PID controller tuning based on disturbance attenuation FRIT using one-shot experimental data due to a load change disturbance

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Abstract: A direct design approaches based on input-output measurements with no need for help from a plant model have attracted attention from several researchers. We have recently proposed such a disturbance attenuation FRIT method using input-output data generated by disturbances. The approach has advantages that it can tune PID gains to improve feedback properties, such as disturbance attenuation. Furthermore, the method has been applied to PID control gain tuning. The method tunes PID gains using one-shot experimental data generated by a known step-type disturbance added at the input signal. However, additive known step-type signal implies a test signal to identify the characteristic of the control systems, so the case where the method can be applied may be restricted. The paper, therefore, gives a tuning method using a one-shot experimental data generated by a load change. Such a disturbance has a possibility that the data can be collected during a full operation. The proposed method can be realized by estimating the magnitude of an impulse and step signal for a load change disturbance from one-shot experimental data. The proposed method realizes the approach by modeling a load change disturbance as a linear combination of a step signal and an impulse signal, where the weighting parameters are unknown. The paper also gives the way how unknown parameters are estimated from the one-shot initial input-output data. Finally, this paper shows the efficiency of the disturbance attenuation FRIT through the experimental result of a helicopter attitude control model when the disturbance is known or correctly estimated.

Keywords: control parameter tuning, FRIT, disturbance attenuation

1. INTRODUCTION

A direct design approaches based on input-output measurements with no need for help from a plant model have attracted attention from several researchers. The method tunes the control parameters, such as PID gains, directly from experimental input-output data with no help from a plant model. System modeling requires much time and cost. Hence, the method can improve efficiency of tuning of control parameters. Especially, when closed loop systems are already set up a full operation, and it just improves the control performance where it is stabilized but not well tuned, the approach is practically useful and efficient.

In the model based approach, the closed loop identification has to be done from the input and output experimental data. However, there are lots of choices to be determined beforehand, such as the system order, structure, representation, and so on. Who knows the best among the various choices? Maybe, the practitioners struggle for finding the answer against the given applications by trial and error. Even if the precise model is obtained, the practitioners have to select the control design approach among the various choices. That is also troublesome tasks.

Furthermore, considering controller parameters tuning, the controller design has to be done on condition that the structure of the controller is fixed. That is not also straightforward. Therefore, the direct controller approach is desirable in the situation where the control parameter tuning is required to improve the control systems performance.

Several approaches on direct control parameter tuning methods have been proposed. H. Hjalmarsson et al. (1998) developed iterative feedback tuning (IFT), M.C. Campi et al. (2002) proposed virtual reference feedback tuning (VRFT), and S. Soma et al. (2004); Soma et al. (2004); Kaneko et al. (2005) proposed fictitious reference iterative tuning (FRIT). Furthermore, several solutions based on IFT, VRFT, and FRIT approaches have been proposed by researchers. O. Lequin et al. (2003) compared IFT with classical tuning methods. A. E. Graham et al. (2007) reported an application of IFT to a mechanical process. A. M. Cecchini et al. (2007) proposed VRFT for controllers with two degrees of freedom. M. C. Campi and S. M. Savarese (2006) introduced the VRFT approach for controller tuning in a nonlinear environment. A. Sala and A. Esparza (2005) applied VRFT to an unstable process by using a high-order controller. O. Kaneko et al. (2005) proposed FRIT in two-degree of freedom control schemes. K. Tasaka et al. (2009) proposed an extended FRIT (E-FRIT), which employs a performance index that includes the cost of
the manipulated variable and adjustable parameters in the reference model. Furthermore, Masuda et al. (2010) proposed a modified FRIT method with simultaneous delay parameter tuning of the reference model for compensating inappropriate reference models.

We have recently proposed such a disturbance attenuation FRIT method using input-output data generated by disturbances. The approach has advantages that it can tune PID gains to improve feedback properties, such as disturbance attenuation. Furthermore, the method has been applied to PID control gain tuning. The method tunes PID gains using one-shot experimental data generated by a known step-type disturbance added at the input signal. However, additive known step-type signal implies a test signal to identify the characteristic of the control systems, so the case where the method can be applied may be restricted. The paper, therefore, gives a tuning method using a one-shot experimental data generated by a load change. Such a disturbance has a possibility that the data can be collected during a full operation.

The proposed method realizes the approach by modeling a load change disturbance as a linear combination of a step signal and an impulse signal, where the weighting parameters are unknown. The paper also gives the way how unknown parameters are estimated from the one-shot initial input-output data. Finally, this paper shows the efficiency of the disturbance attenuation FRIT through the experimental result of a helicopter attitude control model when the disturbance is known or correctly estimated.

2. PROBLEM STATEMENTS

![Fig. 1. Closed loop system in the regulator problem](image)

Consider a single-input, single-output, continuous-time, time-invariant, one-degree-of freedom closed-loop system with a disturbance signal at the input signal, shown in Fig. 1. Let the plant model be denoted by \( P(s) \) in the form of the transfer function. The argument \( s \) stands for a differential operator, and the initial values of transfer functions are assumed to be zero.

Attention is restricted to the feedback controller \( C(\rho, s) \) linearly parametrized in terms of control parameters. That is, the controller \( C(\rho, s) \) can be described as

\[
C(\rho, s) = \rho^T \varphi(s)
\]  

where \( \rho \) is an \( n \)-dimensional control parameter vector, and \( \varphi(s) \) is also an \( n \)-dimensional vector whose elements are rational functions of \( s \) representing a transfer function. Using the description in (1), a PID controller can be represented as

\[
\rho^T = [k_p, k_I, k_D]
\]

\[
\varphi^T(s) = \left[ \frac{1}{\tau}, \frac{1}{\tau(s + 1)} \right], \quad \tau > 0
\]

where \( k_p, k_I, \) and \( k_D \) are proportional, integral, and differential gain, respectively. \( \tau \) is a small positive real number, which stands for a time constant of the approximate differentiation.

\( \rho_0 \) is an initial control parameter vector, and it is assumed that the controller \( C(\rho_0) \) stabilizes the closed-loop system, but \( \rho_0 \) is not well tuned. Now, let \( u_0 \) and \( y_0 \) be the measured one-shot closed-loop experimental input and output signal, respectively when a known step-type signal \( d_0 \) is added as a test signal to the closed-loop system for a certain period (from time 0 to time \( T \)). The control objective is to derive the control parameter vector realizing an ideal response by using one-shot closed-loop experimental input and output signal \( u_0, y_0 \) instead of a plant model. The paper defines the ideal response as outputs of reference model transfer function \( P_{dr}(s) \) as is shown in \( y_{dr} = P_{dr}(s)d \).

Hence, the control problem, which is illustrated by Fig. 2, can be reduced into the optimization problem which minimizes the following evaluation function by only using the one-shot closed-loop experimental input and output signal \( u_0, y_0 \).

\[
J_d(\rho) = \int_0^T (y_d(\rho) - y_{dr})^2 dt
\]

where \( y_d(\rho) \) is the closed-loop response for the disturbance signal \( d \) when the controller \( C(\rho, s) \) is employed for the feedback controller.

\[
y_d(\rho) = \frac{P(s)}{1 + C(\rho, s)P(s)}d
\]

3. FRIT IN THE REGULATOR PROBLEM

FRIT introduces a fictitious reference signal \( r^*(\rho) \) that generates closed-loop input and output signals corresponding to the first experimental input-output data \( u_0 \) and \( y_0 \), which are measured from the closed-loop system employing the controller \( C(\rho, s) \) shown in Fig. 1 when a known step-type signal \( d_0 \) is added as a test signal.

Regarding the control input \( u_0 \) is generated from the fictitious reference signal \( r^*(\rho) \), we can introduce the following equation.

\[
u_0 = C(\rho, s) (r^*(\rho) - y_0)
\]
Using the control input (6), the measured output signal \( y_0 \) can be represented as

\[
y_0 = P(s)(u_0 + d_0) = P(s)\{C(p, s)(r^*(p) - y_0) + d_0 \} = P(s)C(p, s)r^*(p) + P(s)C(p, s)y_0 + P(s)d_0
\]

Hence, the output signal \( y_0 \) becomes

\[
y_0 = \frac{P(s)C(p, s)}{1 + P(s)C(p, s)}r^*(p) + \frac{P(s)}{1 + P(s)C(p, s)}d_0
\]

On the other hand, from (6), the fictitious reference signal \( r^*(p) \) can be described as

\[
r^*(p) = C(p, s)^{-1}u_0 + y_0
\]

Using (7) and (8), we can get

\[
y_0 = \frac{P(s)}{1 + P(s)C(p, s)}u_0 + \frac{P(s)C(p, s)}{1 + P(s)C(p, s)}y_0 + \frac{P(s)}{1 + P(s)C(p, s)}d_0
\]

Now, assuming that there exists the ideal control parameter \( p_d \), which leads to perfect matching of the transfer function from disturbances \( d \) to outputs \( y \) with the prescribed ideal reference transfer function.

Under the assumption, the following equation is satisfied.

\[
P_{dr}(s) = \frac{P(s)}{1 + P(s)C(p_d, s)}
\]

Hence, substituting \( p = p_d \) into (9), it follows from (10) that

\[
y_0 = P_{dr}(s)u_0 + P_{dr}(s)y_0 + P_{dr}(s)d_0
\]

It should be noted that (11) is an identification model in terms of the ideal control parameter \( p_d \). The proposed FRIT for disturbance attenuation is a method for deriving control parameters based on the identification model.

The tuning method for the proposed FRIT is as follows.

**Step 1:** Define the estimated output signal \( \hat{y}(p) \) as

\[
\hat{y}(p) = P_{dr}(s)u_0 + C(p_d, s)P_{dr}(s)y_0 + P_{dr}(s)d_0
\]

using \( u_0 \), \( y_0 \) and \( d_0 \). \( d_0 \) can be utilized to derive (12) because it is a test signal for generating one-shot input-output signal.

**Step 2:** Set the performance index evaluating the integrated square error between \( \hat{y}(p) \) and \( y_0 \).

\[
J_{FD}(p) = \int_0^T (\hat{y}(p) - y_0)^2 \, dt
\]

**Step 3:** Derive the control parameters by solving the optimal value which minimizes the cost function (13).

\[
p^* = \arg \left( \min_p J_{FD}(p) \right)
\]

The following theorem which gives the calculation method for solving optimal value of the evaluation function has been shown in Masuda and Li (2010) in the case where the disturbance signal \( d_0 \) generating the initial input-output data is supposed to be precisely estimated.

**Theorem 1:** It is assumed that there exist an ideal control parameters \( p_d \) which satisfies (10), and the disturbance signal \( d_0 \) which has generated the input-output data \( u_0 \) and \( y_0 \) is known. In addition, the matrix \( A \) \( \in \mathbb{R}^{n \times n} \) defined as

\[
A = \int_0^T \left( \varphi_p(s)P_{dr}(s)y_0 \right) \left( \varphi_p^T(s)P_{dr}(s)y_0 \right) \, dt
\]

is assumed to be a nonsingular matrix. Then, the control parameter vector \( p^* \) which minimizes the performance index (13) can be solved as is shown in the following way.

\[
p^* = A^{-1}b
\]

Furthermore, the optimal solution \( p^* \) corresponds to the ideal control parameter \( p_d \). Here, \( b \in \mathbb{R}^n \) is defined as

\[
b = -\int_0^T \left( \eta \right) \left( \varphi_p(s)P_{dr}(s)y_0 \right) \, dt
\]

\[
\eta = P_{dr}(s)u_0 + P_{dr}(s)d_0 - y_0
\]

The theorem 1 is significant on the ground that the disturbance attenuation FRIT can be derived while keeping the reference signal to be zero value when the experimental input and output data \( u_0 \) and \( y_0 \) is generated by a known test signal \( d_0 \) added at the input signal.

The next section also shows how the disturbance attenuation FRIT applies to PID gains tuning from the initial input-output data.

### 4. PID GAINS TUNING USING THE DISTURBANCE ATTENUATION FRIT

#### 4.1 PID gains tuning method

Now, we will give the design procedure in case of PID gains tuning. Since, the integral gain is set beforehand for designing the disturbance reference model, the proportional gain \( k_P \) and the differential gain \( k_D \) is the tuning parameter. The disturbance reference model is given by (29). Hence, (15), (17), and (18) are given as

\[
A_P = \int_0^T \left( \varphi_p(s)P_{dr}(s)y_0 \right) \left( \varphi_p^T(s)P_{dr}(s)y_0 \right) \, dt
\]

\[
b_P = -\int_0^T \left( \eta_P \right) \left( \varphi_p(s)P_{dr}(s)y_0 \right) \, dt
\]

\[
\eta_P = P_{dr}(s)u_0 + P_{dr}(s)d_0 + k_D \frac{1}{s}y_0 - y_0
\]

\[
\varphi_p(s) = \begin{bmatrix} 1 \\ \frac{1}{s} \end{bmatrix}
\]

Using \( A_P \) and \( b_P \) defined in (19) and (20), the tuned proportional gain \( k_P^* \) and \( k_D^* \) are calculated as

\[
\begin{bmatrix} k_P^* \\ k_D^* \end{bmatrix} = A_P^{-1}b_P
\]
4.2 The disturbance reference model

The disturbance attenuation FRIT firstly sets the disturbance reference model \( P_{dr}(s) \). The subsection gives such disturbance reference model \( P_{dr}(s) \) from the reference model \( P_0(s) \) from the reference signal to the controlled output because the designing flow of \( P_0(s) \) is clearly set from the time constant and the steady state value. From the calculation of the closed loop transfer function, the disturbance reference model \( P_{dr}(s) \) is given from the reference model \( P_0(s) \) as is shown in the next equation.

\[
P_{dr}(s) = \frac{P_0(s)}{C_{PID}(\rho, s)}
\]

where \( C_{PID}(\rho, s) \) is defined as

\[
C_{PID}(\rho, s) = \frac{(k_P \tau + k_D)s^2 + (k_P + k_I \tau)s + k_I}{s(\tau s + 1)}
\]

From (24) and (25), the disturbance reference model \( P_{dr}(s) \) is represented as

\[
P_{dr}(s) = T(s) \cdot \frac{P_0(s)}{s(\tau s + 1)}
\]

Noting that the steady state gain \( P_0(0) \) is 1, so it follows that \( P_0(0) = 1 \). Hence, \( T(0) = 1/k_I \). In addition, when the relative degree of the controlled plant is \( l \), the relative degree of \( P_0(s) \) is more than or equal to \( l \). Hence, it follows that the relative degree of \( T(s) \) is more than or equal to \( l+2 \), and the steady state gain is \( 1/k_I \). Therefore, denoting that the design parameter of \( T(s) \) relating the time constant is \( \gamma \), \( T(s) \) can be given as

\[
T(s) = \frac{1}{k_I} \cdot \frac{\gamma s + 1}{(\gamma s + 1)^{l+2}}
\]

Hence, it follows that the disturbance reference model \( P_{dr}(s) \) is described as

\[
P_{dr}(s) = \frac{1}{k_I} \cdot \frac{s(\tau s + 1)}{(\gamma s + 1)^{l+2}}
\]

From (29), \( P_{dr}(s) \) is characterized using the integral gain \( k_I \), the relative degree of the controlled plant \( l \), the time constant of the disturbance reference signal \( \gamma \), and the time constant of approximated derivative controller \( \tau \). It should be noted that the integral gain \( k_I \) becomes a design parameter if the disturbance reference model is set by (29).

4.3 Estimation of impulse components and step components for load change disturbance

This paper considers the load change disturbance can be modeled by a linear combination of impulse signal and step signal in the following way.

\[
d_0 = \alpha \delta + \beta \cdot 1(t)
\]

Where \( \alpha, \beta \) are the weighting parameter of impulse components and step components, respectively. Subsequent discussions derives the estimation method of the parameters \( \alpha, \beta \) from the one-shot experimental data.

The input signal \( u_0 \) generated from such a disturbance \( d_0 \) can be described as

\[
-u_0 = G(s)(\alpha \delta + \beta \cdot 1(t))
\]

\[
G(s) = \frac{P(s)N_c(s)}{D_c(s) + P(s)N_c(s)}
\]

\[
D_c(s) = s(\tau s + 1),
\]

\[
N_c(s) = (k_P \tau + k_D)s^2 + (k_P + k_I \tau)s + k_I
\]

The Laplace transform of the both side of equation (30) becomes

\[
L[-u_0] = \left\{ G(s)\alpha + \beta G(s)\frac{1}{s} \right\}
\]

Then, noting that the following equation

\[
G(0) = 1, \quad sG(s)|_{s=0} = 0,
\]

we can get the following equation from the final value theorem in the Laplace transform.

\[
\lim_{t \to \infty} (-u_0) = \beta
\]

Subtracting \( \beta \cdot 1(t) \) from the both side of (30), and integrating the both side, and calculating the Laplace transform of the both side of the equation, we can get

\[
\mathcal{L} \left[ \int_0^T -u_0 - \beta \cdot 1(t) \, dt \right] = \frac{1}{s} \left\{ G(s)\alpha + \beta (G(s) - 1)\frac{1}{s} \right\}
\]

Assuming that \( P(0) \neq 0 \) and \( P(0) < \infty \) is satisfied, the following equation is satisfied irrespective of the PID gains \( k_P, k_D \).

\[
G(0) = 1, \quad (G(s) - 1)\frac{1}{s}|_{s=0} = -\frac{1}{k_I P(0)}
\]

Hence, the following equation can be derived from the final value theorem in the Laplace transform

\[
\lim_{T \to \infty} \left[ \int_0^T -u_0 - \beta \cdot 1(t) \, dt \right] = \left\{ G(s)\alpha + \beta (G(s) - 1)\frac{1}{s} \right\}
\]

The value of \( \beta \) is calculated from (36), and the \( k_I \) is known due to the initial PID gains are known. Therefore, (37) enables to estimate the weighting parameter \( \alpha \) of the impulse component.

5. AN EXPERIMENTAL RESULT FOR AN ATTITUDE CONTROL FOR A HELICOPTER EXPERIMENTAL MODEL

This section shows the efficiency of the disturbance attenuation FRIT through the experimental result of a helicopter attitude control model when the disturbance is known or correctly estimated.

In the experiment, the control objective is to keep the elevation angular of the helicopter attitude to the horizontal
The initial PI gains were set to be $k_P = 0.9$, $k_I = 0.18$, and $k_D = 1.5$. After keeping the horizontal angular using a PID control, a step-type signal, the magnitude of which is 0.1, is added at the input signal. The step-type signal works as disturbance signal, which makes the elevation angular deviate from the horizontal angular. Then, the PID control regulates the deviation. The input-output data when the helicopter attitude is stabilized using the initial PID gains are shown in Fig. 3, where the disturbance is added at 40 [sec].

The disturbance reference model is selected based on the way shown in subsection 4.2. The integral gain $k_I$ was doubled from the initial integral gain to improve the disturbance attenuation property. The approximated differential parameter $\tau$ was set to 0.003 considering the sampling period of the experimental model, which is 0.01 [sec.]. The relative degree of the controlled plant was set to $l = 1$. The time constant $\gamma = 0.7$ was set by comparing the simulated disturbance reference model output with various time constant parameters $\gamma$ with the experimental output data. That is how the disturbance reference model was selected in the following way.

$$P_{dr}(s) = \frac{1}{k_I} \frac{s(0.04s + 1)}{(0.7s + 1)^4}, \quad k_I = 0.24 \quad (40)$$

The obtained proportional and differential gains are $k_P = 0.9589$ and $k_D = 0.7366$. Fig. 6 shows that the experimental input and output signal by the tuned PID gains (solid line), the disturbance reference signal (dashed line), and the initial input and output data by the initial PID gains (dotted line).

The above figure in Fig. 6 shows that the elevation angle for a helicopter experimental model controlled by tuned PID gains well follows that the disturbance reference model output. Hence, the obtained PID gains from the proposed method are well tuned ones to improve the initial PID gains, and make the controlled output follow the prescribed disturbance model reference output.

The Fig. 5 shows the disturbance reference model output and the initial disturbance response data.

The disturbance reference model output and the initial disturbance response data.

The fitting data for identified plant model using the initial input output data for demonstrating the advantage of direct control parameter tuning method. The input-output data in Fig. 4 is not well sufficiently rich for identification of the plant model, so the identified model output signal does not fit the observed signal, which implies that the PID gains tuning using the identified
Fig. 6. The experimental result when the tuned PID gains were applied using the proposed method model will not result in a good response. Therefore, we can see that the direct tuning approach is efficient in the case where the input-output data is restricted to the one shot input-output experimental data.

Fig. 7. The fitting data for identified plant model using the initial input output data

6. CONCLUDING REMARKS

The paper gave a tuning method using a one-shot experimental data generated by a load change. Such a disturbance has a possibility that the data can be collected during a full operation. The proposed method can be realized by estimating the magnitude of an impulse and step signal for a load change disturbance from one-shot experimental data. The paper realizes the approach by modeling a load change disturbance as a linear combination of a step signal and an impulse signal, where the weighting parameters are unknown. The paper also gave the way how unknown parameters are estimated from the one-shot initial input-output data. Finally, this paper showed the efficiency of the disturbance attenuation FRIT through the experimental result of a helicopter attitude control model when the disturbance is known or correctly estimated.

As a future work, the stability analysis is a significant topic for the subject of the paper.

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