Data-driven robust PID tuning toolbox

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Abstract: In this paper, a toolbox is presented for the parameter tuning of PID controllers based on the Virtual Reference Feedback Tuning. The VRFT is a data-driven methodology that uses only data to find the parameters of the controller. In order to add robustness to the design, a test is proposed and implemented in the tool that use an approximation of the multiplicative uncertainty in an “Internal Model Control” like framework to check if the controller found is robustly stable.

Keywords: Graphical user interface, internal model control, PID control, virtual reference feedback tuning

1. INTRODUCTION

Data-Driven control is a methodology that skips the modelling step (fundamental in the so called model-based design) and find the parameters of the controller directly using data taken directly from one or more experiments on the plant. Several different methodologies have been presented in the literature: the Iterative Feedback Tuning (IFT) (Gevers, 2002), computes an unbiased gradient of a performance index to improve iteratively the tuning of the parameters of a reduced order discrete time controller. The Correlation-based Tuning (CbT) (Karimi et al., 2005) is a one-shot methodology that attempts to find the values of a restricted order controller that minimizes the correlation between the closed-loop error of the system (based in a desired closed-loop behaviour) and the reference signal. The Virtual Reference Feedback Tuning (VRFT) (Campi et al., 2002) translates the model reference control problem into an identification problem, being the controller the transfer function to identify. The optimization used to find the parameters of the controllers is based on some “virtual signals” computed from a batch of data taken directly from an open-loop experiment.

One drawback of data-driven control is that the “classical” stability and robustness analysis cannot be performed, since a model of the process is needed for those analysis. Concepts like “phase margin” and “gain margin” for example (Skogestad, 2003), are not directly applicable in data-driven design.

One of the model-based control methods that introduces the idea of robustness directly in the controller design stage is the Internal Model Control (IMC) (Garcia and Morari, 1982; Morari and Zafirou, 1989). In this paper, the conjunction of the VRFT methodology with the robustness condition of IMC is presented, unifying the best of two worlds and a MATLAB application is presented aimed to be useful for educational purposes.

The issue of education on control (Kheir et al., 1996) and how to take advantage of the new technologies (Antsaklis et al., 1999) have been subject of several studies. It is clear that students learn better when they test and try different concepts on real plants or even using simulation environments, having a direct experience with the concepts (Bernstein, 1999). For example in Dormido (2004) an extensive analysis of the effect and advantages of the web over the traditional laboratories is presented as well as the different types of remote laboratories and the technologies they are based on. Several interactive tools have been presented in the literature, for example in Guzman et al. (2008), where several modules programmed using the software Sysquake are presented for the analysis and design of PID controllers. Aliane (2008) presented a tool for the analysis of linear system programmed using the Visual Basic for Application support on Microsoft Excel spreadsheets. The motivation of this application was to provide teachers with tools that allow them to concentrate on the subject rather than in the programming details at the same time that the user is allowed to interactively see the effects of the change in some of the parameters.

Not only the classical control theory have been tackled for control purposes, some modern control techniques are starting to be introduced in class and laboratories, as for example the practices and laboratories presented in Jurado et al. (2002). Even more, with the arrival of faster Internet connections and more reliable technologies, it is possible to provide real laboratories with remote access to the students, as the case presented by Dormido et al. (2008) and Fabregas et al. (2011). This kind of application not only help the student for a better understanding of the concepts but it allows a better planning and a wider access to the university resources.

The objective of this paper is to present a toolbox aimed to introduce data driven control techniques applied to PID
controllers. This toolbox fully implements the design of steps for PID tuning based on the virtual reference idea in conjunction with the internal model control framework. In section 2 a short overview on the VRFT is presented while the IMC-VRFT version of the method is presented in section 3. The toolbox is described in section 4.

2. VRFT OVERVIEW

The Virtual Reference Feedback Tuning (VRFT) is a one-shot data-based method for the design of feedback controllers. The original idea was presented in Campi et al. (2002). The methodology can be explained as follows: suppose that the controller belongs to the class \( \mathcal{C}(z; \theta) \) given by \( C(z; \theta) = \beta^T(z) \theta \), where \( \beta(z) = [\beta_1(z) \cdot \cdot \cdot \beta_n(z)]^T \) is a known vector of transfer functions, and \( \theta = [\theta_1, \theta_2, \cdot \cdot \cdot, \theta_n]^T \) is the vector of parameters. The control objective is to minimize the model-reference criterion given by:

\[
J_{MR}(\theta) = \left\| \frac{P(z)C(z; \theta) - M(z)}{1 + P(z)C(z; \theta)} W(z) \right\|_2^2
\]

where \( P(z) \) is the unknown plant model, \( M(z) \) is the desired closed-loop response and \( W(z) \) is a frequency weighting factor. The main idea of the method is that, given a set of input-output data from the plant, taken in open-loop, (i.e. \( u(t) \) and \( y(t) \) respectively), the designer should be able to minimize equation (1), without a model of the plant. This can be achieved by creating a “virtual” signal constructed from the open-loop data. If the ideal open-loop output \( y(t) \) had been taken in open-loop and the closed-loop transfer function were \( M(z) \), one can find a “virtual reference” \( \hat{r}(t) \) that, if applied to the closed loop system, would yield \( y(t) \) as the output. If the output of the plant is \( y(t) \), then the output of the controller should be equal to \( u(t) \). This controller can be found by identifying the transfer function which yields the output \( u(t) \) when the input \( \hat{r}(t) - y(t) \) is applied to the input as depicted in Fig. 1.

The original algorithm, as presented by the authors in Campi et al. (2002), is given as follows: Given a set of measured I/O data \( \{u(t), y(t)\}_{t=1,...,N} \):

1. Compute:
   - a virtual reference \( \hat{r}(t) \) such that \( y(t) = M(z) \hat{r}(t) \), and
   - the corresponding tracking error \( e(t) = \hat{r}(t) - y(t) \)

2. Filter the signals \( e(t) \) and \( u(t) \) with a suitable filter \( L(z) \):

\[
e_L(t) = L(z)e(t) \quad u_L(t) = L(z)u(t)
\]

3. Select the controller parameter vector, say, \( \hat{\theta}_N \), that minimizes the following criterion:

\[
J_{VR}^N(\theta) = \frac{1}{N} \sum_{t=1}^{N} (u_L(t) - C(z; \theta)e_L(t))^2
\]

If \( C(z; \theta) = \beta^T(z) \theta \), the criterion (2) can be rewritten as

\[
J_{VR}^N(\theta) = \frac{1}{N} \sum_{t=1}^{N} (u_L(t) - \varphi_L(t) \theta)^2
\]

with \( \varphi_L(t) = \beta(t)e_L(t) \) and the parameter vector \( \hat{\theta}_N \) is given by

\[
\hat{\theta}_N = \left[ \sum_{t=1}^{N} \varphi_L(t) \varphi_L(t)^T \right]^{-1} \sum_{t=1}^{N} \varphi_L(t) u_L(t)
\]

The authors, also showed that, the filter \( L(z) \) should be the one that approximates the criterion (2) to (1). This filter should accomplish:

\[
|L(e^{j\omega})|^2 = |1 - M(e^{j\omega})|^2 |M(e^{j\omega})||W(e^{j\omega})|^2 \Phi_u(e^{j\omega})^{-1}
\]

where \( \Phi_u \) is the spectral density of \( u(t) \).

3. IMC-VRFT OVERVIEW

Internal Model Control (IMC) is a popular and well known control method that incorporates the model of the process directly into the controller Morari and Zafiriou (1989). The standard structure is depicted in Fig. 2. \( P(z) \) represents the Plant, while \( \hat{P}(z) \) is its model. \( Q(z) \) is the IMC controller. In the absence of uncertainty, the control acts as if it were in open-loop control for the reference tracking but when a disturbance enters into the system, the same controller acts as close-loop for the disturbance rejection. If \( Q(z) \) is designed as \( Q(z) = P(z)^{-1}f(z) \) and \( \hat{P}(z) = P(z) \), the output ideally becomes

\[
y = f(z)r + (1 - f(z))d
\]

It is clear that, to have perfect model matching control (in close-loop, the desired dynamics are given by \( f(z) \)), \( Q(z) \) must try to cancel the dynamics of the plant. This characteristic leads to the well know property that an IMC system would be nominally internally stable if \( Q(z) \) is stable, in case the model is equal to the plant. Finding a perfect model is rarely achievable and in such case, \( Q(z) \) may not be possible to contain the inverse of this model due to physical limitations or because the inverse of the plant may lead to an unstable controller. In Morari and Zafiriou (1989) a two-step design is proposed for this kind of controller:

1. Solve the nominal performance criterion given, for example, by

\[
\min_{Q(z)} \| (1 - \hat{P}(z)Q(z)) W(z) \|_p
\]

Where \( W(z) \) is a filter chosen to give more importance in certain frequencies and \( \| \cdot \|_p \) is a given norm that defines the performance criterion. The optimal solution of this problem yields to a sensitivity function given by \( S^*(z) = 1 - \hat{P}(z)Q(z) \) and the complementary sensitivity function given by \( M^*(z) = \hat{P}(z)Q(z) \), that is, the response to a change in the reference is as if it were in open loop, while the response to a disturbance is in close-loop.
In IMC where the controller is directly derived from the model. This basic idea leads to the following optimization problem which gives the set of optimal parameters $\theta^*$ (in a least squares sense):

$$\theta^* = \arg \min_{\theta} J(\theta) \quad (11)$$

Once $Q(z; \theta^*)$ has been determined, it is easy to compute the approximation of the process model of the plant from (10):

$$P(z; \theta^*) = M(z)Q(z; \theta^*)^{-1} \quad (12)$$

It is important to note that $P(z; \theta)$ is seen just as an “instrumental model”, that results from the determination of the optimal controller. This instrumental model is used as part of the control loop and, as presented in the next section, as the manner to describe a “nominal plant”. In Data Driven Control, there is no nominal model of the plant, therefore, to define a test to check if the controller is robustly stable, it is necessary to approximate the plant by this “instrumental model”. The filter for robust operation presented in (8), is already included in $Q(z; \theta^*)$ since the closed-loop behaviour is expected to be $M(z)$, but it is not possible to know if condition (8) is fulfilled just by solving this optimization problem. It is therefore, desirable to count with a data-based test to check if this condition holds.

### 3.2 Robust Stability for the IMC-VRFT

When the closed-loop system is stable for all perturbed plants around the nominal model up to the worst-case model uncertainty, it is said to be robustly stable (Skogstad and Postlethwaite, 2007). In Data Driven Control it is difficult to find a controller that assures robust stability of the plant since not even a nominal model is available (in van Heusden et al. (2011) the stability problem is addressed by adding some constraints in the frequency domain directly into the optimization problem). However, it is possible to use (8) and the batch of input-output data, to test if the controller is robustly stabilizing the plant, before the actual controller is implemented, by approximating the uncertainty function $l_m(\omega)$ from the instrumental model $P(z; \theta^*)$.

Using the results on Empirical Transfer Function Estimate (ETFE) from Ljung (1999), given an input-output set of $N$ points of data $\{u(t), y(t)\}_N$ from a plant $G(z)$ which transfer function is assumed to be unknown, the estimate of the frequency response is given by

$$\hat{G}_N(e^{j\omega}) = \frac{Y_N(\omega)}{U_N(\omega)} \quad (13)$$

where $U_N(\omega)$ and $Y_N(\omega)$ are given by

$$U_N(\omega) = \frac{1}{\sqrt{N}} \sum_{t=1}^{N} \{u(t)e^{-j\omega t}\} \quad (14)$$

$$Y_N(\omega) = \frac{1}{\sqrt{N}} \sum_{t=1}^{N} \{y(t)e^{-j\omega t}\} \quad (14)$$

The essential frequency points are given at $\omega = 2\pi k/N$, $k = 0, 1, \ldots, N-1$. Other points are obtained by interpo-
Fig. 3. Graphical interpretation of the robust stability test. According to Ljung (1999), the ETFE is an asymptotically unbiased estimate of the transfer function at increasingly (with \( N \)) many frequencies, but the variance of the ETFE do not decrease as \( N \) increases. To tackle this problem, the use of filtering windows is recommended to smooth the ETFE.

**Approximation of \( \hat{I}_m(\omega) \) and the robust stability test**  If the uncertainty bound \( \hat{I}_m(\omega) \) can be approximated using the ETFE, it will be possible to perform a data-based test to check Robust Stability using (8). In the case of the IMCVVRT, the assumption on the instrumental model \( \hat{P}(z) \) is that it is close enough to the real plant transfer function, in order to left the “nominal” stability depending on \( Q(z) \): if \( Q(z) \) is stable, the “nominal” closed-loop system is stable given that the plant is stable. If the controller has been found using the proposed approach, the filter \( f(z) \), is assumed to be already included in \( Q(z) \) and (8) can be rewritten as

\[
\left| \frac{\hat{P}(e^{j\omega}, \theta^*) \hat{Q}(e^{j\omega}, \theta^*) \hat{I}_m(\omega)}{1 - \tau z^{-1}} \right| \leq 1
\] (15)

or, if a “security” constant \( \alpha \geq 0 \) is added to cope with possible errors when approximating \( \hat{I}_m \)

\[
\left| \frac{\hat{P}(e^{j\omega}, \theta^*) \hat{Q}(e^{j\omega}, \theta^*) \hat{I}_m(\omega)}{1 - \alpha} \right| \leq 1
\] (16)

A graphical interpretation of (16) can be seen in Fig. 3: if at some point the dashed line falls below the solid line (which represents the complementary sensitivity function if the instrumental model is close enough to the transfer function of the plant) it means one is trying to extend the system beyond the uncertainty limits. At this point (16) fails, and it is not possible to assure robust stability with controller \( Q(z, \theta^*) \) and given value of \( \alpha \).

To approximate (16), the frequency response of \( Q(z, \theta^*) \) and \( \hat{P}(z, \theta^*) \) can be calculated using the results of the optimization. \( \hat{I}_m(\omega) \) is approximated using the definition of the multiplicative uncertainty and the ETFE approximation. If the test fails, the designer has two options: it is possible to increase the number of parameters of the controller to relax the closed-loop specification \( M(z) \). Once the new controller is found, the test can be performed again to check if the robust condition holds for the new setting.

### 3.3 Particular case for PID controllers

It is possible to directly find the parameters of PID controllers using the IMCVVRT method. It is common to set the desired closed-loop dynamics as a first order transfer function:

\[
M(z) = \frac{(1 - \tau)z^{-1}}{1 - \tau z^{-1}}
\] (17)

Also the controller structure is already fixed as a PID controller:

\[
C(z) = \frac{a + bz^{-1} + cz^{-2}}{1 - z^{-1}}
\] (18)

Then it is easy to find that for such particular case, the structure of the IMC controller is given by

\[
Q(z) = \frac{a + bz^{-1} + cz^{-2}}{1 - \tau z^{-1}}
\] (19)

From (19), it is clear that \( Q(z) \) is linear in the parameters and therefore, the optimization problem (11) can be solved using least squares techniques. Also, once the parameters of \( Q(z) \) are computed, both the feedback controller and the instrumental model can be found at once. Then, the robustness test presented above can be applied directly.

### 4. THE PID-IMCVVRT TOOLBOX

The main objective of this paper is to introduce a tool that incorporates the data-driven methodology described above. An snapshot of the interface of the tool is presented in Fig. 4. The components are enumerated in the figure and describe as follows:

1. Selection of the input method for the data. It is possible to select four different input-data sources:
   - a variable from the workspace.
   - a .csv or a .mat file.
   - from a model that is simulated using Simulink.
   - defining the transfer function of the plant in discrete time, and simulated with a PRBS signal.
2. Sampling time of the data. It is supposed that the data is evenly sampled.
3. The name of the matrix that contains the data if the “Workspace data” option is chosen.
4. The name of the .csv or .mat file that contains the data, in case the “File data” option is chosen.
5. The Simulink model that contains the experiment if the “Simulink Model” option is chosen.
6. In case a transfer function is selected as the data input method, the numerator and denominator are introduced using this text fields.
7. Once the input option is selected the “Load Data” button loads the data to the toolbox.
8. Closed-loop selection. The value of \( \tau \) can be selected here to select the constant time of the closed-loop.
9. The “Find Controller” button computes the optimization problem presented in section 3.1 as well as the robustness test.
10. The controller, the data and the closed-loop specification can be exported to the workspace with the “Export Controller” button.
11. The input data is plotted in these two graphics.
12. The parameter of the controller are presented to the user within the same window of the toolbox.
13. The result of the robustness test is presented in this graphs. The first graph shows the approximation of the uncertainty while the second graph shows the frequencies where the condition is violated.

One of the interesting characteristics of the toolbox is that it is possible to load data from a variety of sources. If the data is imported from the workspace, the input and output data in column vector format have to be bounded in a single matrix. The user is also allowed to directly import
data from a .csv or .mat file. In the case of .csv files, the data need to have a similar structure as in the “variable from workspace” case, but it is supposed that the data contains a header of one single line. In the case of the .mat file, the variables $u$ and $y$ have to be defined as the vectors that containing the input and output respectively. All these cases are useful if the data has been taken from a real plant or from other applications outside MATLAB.

The user is also able to define a Simulink model with its own experiment. The only requisite is that the model needs to have no inputs and two outputs: one that contains the input data to the experiment and the other with the corresponding output. It is important that the data is sampled at a constant rate and therefore, precautions has to be taken into account. The last input method is from a used-defined transfer function. In this case, the input data is considered to be a pseudo random binary signal (PRBS) covering all the possible frequencies from zero to the Nyquist frequency. In an interactive way, the user is capable to vary the desired closed-loop response and to see the effect on the controller parameters and the robustness test automatically. Once the controllers are found, the user is able to export the results directly to the workspace for further tests and processing. An standard export windows (Fig. 5) is open when the user clicks on the export button.

Fig. 5. Pop-up window during the export to workspace action.

4.1 Example

A simple example is presented to show the operation of the toolbox. A second degree plant is considered, which transfer function is given by:

$$P(z) = \frac{3.16z^{-1} + 1.907z^{-2}}{1 + 0.01666z^{-1} + 0.2502z^{-2}}$$

In this case, it is not possible to find the ideal controller using a PID parametrization, and therefore certain error is expected in the obtained closed-loop response. The snapshot of the toolbox once the data is loaded and the controller’s parameters are computed is given in Fig. 6. The obtained controller is given by

$$C(z) = \frac{0.1454 - 0.06543z^{-1} + 0.0581z^{-2}}{1 - z^{-1}}$$

The comparison of the step response in closed-loop is given in Fig. 7. As it can be seen, since it is not possible to obtain the ideal controller with a PID structure, the obtained closed-loop response is not exactly the same as the target transfer function. According to the robustness test in Fig. 6 (down right in the user interface) the obtained controller is robust in all the frequency range, since the condition is not violated at any moment. If the user change the closed-loop specification, automatically the controller’s parameters are re-computed and the robustness test is updated. When the closed loop specification is made faster, the uncertainty increases, but it never violates the robust condition.
The toolbox presented is capable of finding the parameters of a discrete time PID controller in an IMC framework directly from data using the virtual reference concept. However, the toolbox is still a work in progress. Some of the functionalities that are in the way of being added are:

- Support for two degrees of freedom PID controllers.
- Support for other controller structures and target closed-loop functions orders.
- In the cases when the input data is found based on a simulation (with Simulink or a transfer function), the capability of testing the controller on closed-loop should be added.
- To improve the interaction between the user and the tool, it is expected to migrate the program from MATLAB to SyQuake. This is also helpful in order to distribute the tool to a wider range of users that may not have a MATLAB license.

5. CONCLUSIONS

A tool for the tuning of discrete time PID controllers was presented. The tuning methodology implemented was the Virtual Reference Feedback Tuning within an “Internal Model Control” like framework. To ensure robustness, a test based on an approximation of the multiplicative uncertainty was proposed and implemented in the tool. This tool is work in progress and is intended to be a useful resource for teaching these new trends in control theory.

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