Blackboard Lecture

15 October 2020

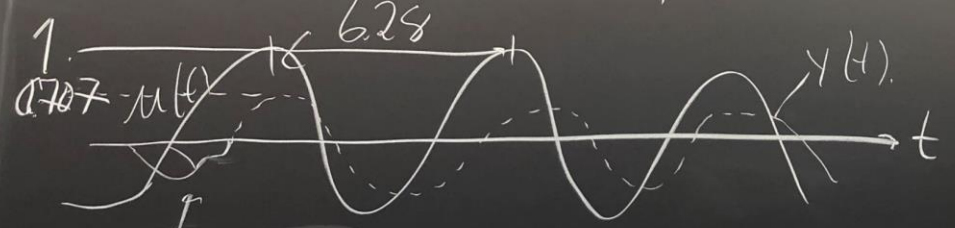
This material is uploaded to compensate for the technical problems with the video recording

$u = u_0 \sin(\omega t)$

$\omega = 1$ rad/s

$\tau = 1$ s

$y(t)$



$\Delta t = 0.78$ s

$$g(j\omega) = \frac{1}{j\underbrace{\omega\tau}_{=1}+1} = \frac{1}{1+j}$$

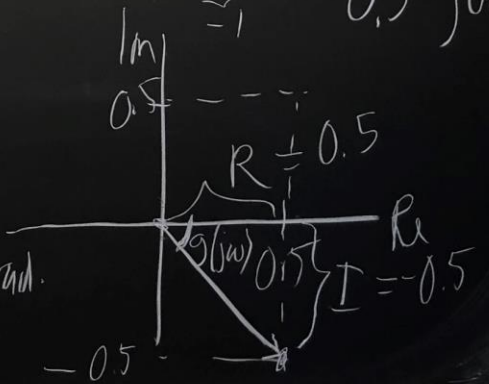
Two ways.

$$g(j\omega) = \frac{1-j}{\underbrace{(1+j)(1-j)}_{1-\underbrace{j^2}_{=1}}} = \frac{1-j}{2} = 0.5 - j0.5$$

$$AR = |g(j\omega)| = \sqrt{0.5^2+0.5^2} = \sqrt{\frac{1}{2}} = 0.707$$

$$\angle g(j\omega) = \arctan\left(\frac{I}{R}\right) = \arctan(-1) = -45^\circ = -\frac{\pi}{4} \text{ rad.}$$

$$\Delta t = \frac{-\varphi}{\omega} = \frac{\pi/4 \text{ rad}}{1 \text{ rad/s}} = \frac{\pi}{4} \text{ s} = 0.78 \text{ s}$$



NB!

$$G(s) = \frac{K}{\tau s + 1}$$

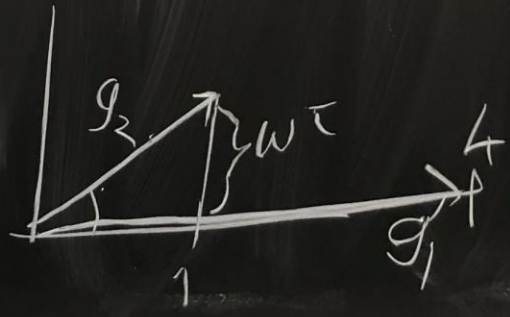
$$G(j\omega) = \frac{K}{j\omega\tau + 1} = \frac{g_1}{g_2}$$

$$AR = |G(j\omega)| = \frac{|g_1|}{|g_2|} = \frac{K}{\sqrt{(\omega\tau)^2 + 1}}$$

$$\varphi = -\omega \cdot \Delta t$$

rad rad/s s

$$g_2 = 1 + j\omega\tau$$

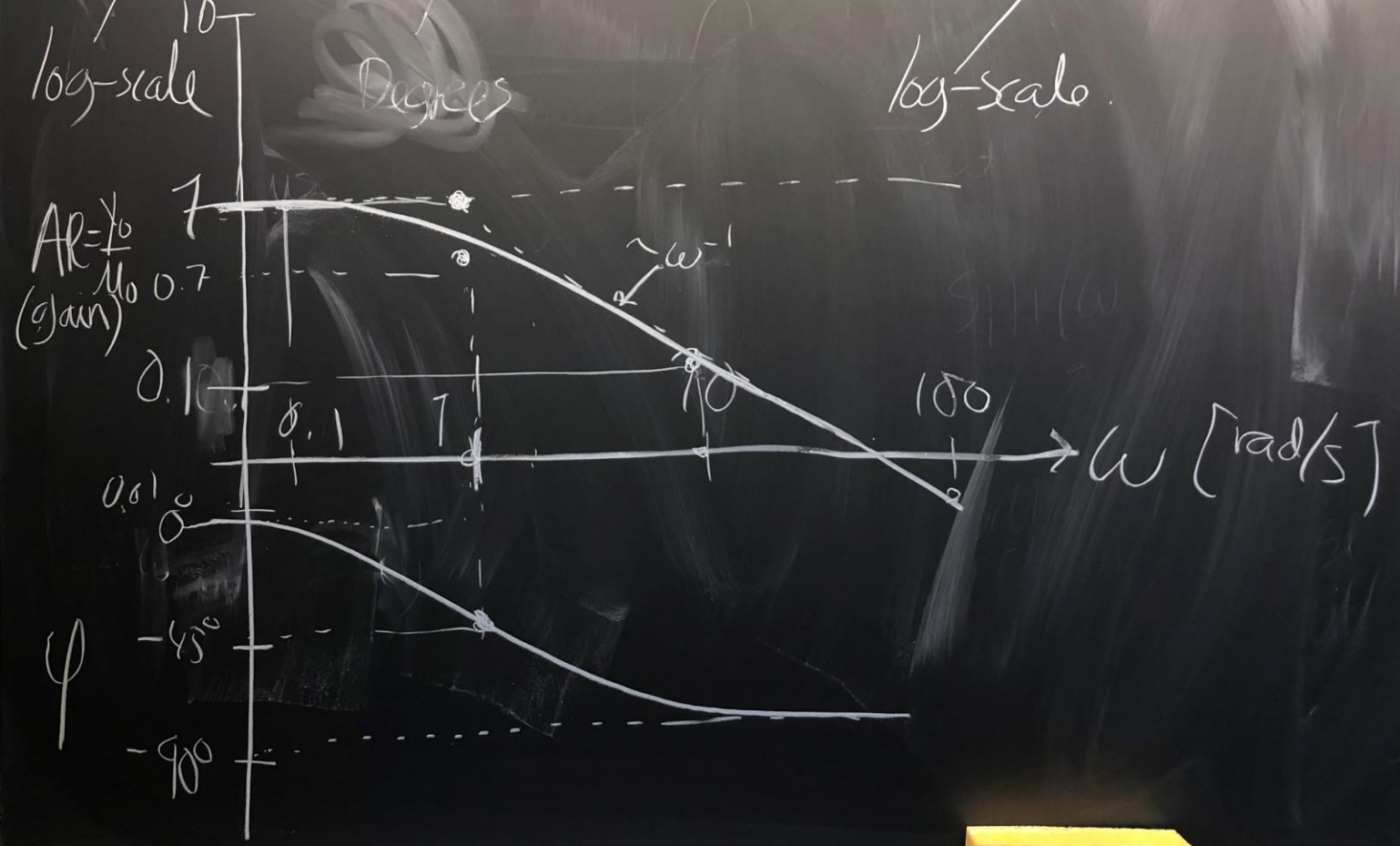


$$\varphi = \angle g(j\omega) = \angle g_1 - \angle g_2 = 0^\circ - \arctan\left(\frac{\omega\tau}{1}\right)$$

$$= -\arctan(\omega\tau)$$

Frequency ↓

$\omega\tau$	$\lvert g(j\omega)\rvert$	$\angle g(j\omega)$
0	k	0°
0.1	$k/\sqrt{1.01}$	$-\arctan(0.1) = -5.7^\circ = -0.01$
1	$k/\sqrt{2}$	$-\arctan(1) = -0.78\,\text{rad} = -45^\circ$
2	$k/\sqrt{5}$	
10	$\frac{k}{\sqrt{101}} \approx \frac{k}{10}$	$-\arctan(10) = -84.3^\circ$

Bode plot.

AR and φ as function of ω