2003/05-001: Dynamics / Dynamikk

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English:

Given the transfer function

$$g(s) := \frac{(-1\,s+1)\,e^{-1\,s}}{(20\,s+1)\,(4\,s+1)}$$

(a) (5 points) List zeros, poles and steady-state gain as well as delay.

(b) (3 points) Is this system stable?

(c) (3 points) What is special about this system?

(d) (4 points) Sketch a unit step response for this system.

(e) (5 points) Design a PI controller using the SIMC rules.

(f) (5 points) Show in terms of asymptotes the behaviour of the system with the controller attached in open loop (product of process and controller transfer function) in a Bode plot, that is the amplitude and the phase plot.

Norsk:

Gitt transferfunksjonen

$$g(s) \quad := \quad \frac{(-1\,s+1)\,e^{-1\,s}}{(20\,s+1)\,(4\,s+1)}\,.$$

(a) (5 points) Finn nullpunkter, poler, stasjonær forsterkning og dødtid.

(b) (3 points) Er systemet stabilt?

(c) (3 points) Hva er spesielt med dette systemet?

(d) (4 points) Skisser systemets sprangrespons.

(e) (5 points) Lag en PI regulator ved hjelp av SIMC regler.

(f) (5 points) Vis oppførselen til systemet med regulatoren tilknyttet i åpen sløyfe (produkt av prosess- og regulatortransferfunksjonen) ved hjelp av asymptoter i et Bode-diagram som viser forsterkningen og fasen.

Suggested Solution – Løsningsforslag:

The transfer function is given in the standard form, that is with time-constants. Transforming it into the zero-pole representation is straightforward:

$$\begin{aligned} g(s) &:= \frac{(-1\,s+1)\,e^{-1\,s}}{(20\,s+1)\,(4\,s+1)}, \\ &:= \left(\frac{-1}{20\cdot 4}\right)\frac{(s-1)\,e^{-1\,s}}{(s+1/20)\,(s+1/4)}. \end{aligned}$$

(a) steady state gain, zeros, poles, and delay

steady state gain :	1
zeros	+1
poles	-1/20, -1/4
delay	1

(b) stability: yes system is stable; all poles are in the left-half plain

(c) what's special: right-half-plain zero: non-minimal phase behaviour

(d) sketch step response:

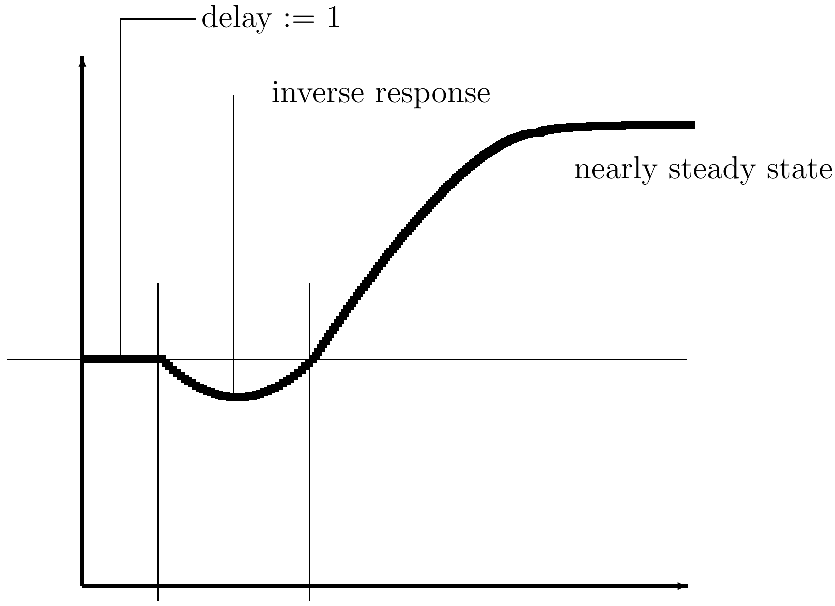

Figure 1: *Step response*

(e) PI controller using SIMC The model is of second order and has a non-minimal phase zero. For the PI controller we need to reduce to a first order model. The time constant takes the dominant one plus half of the neglected one. The delay is made up of the original delay, the other half of the neglected pole and the positive zero and the gain is then simple to calculate:

$$\begin{aligned} \tau_1 &:= 20 + 4/2 &&:= 22\,, \\ \theta_1 &:= 1 + 4/2 + 1 &&:= 4\,. \end{aligned}$$

$$K_C := \frac{0.5\,\tau_1}{k\,\theta_1} := \frac{0.5\cdot 22}{1\cdot 4} := \frac{11}{4}\,,$$
$$\tau_I := \min\,(22, 8\cdot 4) := 22\,,$$

(f) Open loop transfer function g_{ol}:

$$g_{ol}(s) := \frac{(-1\,s+1)\,e^{-1\,s}}{(20\,s+1)\,(4\,s+1)}\,\frac{11}{4}\,\frac{22\,s+1}{22\,s}\,,$$
$$:= \frac{-0.034\,(s-1)\,(s+0.045)\,e^{-1\,s}}{s\,(s+0.05)\,(s+0.25)}\,,$$

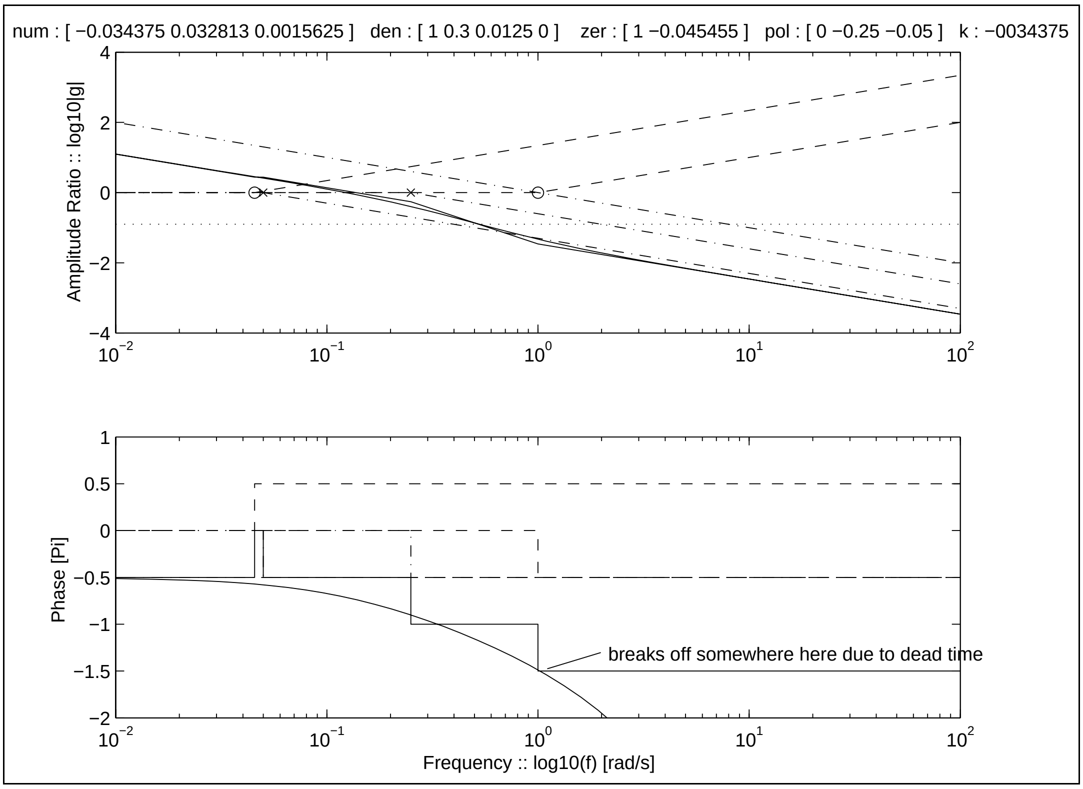

Figure 2: *Approximate Bode Plot*

2003/05-003: Basic Skills / Grunnleggende ferdigheter

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English:

Answer the following questions precisely, but briefly.

(a) (7 points) Show a block diagram of the following transfer function expanded to the primitive blocks $g_d(s)$, $g_p(s)$ and $g_c(s)$

$$y(s) := g_d(s)\, d(s) + \frac{g_p(s)\, g_c(s)}{1 + g_p(s)\, g_c(s)}\, u(s)$$

(b) (7 points) Calculate the transfer function g_1 from y_s to y and the transfer function g_2 from y_s to z for the system shown in the block diagram 1.

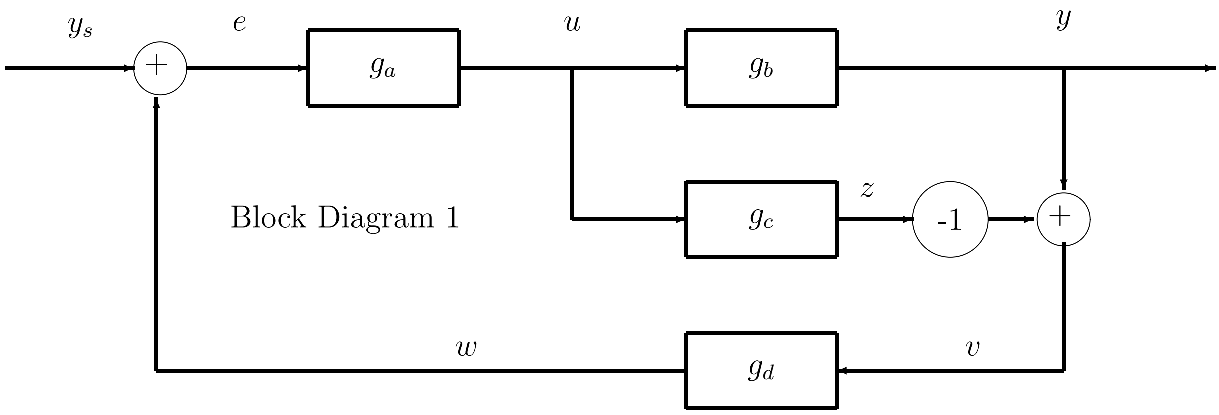

Figure 1: *Block diagram*

(c) (6 points) Compute the steady-state RGA of the following 2x2 system and argue pairing

$$\underline{\underline{\mathbf{G}}}(s) := \begin{bmatrix} \frac{12}{20s+1} & \frac{2}{10s+1} \\ \frac{5(s+1)}{(2s+1)(10s+1)} & \frac{2}{(3s+1)(10s+1)} \end{bmatrix}.$$

(d) (5 points) Show in the Bode plot (attachment 1) the gain margin and phase margin and do not forget to turn it in at the end.

Norsk:

Svar kort og presist på følgende spørsmål:

(a) (7 points) Vis et blokkdiagram med de elementære blokkene $g_d(s)$, $g_p(s)$ og $g_c(s)$ for følgende transferfunksjon:

$$y(s) := g_d(s)\,d(s) + \frac{g_p(s)\,g_c(s)}{1 + g_p(s)\,g_c(s)}\,u(s)$$

(b) (7 points) Beregn transferfunksjonen g_1 fra y_s til y og transferfunksjonen g_2 fra y_s til z for systemet vist i blokkdiagram 1.

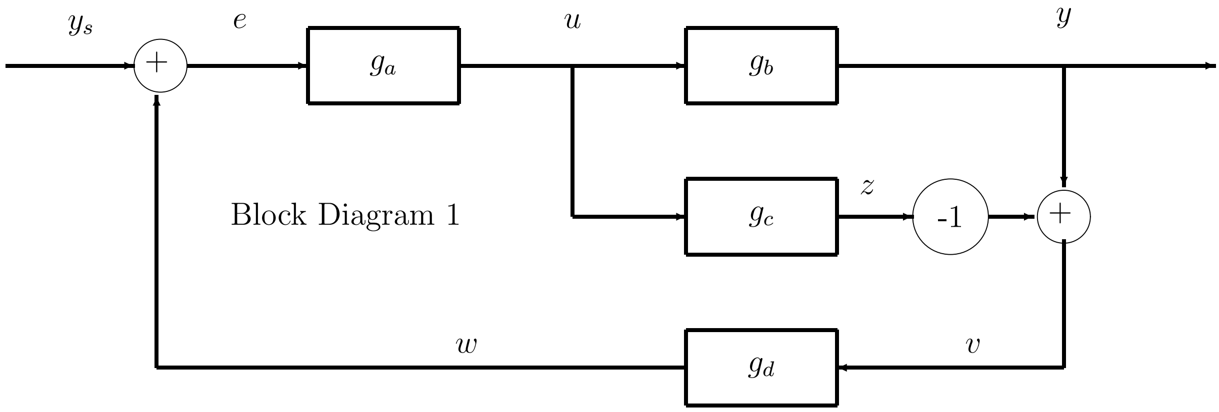

Figure 2: *Block diagram*

(c) (6 points) Beregn stasjonær RGA for følgende 2x2 system. Diskuter parring.

$$\underline{\underline{\mathbf{G}}}(s) := \begin{bmatrix} \frac{12}{20s+1} & \frac{2}{10s+1} \\ \frac{5(s+1)}{(2s+1)(10s+1)} & \frac{2}{(3s+1)(10s+1)} \end{bmatrix}.$$

(d) (5 points) Vis forsterkningsmarginen og fasemarginen i Bode-plottet (vedlegg 1). Ikke glem å levere inn plottet!

Suggested Solution – Løsningsforslag:

(a) Transfer function to block diagram:

Firstly observe that the transfer function has two parts, one describing the effect of the disturbance d and the second describing the effect of the controlled input u. The two effects are added, thus run in parallel. Second, one observes that the transfer of the controlled input is a straightforward feedback loop. Thus

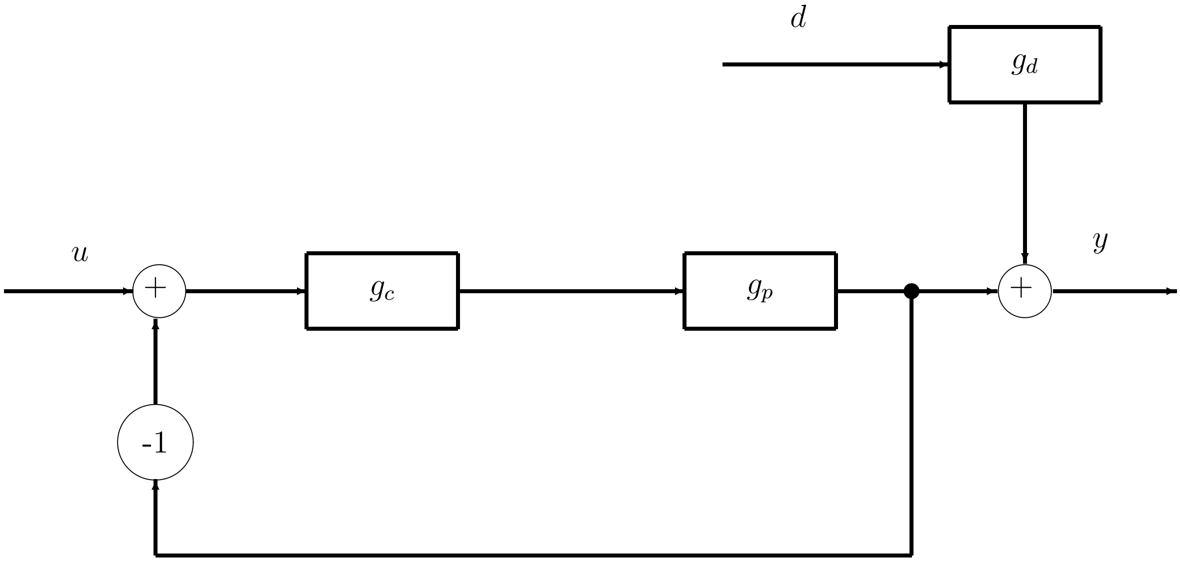

Figure 3: *Block diagram*

(b) Block diagram to transfer function: Put the equations down and then eliminate step by step.

$$
\begin{aligned}
y &:= g_b\, u \\
u &:= g_a\, e \\
e &:= y_s + w \\
w &:= g_d\, v \\
v &:= y - z \\
z &:= g_c\, u\,.
\end{aligned}
$$

First the transfer function from $y_s \Rightarrow y$:

$$\begin{aligned}
z &:= g_c\, u \\
v &:= y - g_c\, u \\
w &:= g_d\,(y - g_c\, u) \\
e &:= y_s + g_d\,(y - g_c\, u) \\
u &:= g_a\,(y_s + g_d\,(y - g_c\, u)) \\
&:= g_a\, y_s + g_a\, g_d\, y - g_a\, g_d\, g_c\, u \\
(1 - g_a\, g_d\, g_c)\, u &:= g_a\, y_s + g_a\, g_d\, y \\
g_S &:= (1 + g_a\, g_d\, g_c)^{-1} \\
u &:= g_S\, g_a\,(y_s + g_d\, y) \\
y &:= g_b\, g_S\, g_a\,(y_s + g_d\, y) \\
&:= g_b\, g_S\, g_a\, y_s + g_b\, g_S\, g_a\, g_d\, y \\
(1 - g_b\, g_S\, g_a\, g_d)\; y &:= g_b\, g_S\, g_a\, y_s \\
y &:= (1 - g_b\, g_S\, g_a\, g_d)^{-1}\; g_b\, g_S\, g_a\, y_s\,.
\end{aligned}$$

This can be simplified to

$$y \;:=\; g_L\, gb\, gay_s\,.$$

with g_L as defined in the next block of equations.

Next the transfer function from $y_s \Rightarrow z$:

$$\begin{aligned}
u &:= g_a\, e \\
&:= g_a\,(y_s + w) \\
&:= g_a\,(y_s + g_d\, v) \\
&:= g_a\,(y_s + g_d\,(y - z)) \\
&:= g_a\,(y_s + g_d\,(g_b\, u - g_c\, u)) \\
&:= g_a\, y_s + g_a\, g_d\,(g_b - g_c)\, u \\
(1 - g_a\, g_d\,(g_b - g_c))\; u &:= g_a\, y_s \\
g_L &:= (1 - g_a\, g_d\,(g_b - g_c))^{-1} \\
u &:= g_L g_a\, y_s \\
z &:= g_c\, u \\
z &:= g_c\, g_L g_a\, y_s\,.
\end{aligned}$$

(c) RGA

$$\begin{aligned}
\underline{\underline{\mathbf{G}}}(s) &:= \begin{bmatrix} \frac{12}{20s+1} & \frac{2}{10s+1} \\ \frac{5(s+1)}{(2s+1)(10s+1)} & \frac{2}{(3s+1)(10s+1)} \end{bmatrix} \\
\underline{\underline{\mathbf{G}}}(0) &:= \begin{bmatrix} 12 & 2 \\ 5 & 2 \end{bmatrix}.
\end{aligned}$$

$$\underline{\underline{\mathbf{\Lambda}}} := \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}$$

$$\lambda := \frac{1}{1-\frac{k_{12}k_{21}}{k_{11}k_{22}}} := \frac{1}{1-\frac{2\cdot 5}{12\cdot 2}} := \frac{12}{7} := 1.7$$

$$\underline{\underline{\mathbf{\Lambda}}} := \begin{bmatrix} 1.7 & -0.7 \\ -0.7 & 1.7 \end{bmatrix}.$$

The only feasible pairing is 1 - 1 and 2 - 2 as the other elements of the RGA matrix are negative.

(d) Stability margins

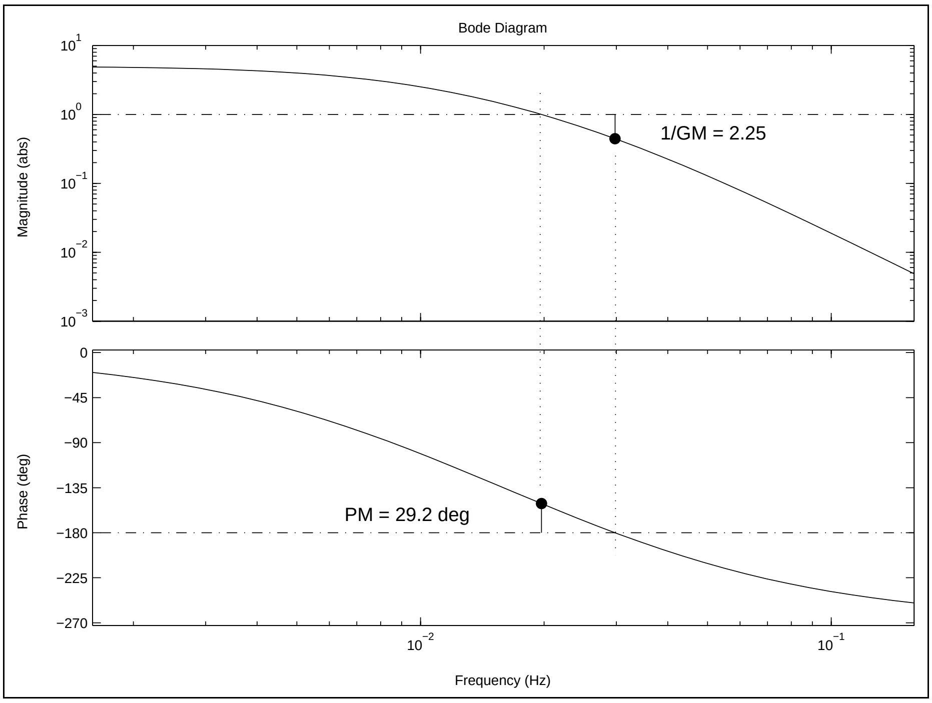

Figure 4: *Stability margins*

2003/05-004: Controllability / Regulerbarhet

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English:

Given the plant

$$y(s) \quad := \quad \frac{200}{(10s+1)(50s+1)}\, u(s) + \frac{30}{(10s+1)}\, d(s)$$

with maximum variations of the input $\Delta u := \pm 5$, maximum variation of the disturbance $\Delta d := \pm 6$ and a maximum allowed variation in the output $\Delta y := \pm 2$, discuss controllability.

(a) (8 points) Norm all signals to ± 1.

(b) (8 points) Show plant behaviour in approximate Bode plot.

(c) (5 points) Problem with input saturation?

(d) (4 points) Can you design a controller that meets the performance conditions? Do not forget to argue the case!

Norsk:

Gitt prosessen

$$y(s) \quad := \quad \frac{200}{(10s+1)(50s+1)}\, u(s) + \frac{30}{(10s+1)}\, d(s)$$

med maksimal variasjon i pådrag $\Delta u := \pm 5$, maksimum forventet variasjon i forstyrrelser $\Delta d := \pm 6$ og maksimum tillatt variasjon i utgangene $\Delta y := \pm 2$, diskuter regulerbarheten.

(a) (8 points) Normaliser alle signaler til ± 1.

(b) (8 points) Vis prosessens oppførsel i et tilnærmet Bode-plott.

(c) (5 points) Er det problemer med pådragsmetning?

(d) (4 points) Kan du lage en regulator som oppfyller kravene til ytelse? Ikke glem begrunnelse!

Suggested Solution – Løsningsforslag:

(a) Scaling

$$y(s) := \frac{200}{(10s+1)(50s+1)}\,u(s) + \frac{30}{(10s+1)}\,d(s) \quad (1)$$

$$\tilde{y}(s) := \frac{y}{y_{max}} \qquad \tilde{u}(s) := \frac{u}{u_{max}} \qquad \tilde{d}(s) := \frac{d}{d_{max}} \quad (2)$$

$$\tilde{y}(s) := \frac{u_{max}}{y_{max}}\left(\frac{200}{(10s+1)(50s+1)}\right)\tilde{u}(s) + \frac{d_{max}}{y_{max}}\left(\frac{30}{(10s+1)}\right)\tilde{d}(s) \quad (3)$$

$$:= \frac{5}{2}\left(\frac{200}{(10s+1)(50s+1)}\right)\tilde{u}(s) + \frac{6}{2}\left(\frac{30}{(10s+1)}\right)\tilde{d}(s) \quad (4)$$

$$:= \frac{500}{(10s+1)(50s+1)}\,\tilde{u}(s) + \frac{90}{(10s+1)}\,\tilde{d}(s) \quad (5)$$

$$:= g_p\,\tilde{u}(s) + g_d\,\tilde{d}(s)\ . \quad (6)$$

(b)

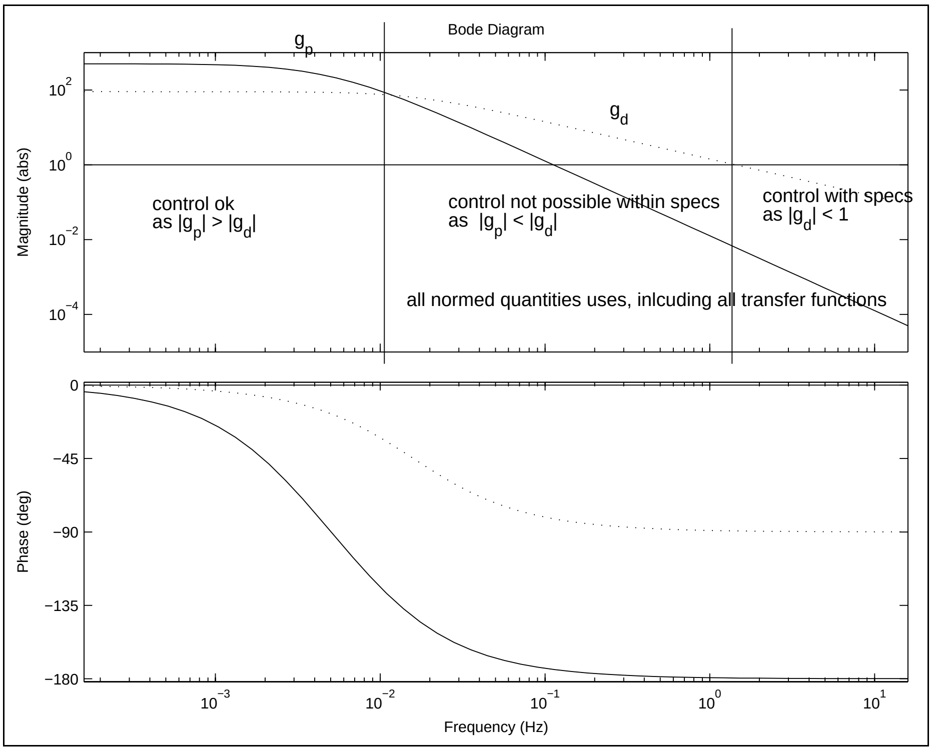

Figure 1: *Bode Plot*

(c) yes there are problems: see Bode plot

(d) No the controller will have problems to meet specs in the middle frequency range as indicated in the Bode plot. One has to either change the performance requirements, thus change increase the acceptance range of the output, or increase the gain of the normed process by either adjusting the gain of the original process or increase the input range, or, as a last possibility, add a low-pass filter to the disturbance dynamics, which could be done by adding a capacity into that stream. Latter would be an additional tank if the disturbance is a mass flow.