Inverse Kinematics Analysis and Path Planning for 6DOF RSS Parallel Manipulator

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Abstract—Spatial parallel manipulators have a number of practical applications due to their high reliability, accuracy, and performance. In this paper we deal with an inverse kinematic problem for six-degree-of-freedom (6DOF) parallel manipulator known as modified Stewart platform with rotative-spherical-spherical (RSS) structure. An effective analytic method for solving the inverse kinematic problem for given terminal state is proposed. This method is used for trajectory planning of 6DOF parallel manipulator. Numerical simulation is considered.

Index Terms—path planning, manipulators, parallel robots

I. INTRODUCTION

In the past years, parallel mechanisms have been case of study of many researchers due to their applications in industry research and machinery design. Currently, devices based on mechanisms of parallel kinematics are widely used as positioning devices, manipulators and micromanipulators, vibration stands, simulators, measuring systems, etc. The multidirectional closed kinematic loop of the mechanism leads to a decrease in the dimensions and masses of the moving links. Such devices apply the load like space trusses, which determine their increased accuracy and carrying capacity.

Parallel manipulators have several advantages: better load capacity, high accuracy of positioning of the working element, higher rigidity of the system, high speeds and accelerations of the working element, high degree of unification of mechatronic knots. Spatial precision positioning devices are often based on hexapods or tripods. To overcome the limitations of the serial manipulator many researchers proposed spatial parallel manipulator including Stewart platform. Many kinds of literature are available in the field of spatial parallel manipulator [1], [2].

The hexapod is one form of parallel manipulator that is used increasingly in manufacturing, inspection and research. The ultimate hexapod would provide large motions for massive payloads in up to six degrees of freedom with high accuracy, resolution and repeatability.

Some of the works related to the derivation of the inverse kinematics, forward kinematics, workspace analysis and singularity analysis of the spatial platform are listed in the references [4]–[7]. A number of related papers is dedicated to the inverse kinematics problem for parallel manipulator structures [8]–[10].

Singular point and behavior of the parallel platforms at their neighborhood are investigated in papers [11], [12]. In [11] a method for analysis of numerical location of singular points of in-parallel actuated manipulators has been proposed, and the neighborhood of the singular points has been determined based on the kinematic and static characteristics of in-parallel manipulators. The operating area of in-parallel actuated manipulators has been obtained using transmission index. For details, see [11], [12].

In this paper we propose a simple solution of the inverse kinematic problem for hexapod with rotative-spherical-spherical (RSS) structure. This solution allows to find operating area zone of hexapod and to solve path planning problem. The novelty of proposed method consists of trigonometric function approach. The solution of inverse kinematics is obtained in analytic form. In addition, a problem of multiple choice of angular orientation of actuators is removed.

The rest paper is organized as follows. In Section II description of the platform, necessary notations, and problem statement are considered. Section III establishes the main results: inverse kinematics analysis, operating area zone estimation, and path planning. Conclusive remarks are given in Section IV.

II. PLATFORM DESCRIPTION AND PROBLEM STATEMENT

The mechanism of the parallel structure with rotary drives (modified Stewart platform) has 6 degrees of freedom and consists of six kinematic chains containing one rotative and two spherical pairs (see Fig. 1). Construction pattern of the manipulator is presented in Fig. 2.

Denote input rotative pairs of the section as \( A_i \), where \( i = 1,6 \) is a number of rotative pairs. Output spherical pair of the section are denoted as \( B_i \). Medium spherical pairs are denoted as \( C_i \). Input and output links are assumed to be ideal discs. Now we assign local coordinate systems with input and output sections. Moreover, center of the local coordinate system is assumed to be in the center of the disc. Axes \( o_jx_j \).
hexagons with radiuses $R$ and $r$ respectively. Lengths of the segments $A,C_i$ and $C_iB_i$ are denoted $l_1$ and $l_2$ respectively.

Joint coordinates are defined through angular coordinate of the first joint $\varphi_b$ and angular coordinate of the moving platform $\varphi_{mp}$. We will use the angles between the coupling link connecting the input kinematic pair $A_i$ with the medium spherical pair $C_i$, and the normal to the plane of the platform base as the generalized coordinates of this mechanism. Generalized coordinates are denoted as $\theta_i$, $i = 1,6$.

Inverse kinematic analysis problem for parallel structure manipulator can be formulated in the following way.

**Problem:** for a given position of the center point and angle position of the output section find generalized coordinates of the platform, i.e. angle positions $\theta_i$.

III. MAIN RESULTS

A. Inverse Kinematic Analysis

In order to solve this problem, we write uniform coordinates of the joints in fixed coordinate system as follows:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \end{bmatrix}$$

where

$$a_{1,1} = R \cos \varphi_b, \quad a_{2,1} = R \sin \varphi_b,$$

$$a_{1,2} = R \cos \left(\frac{\pi}{3} - \varphi_b\right), \quad a_{2,2} = R \sin \left(\frac{\pi}{3} - \varphi_b\right),$$

$$a_{1,3} = R \cos \left(\frac{\pi}{3} + \varphi_b\right), \quad a_{2,3} = R \sin \left(\frac{\pi}{3} + \varphi_b\right),$$

$$a_{1,4} = R \cos \left(-\varphi_b\right), \quad a_{2,4} = R \sin \left(-\varphi_b\right),$$

$$a_{1,5} = R \cos \left(\frac{\pi}{3} + \varphi_{mp}\right), \quad a_{2,5} = R \sin \left(\frac{\pi}{3} + \varphi_{mp}\right),$$

$$a_{1,6} = R \cos \varphi_{mp}, \quad a_{2,6} = R \sin \varphi_{mp},$$

and $a_{3,1} = a_{3,2} = a_{3,3} = a_{3,4} = a_{3,5} = a_{3,6} = h_b$.

Uniform coordinates of the moving platform in moving coordinates are equal to

$$\tilde{B} = \begin{bmatrix} \tilde{b}_{1,1} & \tilde{b}_{1,2} & \tilde{b}_{1,3} & \tilde{b}_{1,4} \\ \tilde{b}_{2,1} & \tilde{b}_{2,2} & \tilde{b}_{2,3} & \tilde{b}_{2,4} \\ \tilde{b}_{3,1} & \tilde{b}_{3,2} & \tilde{b}_{3,3} & \tilde{b}_{3,4} \end{bmatrix}$$

where

$$\tilde{b}_{1,1} = R \cos \varphi_{mp}, \quad \tilde{b}_{2,1} = R \sin \varphi_{mp},$$

$$\tilde{b}_{1,2} = R \cos \left(\frac{\pi}{3} - \varphi_{mp}\right), \quad \tilde{b}_{2,2} = R \sin \left(\frac{\pi}{3} - \varphi_{mp}\right),$$

$$\tilde{b}_{1,3} = R \cos \left(\frac{\pi}{3} + \varphi_{mp}\right), \quad \tilde{b}_{2,3} = R \sin \left(\frac{\pi}{3} + \varphi_{mp}\right),$$

$$\tilde{b}_{1,4} = R \cos \left(-\varphi_{mp}\right), \quad \tilde{b}_{2,4} = R \sin \left(-\varphi_{mp}\right),$$

$$\tilde{b}_{1,5} = R \cos \left(\frac{\pi}{3} + \varphi_{mp}\right), \quad \tilde{b}_{2,5} = R \sin \left(\frac{\pi}{3} + \varphi_{mp}\right),$$

$$\tilde{b}_{1,6} = R \cos \varphi_{mp}, \quad \tilde{b}_{2,6} = R \sin \varphi_{mp},$$

and $\tilde{b}_{3,1} = \tilde{b}_{3,2} = \tilde{b}_{3,3} = \tilde{b}_{3,4} = \tilde{b}_{3,5} = \tilde{b}_{3,6} = -h_b$.

Uniform transformation matrix describing the transition between moving and fixed coordinate systems is considered to be known, and equal to

$$T = \begin{bmatrix} d_{1,1} & d_{1,2} & d_{1,3} & x_0 \\ d_{2,1} & d_{2,2} & d_{2,3} & y_0 \\ d_{3,1} & d_{3,2} & d_{3,3} & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $o_jy_j$, $j = 0, 1$ lie on the surface of lower and upper discs respectively. Axes $o_zj$ are orthogonal to these surfaces. Let input and output pairs be located on the vertexes of regular
where \( d_{i,j} \), \( i,j = 1,3 \) are direction cosines of unit normal vector, and \( x_0, y_0, z_0 \) are coordinates of the displacement vector.

Taking into account (2) and (3), we get coordinates of joints of the moving platform as

\[
B = T \bar{B}. \tag{4}
\]

To solve inverse kinematic problem, we consider \( i \)-th kinematic chain of the platform. Let the positive direction of rotation be clockwise direction. Link \( A_iC_i \) rotates in both clockwise and counterclockwise directions. This leads to multiple solutions of the problem. Possible ways of rotation are depicted in Fig. 3. Variants of rotation in clockwise direction corresponds to pictures 3) and 4) in Fig. 3. Variants of rotation in counterclockwise direction corresponds to pictures 1) and 2) in Fig. 3. It is easy to see, that generalized coordinates \( \theta_i \) are defined as

\[
\theta_i = \theta_i' + \alpha_i \tag{5}
\]

or

\[
\theta_i = \theta_i' - \alpha_i \tag{6}
\]

for the cases 1), 4) and 2), 3) respectively.

Angle \( \theta_i' \) of the triangle \( A_iC_iB_i \) can be found from cosine theorem as follows

\[
\theta_i' = \arccos \left( \frac{L_2^2 - L_1^2 - \|A_iB_i\|^2}{-2L_1 \|A_iB_i\|} \right) \tag{7}
\]

where \( \|A_iB_i\|^2 = (b_{i,1} - a_{i,1})^2 + (b_{i,2} - a_{i,2})^2 + (b_{i,3} - a_{i,3})^2 \).

\( \alpha_i \) are defined as

\[
\alpha_i = \arctan \left( \frac{b_{i,2} - \tilde{b}_{i,2}}{b_{i,3}} \right). \tag{8}
\]

Here \( b_{i,j} \) are elements of matrix \( B \) from (4).

**B. Operating area zone analysis**

We consider a 6DOF parallel manipulator with the following parameters (see Table I).

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base radius, ( R )</td>
<td>170 mm</td>
<td></td>
</tr>
<tr>
<td>Moving platform (MP) radius, ( r )</td>
<td>160 mm</td>
<td></td>
</tr>
<tr>
<td>Angle coordinate of first base joint, ( \phi_n )</td>
<td>( \pi/6 ) rad</td>
<td></td>
</tr>
<tr>
<td>Angle coordinate of first MP joint, ( \phi_{mp} )</td>
<td>( \pi/18 ) rad</td>
<td></td>
</tr>
<tr>
<td>Distance from base joints to rotation axis, ( h_n )</td>
<td>45 mm</td>
<td></td>
</tr>
<tr>
<td>Distance from MP joints to rotation axis, ( h_{mp} )</td>
<td>15 mm</td>
<td></td>
</tr>
<tr>
<td>Length of the link, ( l_1 )</td>
<td>68 mm</td>
<td></td>
</tr>
<tr>
<td>Length of the link, ( l_2 )</td>
<td>168 mm</td>
<td></td>
</tr>
</tbody>
</table>

Angle \( \alpha_i \) are defined as

\[
\alpha_i = \arctan \left( \frac{b_{i,2} - \tilde{b}_{i,2}}{b_{i,3}} \right). \tag{8}
\]

Here \( b_{i,j} \) are elements of matrix \( B \) from (4).

**C. Path planning**

Results, obtained above, can be successfully used in path planning problem. In this case the problem is to find a set of \( \theta_i \), \( i = 1, \ldots, 6 \), such that

\[
\theta_i \in \Theta_i. \tag{1}
\]

where \( \Theta_i \) is set of possible joint position and \( \Theta_i \) is set of possible rotation angles. So we have to solve the following inverse kinematic problem

\[
\begin{align*}
\theta_1 & = \alpha_1, \\
\theta_2 & = \alpha_2, \\
\theta_3 & = \alpha_3, \\
\theta_4 & = \alpha_4, \\
\theta_5 & = \alpha_5, \\
\theta_6 & = \alpha_6.
\end{align*} \tag{2}
\]

Here \( \alpha_i \) are defined as

\[
\alpha_i = \arctan \left( \frac{b_{i,2} - \tilde{b}_{i,2}}{b_{i,3}} \right). \tag{8}
\]

Here \( b_{i,j} \) are elements of matrix \( B \) from (4).
angle positions of the rotary drives that allows to follow given
spatial curve given as a set of points. Clearly, given trajectory
should lie inside operating area.

To illustrate effectiveness of the proposed inverse kinematic
problem solution in trajectory planning problem, we consider
the following reference trajectory, describing spatial spiral
curve represented in Fig. 7:

\[
\begin{align*}
x &= 30 \sin 0.4s, \\
y &= 30 \cos 0.4s, \\
z &= 1.5s + 225
\end{align*}
\] (9)

where \( s = \frac{1}{20} \) is a parameter.

Since trajectory (9) lies inside operating area zone, the
solution (7)–(8) satisfy constraints C1–C2. The solution to the
path following problem (9) is presented in Fig. 8.

IV. CONCLUSION

The paper proposes a method for solving the inverse
kinematic problem for the modified Stewart platform with
rotational kinematic pairs (six degrees of freedom) using
matrix algebra. The advantage of this solution is that it can
be generalized to other configurations of the Stewart platform.
It is only necessary to determine the geometric parameters of
the robot, which give us a solution to the problem taking into
account the desired position and orientation.

This method was successfully applied to the estimation of
the operating area zone of the 6DOF parallel manipulator and
path planning problem. Obtained results can be applied in
industrial and human robotics.
REFERENCES


