Design and Analysis of IMC based PID Controller for Unstable Systems for Enhanced Closed Loop Performance

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Abstract: In this paper, design of PID controller is analyzed for unstable second order processes with time delay based on IMC method and H₂ minimization. A new desired closed loop transfer function is obtained based on which the PID controller is designed. Maclaurin series is used to approximate the controller expression as a PID controller. Improved closed loop performances are achieved with the proposed method when compared to the recently reported methods in the literature. Comparative analysis has also been carried out with modified Smith predictor schemes and showed that the proposed method is superior. Further, an analysis is carried out based on maximum sensitivity for arriving at systematic guidelines for selection of the closed loop tuning parameter which is essential for unstable systems. The bounds for this tuning parameter are analyzed using the maximum sensitivity.

Keywords: Unstable process, steady state, sensitivity analysis, robustness, PID controller.

1. INTRODUCTION

Open loop unstable systems exist in many chemical and biological processes and these systems are fundamentally difficult to control than that of stable systems. The difficulty increases if the system has time delay. The difficulty further increases when the system contains a positive zero. In unstable systems, controlling a first order system is comparatively easy than that of a second order one (system with two unstable poles). These exists many PID design methods (Nasution et al., 2011; Shamsuzzoha and Lee, 2008a; Sree et al. 2004; Lee et al., 2000) for controlling unstable first order systems. However, very few methods (Shamsuzzoha and Lee, 2008b; Rao and Chidambaram, 2006) are available for unstable second order systems. Existence of second order unstable systems is well described by Sree and Chidambaram (2006). The PID controller cannot provide stabilized responses when the time delay to time constant ratio is greater than 1.2 for unstable systems. Modified Smith predictor based control schemes (Liu et al., 2005) and modified IMC based control schemes (Tan et al., 2003) are also developed for unstable first order and second order systems. However, these modified schemes also are not applicable when the time delay to time constant ratio exceeds 1.2. Also, more control effort is required for these schemes. Hence, keeping the simplicity into account, properly designed PID controller is better than these modified schemes. However, the designed PID controller should provide good nominal and robust closed loop responses and smooth manipulated variable responses. To achieve these objectives, in this work, IMC is used to design PID controller for second order unstable processes based on H₂ minimization. Here, the design is addressed for second order unstable systems because there already exist many design methods for first order unstable systems and also controlling a second order unstable systems is difficult than that of first order one. Once the controller is designed, an analysis is carried out for proper selection of the IMC tuning parameter. The present design method is based on the work described in Nasution et al. (2011). For clear illustration, the design is addressed in the next section followed by simulation results and conclusions.

2. PROPOSED DESIGN METHOD

The block diagram of the IMC scheme is shown in Fig. 1 where \( G_{mc} \) is the IMC controller, \( G_p \) is the process and \( G_m \) is the process model.

![Fig. 1. IMC control scheme](image)

The second order unstable process considered here is

\[
G_p(s) = ke^{\theta} / (\tau_s - 1)(\tau_s - \tau)
\]  

The equivalent controller in a conventional feedback form can be written as \( G_c = G_{mc} / (1 - G_m G_p) \). According to H₂ optimal controller design (Morari and Zafiriou, 1989), the
IMC controller is designed as \( G_{imc}(s) = \hat{G}_{imc}(s)F(s) \) where \( F \) is the filter to make \( G_{imc} \) as a realizable controller and also to maintain robustness. \( \hat{G}_{imc}(s) \) is designed to achieve H2 optimal performance for a specific input type, \( v(s) \). The process model and the input are divided into minimum phase part and non-minimum phase part as \( G(s) = G_m(s)G_n(s) \) and \( v(s) = v_p(s)v_n(s) \) where "−" refers to the minimum phase part and "+" refers to non-minimum phase part. Further, the Blachke product of RHP poles of \( G_n(s) \) and \( v(s) \) are introduced as

\[
b_n(s) = \prod_{i=1}^{k}(s + p_n) (s + \overline{p_n})
\]

Where \( p \) and \( \overline{p} \) are the RHP pole and its conjugate. Then the H2 optimal controller is derived using

\[
\hat{G}_{imc}(s) = b_1\left( \hat{G}_{imc} - \hat{v}_v \right)^{-1} = \left[ b_1 \hat{G}_{imc} \right]^{-1} b_v
\]

(2)

Where \( \left[ ... \right] \) is the operator obtained after omitting all terms involving the poles of \( \hat{G}_{imc}^{-1} \) after taking partial fraction expansion. In the present work, for the second order unstable process \( (1) \), the required quantities for the operator are obtained as

\[
\hat{G}_{imc}(s) = \frac{k}{\tau_1 \tau_2 (s + \frac{1}{\tau_1}) (s + \frac{1}{\tau_2})} \hat{G}_{imc}(s) = e^{-\sigma_i} \hat{G}_{imc}(s)
\]

\[
\hat{v}_v(s) = \frac{k}{\tau_1 \tau_2 (s + \frac{1}{\tau_1}) (s + \frac{1}{\tau_2})} \hat{v}_v(s) = 1
\]

\[
b_1(s) = \left( -s + \frac{1}{\tau_1} \right) \left( -s + \frac{1}{\tau_2} \right) \left( s + \frac{1}{\tau_1} \right) \left( s + \frac{1}{\tau_2} \right)
\]

\[
b_2(s) = \left( -s + \frac{1}{\tau_1} \right) \left( -s + \frac{1}{\tau_2} \right) \left( s + \frac{1}{\tau_1} \right) \left( s + \frac{1}{\tau_2} \right)
\]

Substituting in (2) IMC controller is obtained as

\[
G_{imc}(s) = \frac{\alpha s^2 + \alpha s + 1}{\lambda (s + 1)^2} \left[ \frac{\tau_1 \tau_2 \frac{1}{\tau_1} \tau_2 \frac{1}{\tau_1} \frac{1}{\tau_2} \left( \frac{1}{\tau_1} s + \frac{1}{\tau_2} s \right)}{\frac{1}{\tau_1} \tau_2 \frac{1}{\tau_2} \frac{1}{\tau_1} \tau_2 \left( \frac{1}{\tau_1} s + \frac{1}{\tau_2} s \right)} \right]
\]

Considering the filter as \( F(s) = (\alpha s^2 + \alpha s + 1)/(\lambda s + 1) \), the IMC controller is obtained as \( G_{imc}(s) = \hat{G}_{imc}(s)F(s) \), where \( \lambda \) is the tuning parameter which should be selected carefully so that good nominal and robust closed loop performances are achieved. With that the desired closed loop transfer function is obtained as

\[
\frac{H(s)}{\gamma(s)} = \left[ \frac{\tau_1 \tau_2 \frac{1}{\tau_1} \tau_2 \frac{1}{\tau_1} \frac{1}{\tau_2} \left( \frac{1}{\tau_1} s + \frac{1}{\tau_2} s \right)}{\frac{1}{\tau_1} \tau_2 \frac{1}{\tau_2} \frac{1}{\tau_1} \tau_2 \left( \frac{1}{\tau_1} s + \frac{1}{\tau_2} s \right)} \right]
\]

Then the equivalent controller in a conventional feedback form is obtained from IMC structure as \( G_{imc} = G_m/(1 - G_mG_n) \). After substituting \( G_{imc} \) and \( G_m \) will be obtained as a higher order numerator and denominator expression. To simplify this expression to a PID controller form, Maclaurin series is used here. To do that, let us define \( J(s) = sG(s) \). Expand \( J(s) \) using Maclaurin series expansion to obtain the controller \( G_c \) as

\[
G_c(s) = \frac{1}{s} \left( J(0) + J'(0)s + \frac{J''(0)}{2!} s^2 + \ldots \right)
\]

By considering this as a PID controller in the form

\[
G_c(s) = k \left( 1 + \frac{1}{\tau_i} s + \tau_1 s \right)
\]

the PID controller parameters are obtained as \( k = J(0) \), \( \tau_i = J(0)/J(0) \) and \( \tau_d = J(0)/2J(0) \).

Where

\[
J(0) = \frac{1}{p_n} D(0)
\]

\[
J_i = \frac{-p_n(0)D(0) + p_n(0)D(0) + p_n(0)D(0) - 2J(0)}{p_n(0)D(0) + p_n(0)D(0)}
\]

\[
D(0) = 4\lambda - p_n(0); D(0) = 2[24\lambda^3 - p_n(0)]^2
\]

\[
p_n(0) = \frac{(r_1 - r_2)(-\theta + \alpha) + \tau_i b - \tau_i a}{r_1 - r_2}
\]

\[
(r_1 - r_2)(\theta^2 - 2\alpha\theta + 2\alpha^2) + 2(\tau_i b - \tau_i a)(-\theta + \alpha)
\]

\[
p'_n(0) = \frac{2\tau_i r_3 (a-b)}{r_1 - r_2}
\]

\[
(r_1 - r_2)(-\theta^3 + 3\alpha \theta^2 - 6\alpha \theta + 6\alpha^2) + (\tau_i b - \tau_i a)(3\theta^2 - 6\alpha \theta + 6\alpha^2)
\]

\[
p''_n(0) = \frac{2\tau_i r_3 (a-b)(-3\theta + 3\alpha)}{r_1 - r_2}
\]

\[
p_n(0) = k; p''_n(0) = -k \left[ (r_1 - r_2) \left( -r_2 + \alpha + r_i b - \tau_i a \right) \right]
\]

\[
(r_1 - r_2)
\]

\[
-k \left[ (r_1 - r_2) \left( -r_2 + \alpha + r_i b - \tau_i a \right) \right]
\]

\[
(k(r_1 - r_2)^2)
\]

\[
(k(r_1 - r_2)^2)
\]
In which $\alpha_1$ and $\alpha_2$ values are obtained from the requirements to satisfy internal stability in IMC based control schemes. The condition for internal stability is

$$(1 - G_{mc}(s)) |_{s=1}^{\tau_1} = 0$$

Thus $\alpha_1 = \left( \frac{\lambda}{\tau_1} + 1 \right)^2 \frac{r_1^2 e^{\beta_1}}{x} \left( \frac{\lambda}{\tau_2} + 1 \right)^2 \frac{r_2^2 e^{\beta_2}}{y} - r_1 - r_2$

$\alpha_2 = \left( \frac{\lambda}{\tau_1} + 1 \right)^2 \frac{(r_1 - r_2)}{x} r_1^2 e^{\beta_1} - r_1^2 - \alpha_1 r_1$

$x = \frac{r_1^2}{\tau_1} (a - b) + \frac{(r_1^2 - r_2)}{r_1} + r_1 - r_2$

$y = \frac{r_1}{\tau_2} (a - b) + \frac{(r_1^2 - r_2)}{r_2} + r_1 - r_2$

$a = r_1 (e^{\beta_1}) - 1); b = r_1 (e^{\beta_2}) - 1)$

3. SIMULATION STUDIES

To analyze the performance of the proposed design method, three examples are considered.

3.1 Example-1: An unstable second order process $G_p(s) = 2e^{-0.1s} / (3s - 1)(s - 1)$ is considered here (Tan et al., 2003). To select the tuning parameter ($\lambda$), an analysis is carried out here based on maximum sensitivity. Fig. 2 shows the variation of $Ms$ with respect to $\lambda$. There exist a large value for $Ms$ corresponding to $\lambda = 0.62$ after which the $Ms$ decreases up to $\lambda = 1.45$ ($Ms = 3.1$) and then $Ms$ increases. Hence $\lambda$ should be selected in the range of $0.72 - 2.7$. Within this range of $\lambda$, the maximum value for $Ms$ will be 10. If $\lambda$ is selected outside this range the closed loop performance is not good or is not stable. Note that the minimum value of $Ms$ achievable in this range of $\lambda$ is 3.1. It can be observed from the figure that one should not select $\lambda$ without proper analysis. For the purpose of comparison, method proposed by Panda (2009) is considered. The PID settings obtained by Panda (2009) are $k_c = 0.881$, $\tau_1 = 5.11$, and $\tau_2 = 3.42$ for a tuning parameter value 0.948 which corresponds to $Ms$ of 4.82. To have fair comparison, for the proposed method also, $\lambda$ is selected as 0.9 which gives $Ms$ value of 4.82. Note that this $Ms$ value is after the peak in the graph in Fig. 1. The PID controller parameters are obtained as $k_c = 0.9718$, $\tau_1 = 2.4591$, $\tau_2 = 4.066$. With these controller settings, the methods are simulated by giving a unit step change in set point and a unit negative input in the load at $t = 50$ sec respectively. The closed loop performance for perfect parameters is shown in Fig. 3. Perturbations of +10% in time delay and -10% in process gain are given and the corresponding closed loop responses are shown in Fig. 4. It can be observed that the proposed design performs better and also the control action responses are smooth for the proposed method that that of Panda (2008). The same improvement is also shown in terms of the performance index integral absolute error (IAE). To further analyze the robustness, $Ms$ values are plotted for different values of the time delay and $\lambda$ and is shown in Fig. 5. It can be observed that as $\theta$ increases, $\lambda$ need to be increased so that the system robustness is maintained as per the requirements and within the possible limits.
3.2 Example-2: A second order process with one integrator is considered here (Liu et al., 2006; Rao and Chidambaram, 2006). The process transfer function is described as $G_p(s) = e^{-0.2s} / (s - 1)$. To use the proposed method, this process is converted for convenience as $G_p(s) = e^{-0.2s} / (s - 0.01(s - 1))$ which in turn can be written as $G_p(s) = 100e^{-0.2s} / (100s - 1)(s - 1)$. For this process, the proposed method is applied. To understand the selection of $\lambda$, $M_s$ values are plotted against $\lambda$ and the graph is shown in Fig. 6. It can be observed from Fig. 6 that the selection of $\lambda$ should be carried out as specified within the bracketed section so that the system robustness is maintained. Based on this understanding $\lambda$ is selected as $0.65$ (after the peak in the Fig. 6). With that the controller settings are obtained as $k_c = 2.0662$, $\tau_i = 2.6451$, $\tau_d = 1.5062$.

For the purpose of comparison, method of Liu et al. (2005) is considered here which is a two degrees of freedom control scheme based on modified form of Smith predictor with three controllers. As this method uses a set point controller, for fair comparison, in the proposed method also, set point weighting (0.4) is considered. These two methods are simulated by giving a unit step change in the set point and a negative step change in the load at $t = 50$ sec respectively. Fig. 7 shows the closed loop responses for perfect model. The proposed method performs better. With set point weighting, the set point response is smooth and performs further better. Fig. 8 shows the responses for perturbation of +30% in time delay. It can be observed that the proposed method performs better. Note that the set point response is smoothened by using the set point weighting in the proposed method.

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To analyze the robustness further, sensitivity and complementary sensitivity functions are plotted for different values of $\lambda$ and shown in Fig. 9. It can be observed that as $\lambda$ increases, the peak values of the sensitivity functions decreases indicating more robust behaviour.

Fig. 9. Sensitivity and complementary sensitivity functions for different values of $\lambda$, solid - $\lambda = 0.4$; dash - $\lambda = 0.65$, dot - $\lambda = 0.9$.

3.3 Example-3: An unstable process with one stable pole is considered here (Lee et al., 2000, Tan et al., 2003). The process is described as $G_p(s) = e^{-0.939s}/(5s - 1)(2.07s + 1)$. To apply the proposed method, this process is converted as $G_p(s) = e^{-0.939s}/(5s - 1)(-2.07s - 1)$ with $k = -1, \theta = 0.939, \tau_1 = 5$ and $\tau_2 = -2.07$. To analyze the selection of the tuning parameter, $M_s$ values are plotted for various values of $\lambda$, and corresponding graph is shown in Fig. 10.

Fig. 10. $M_s$ versus $\lambda$ for example-3.

As explained earlier, based on this Fig. 10, $\lambda$ is selected as 1.9. With this the PID controller parameters are obtained as $k_c = 4.4432, \tau_1 = 10.25, \tau_d = 1.932$. Set point weighting (0.4) is used for the proposed. For the purpose of comparison, method of Tan et al. (2003) is considered which is based on the modified IMC scheme with three controllers. Fig. 11 shows the closed loop responses for perfect model for a unit step change in the set point and a negative step input at $t = 75$ sec respectively. Fig. 12 shows the closed loop responses for perturbations. The proposed method performance is marginally lower for perfect conditions but the proposed method performs better for perturbations. Note that the control action responses are not better for Tan et al. in both cases. This is evident from Fig. 11 that for Tan et al. method, to reach a step change from 0 to 1, the control action needs an effort to change from 25 to -1. This is not recommended.

Fig. 11. Closed loop responses for perfect model for example-3, solid-proposed method (IAE = 6.68), dash-Tan et al. (IAE = 4.47).

Fig. 12. Closed loop responses for perturbations of $\pm 15\%$ in gain and time delay & $-15\%$ in $\tau_1$ for example-3, solid-proposed method (IAE = 7.92), dash-Tan et al. (IAE = 33.19).
To analyze the effect of time delay on Ms, a graph is plotted for three different values of T and shown in Fig. 13. It can be observed that as T increases, λ need to be increased to maintain the system robustness.

![Graph showing Ms versus λ for different T values for example-3](image)

Fig. 13. Ms versus λ for different T values for example-3

4. CONCLUSIONS

In this study, simple PID controller design using IMC method and H₂ minimization is presented. The following are the observations from this study.

1. For unstable systems, the selection of the tuning parameter is crucial for obtaining stable responses. Ms versus λ plot gives good understanding about this. For the same Ms value, there exist two λ values (one before the peak and another after the peak). If λ value is selected before the peak, then the closed loop response will be poor and also the resulting controller will not be robust (may lead to unstable behaviour). In the literature, for performance comparison of various methods, same Ms value is normally chosen as the proper comparative measure. However, if there exists two λ values for the same Ms value, the λ value after the peak in the Ms versus λ plot should only be considered.

2. With the present PID controller design, improved closed loop performance is achieved when compared to recently reported PID design methods in the literature.

3. When the performance is compared with complex control schemes (modified Smith predictor or modified IMC) which consist of more than two controllers, the proposed method still provides improved performances. Note that the improvement in the manipulated variable responses is significant. Even if there is a marginal improvement with the complex control schemes, PID will be preferred due to simplicity for practical usage.

4. The proposed method is also applied to second order unstable processes with RHP zeros. If RHP zero process is converted as \( G_p(s) = ke^{(\rho+\psi)T}/(\tau_s - 1)(\tau_s - 1) \) and for this modified process the proposed design is applied and obtained good closed loop performances (graphs not shown here because of space limitation) when the p value is small (p < 0). A straightforward design is required to take into account large values of p.

5. Here, the analysis is carried out for second order unstable processes only. In our future research, a generalized procedure will be developed for all classes of unstable and integrating systems with and without RHP zeros for effective and analytical formulae for selection of the tuning parameter.

REFERENCES


