptation method almost accomplished until the second period of the \( \tilde{n}_e \) change and satisfied the setting settling times \( \tilde{T}_i(2)=3 \) and 4s, although some parameter deviations remained.

These results also demonstrate that the maximum overshoot becomes less than \( M\% \).

5.3 Experimental Results

The physical experiments were carried out under similar conditions to the simulation study. The results are shown in Fig. 14. On average, the time responses of the input \( I_u \) and output \( T_e \) (Fig. 14(A)) reasonably agreed with those of the \( \tilde{n}_e \). The fluctuation of \( n_e \) near \( \tilde{n}_e \) are attributable to combustion variations. The behavior of the reference model (\( u_r \) and \( y_r \)) is also depicted in the
Figure. Ig(exp.) deviated from ur because the engine parameters varied with the environmental conditions. This deterioration is seen during periods 3-4, especially noticeable during the third period. The large variation was caused by the combustion factor such as combustion. Figure 14(A) illustrates the time responses of the parameters and the inequality constraint conditions. Although it settled within the constraint range, the on-line real-time NLSSQP performed well in the experiment. Further improvement can be expected by considering the combustion variation. Figure 14(B) illustrates the time responses of the inputs and outputs. The designed system was applied to a gasoline engine speed control system. From these results, we conclude that the developed adaptive control system satisfies the setting setting time in the experiment. Further improvement can be expected by considering the combustion variation.