NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
INSTITUTT FOR KJEMISK PROSESSTEKNOLOGI

Faglig kontakt under eksamen :
Sigurd Skogestad (94154)

EKSAMEN I FAG 52041 – PROSESSREGULERING
Onsdag 24. november 1999
Tid: kl. 0900 - 1400

Hjelpemidler: Typegodkjent kalkulator med tomt minne. Godkjent matematisk formelsamling. (Ingen andre trykte eller håndskrevne hjelpemidler tillatt.)
Hjelpemedel: Typegodkjend kalkulator med tomt minne. Godkjend matematisk formelsamling. (Ingen andre trykte eller handskrevne hjelpemedel tillate.)

For alle oppgavene gjelder: Angi klart ytterligere antagelser du gjør. Begrunn alle svar.
For alle oppgåvene gjeld: Skriv klart ytterlegare antakelsar du gjer. Grunngjev alle svar.

Oppgave 1 / Oppgåve 1.

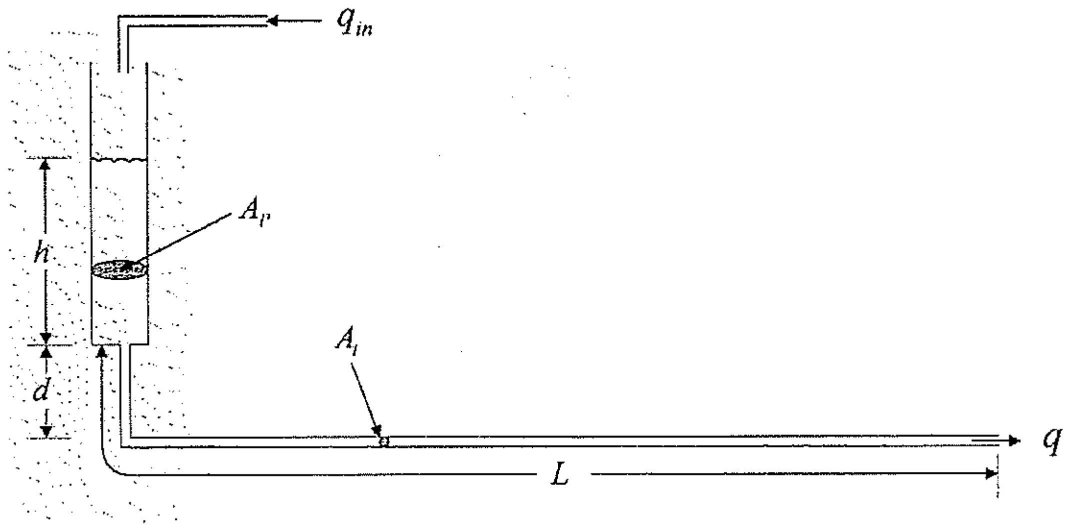


Figure 1: Utstrømning fra tank gjennom langt rør

Vann strømmer inn i en sylinderisk tank og ut gjennom et langt rør. Utstrømningen fra tanken (gjennom røret) kan beskrives ved ligningen

$$\frac{dq}{dt} = \frac{A_t g}{L}(d+h) - f\frac{q^2}{2LA_t} \tag{1}$$

der f er en friksjonskoeffisient. Leddet på venstre side, dq/dt, tar hensyn til at noe kraft trenges for å akselrere væsken i røret. (Merk: Ved stasjonær tilstand er $dq/dt = 0$ og (1) kan omformes til

$$q = A_t\sqrt{2g/f} \cdot \sqrt{d+h} \tag{2}$$

som er uttrykket for turbulent utstrømning som vi har benyttet tidligere i faget.)

(a) Formuler massebalansen for systemet og utled et uttrykk for dh/dt.

(b) Lineariser modellen, og innfør avviksvariable for h, q og q_{in}.

(c) Utled transferfunksjonen $h(s)/q_{in}(s)$ (ingen tallverdier her).

(d) Innfør tallverdier og skisser responsen for h til et sprang i q_{in}.

(e) Hvordan kan du endre designen av prosessen for å unngå svingninger?

Parameters and steady-state data. $A_p = 4.56 \cdot 10^{-3}$ m^2; $A_t = 1.14 \cdot 10^{-3}$ m^2; $d = 0.25$ m; $L = 9$ m; $f = 5.2$; $g = 9.8$ m/s^2; $h = 0.1$ m; $q = 1.309 \cdot 10^{-3}$m^3/s.

English text. Water flows into a cylinderical tank and exits through a long tube. The outflow from the tank (through the tube) can be described by equation (1), where f is a coeffisient of friction. The term on the left of (1), dq/dt, accounts for force needed to accelerate the fluid in the tube. (Note: At steady state (1) can be rearranged to give (2) which is the expression for turbulent outflow that we have used previously in this course). (a) Formulate the mass balance for the system and derive an equation for dh/dt. (b) Lineariize the model, and introduce deviation variables for h, q og q_{in}. (c) Derive the transfer function $h(s)/q_{in}(s)$ (No numerical values here). (d) Introduce parameter values and sketch the expected response for h to a step change in q_{in}. (e) How would you change the design of the process to avoid the oscillations?

Oppgave 2 / Oppgåve 2.

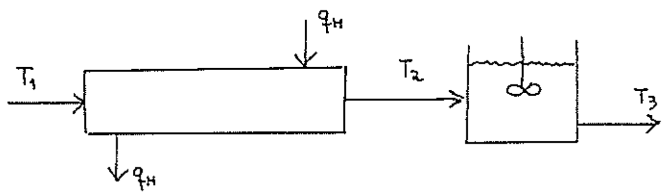


Figure 2: Varmeveksler og tank

En prosess-strøm varmeveksles med en varm strøm q_H (til temperaturen T_2) og sendes til mellomlagring der vi ønsker å holde temperaturen T_3 konstant. Responsen i T_2 til en endring i q_H har tidskonstant 2.5 min, dødtid 0.5 min og forsterkning 50K/ kg/s. Reponsen i T_3 til en endring i T_2 har tidskonstant 10 min (oppholdstiden i tanken), dødtid 0.2 min og forsterkning 1 K/K. Alle temperaturmålinger har en tidskonstant på 0.2 min.

(a) Tegn et blokkdiagram for prosessen med transferfunksjoner inntegnet.

(b) q_H (pådrag) skal brukes til å regulere T_3. Foreslå innstillinger for en PI-regulator.

(c) Foreslå istedet et reguleringsopplegg basert på kaskaderegulering (tegn inn på et flytskjema tilsvarende Fig.2). Trenges integralvirkning for begge regulatorene? Foreslå innstillinger for to PI-regulatorer.

(d) Diskuter forventet forskjell i respons for de to tilfellene i (b) og (c) (hva blir anslagvis lukket sløyfes responstid?)

English text. A process stream exchanges heat with a hot stream q_H (so it reaches temperature T_2) and is send to a buffer tank in which we want the temperature T_3 to be constant. The response in T_2 to a change in q_H has time constant 2.5 min, delay 0.5 min og gain 50K/ kg/s. The response in T_3 to a change in T_2 has time constant 10 min, delay 0.2 min og gain 1 K/K. All temperature measurements have a time constant of 0. 2 min. (a) Make a block diagram for the process and put the transfer functions in the blocks. (b) q_H (manipulated input) shall be used to control T_3. Suggest tunings for a PI-controller. (c) Suggest instead a control scheme based on cascade control (show on a flowsheet of the kind in Fig.2). Is integral action needed in both controllers? Suggest tunings for two PI-controllers. (d) Discuss how you expect the responses to differ in cases (b) and (c) (what is the approximate closed-loop time constant?)

Oppgave 3/ Oppgåve 3.

For en prosess er følgende transferfunksjoner bestemt

$$T(s) = 20\frac{(0.1s+1)e^{-1.6s}}{(6s+1)(2s+1)(0.4s+1)}q_1(s) + 6\frac{e^{-4.1s}}{6s+1}q_2(s) \tag{3}$$

Her er T [K] temperaturen som ønskes holdt konstant med maksimum avvik ± 0.5 K, q_1 [kg/s] er pådraget og q_2 [kg/s] er den viktigste forstyrrelsen. Pådraget q_1 kan maksimalt endres med ± 0.7 kg/s, og forventet maksimal endring i q_2 er ± 2 kg/s. Er prosessen regulerbar? Vil det kunne hjelpe å bruke foroverkobling?

English text. The transfer function model in (3) has been derived for a process. Here T [K] is the temperature which should be held constant with a maximum deviationi of ± 0.5 K, q_1 [kg/s] is the input and q_2 [kg/s] is the main disturbance. The maximum change allowed in the manipulated input q_1 is ± 0.7 kg/s, og expected maximum change in q_2 is ± 2 kg/s. Is the process controllable? Would there be any advantage in using feedforward control?

Oppgave 4/ Oppgåve 4.

Betrakt en integrerende prosess

$$g(s) = k'\frac{e^{-\theta s}}{s} \tag{4}$$

(for å forenkle beregningene kan du om ønskelig sette $\theta = 1$ i det videre).

(a) Utled PI-innstillinger for denne prosessen med (i) Skogestads IMC-regler, og (ii) Ziegler-Nichols regler.

(b) Bestem for de to alternativene frekvensen ω_{cg} der $|L| = 1$ Hva er lukket sløyfes responstid (den er tilnærmet lik $1/\omega_{cg}$)?

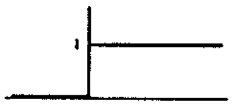
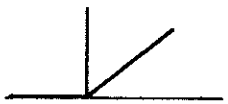
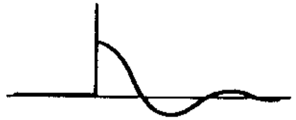
(c) Bestem for de to alternativene forsterkningsmargin (GM), fasemargin (PM) og maksimal tillatt ekstra dødtid (i prosent av θ).

(d) Kommenter resultatene (hvilket alternativ vil du foretrekke i hvilket tilfelle?).

English text. Consider the integrating process in (4) (to simplify the calculations you may set $\theta = 1$ in the following). (a) Derive PI-tunings for this process with (i) Skogestad's IMC-rules and (ii) Ziegler-Nichols' rules. (b) Find for the two alternatives the frequency ω_{cg} where $|L| = 1$ What is the closed-loop response time (it is approximately $1/\omega_{cg}$)? (c) Find for the two alternatives the gain margin (GM), phase margin (PM) og maximum allowed extra delay (in precent of θ). (d) Comment on your results (which alternativ would you prefer in which case)?

Formelverk i prosessregulering

Laplace transformasjoner

Time function ($t \geq 0$)	Laplace transform	
Unit impulse, $\delta(t_0)$	1	
Unit pulse, $\delta_A(t)$	$\frac{1}{A}\frac{1-e^{-sA}}{s}$	
Unit step	$\frac{1}{s}$	
Ramp, $f(t) = t$	$\frac{1}{s^2}$	
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	
$e^{-at}\cos(\omega t)$	$\frac{(s+a)}{(s+a)^2+\omega^2}$	
e^{-at}	$\frac{1}{s+a}$	

Invers Laplace

TABLE
INVERSE LAPLACE TRANSFORMS OF SELECTED EXPRESSIONS

$\frac{1}{\tau}e^{-t/\tau}$	$\frac{1}{\tau s+1}$
$1-e^{-t/\tau}$	$\frac{1}{s(\tau s+1)}$
$1+\frac{1}{\tau_2-\tau_1}(\tau_1 e^{-t/\tau_1}-\tau_2 e^{-t/\tau_2})$ $(\tau_1 \neq \tau_2)$	$\frac{1}{s(\tau_1 s+1)(\tau_2 s+1)}$
$1+\frac{\tau_3-\tau_1}{\tau_1-\tau_2}e^{-t/\tau_1}+\frac{\tau_3-\tau_2}{\tau_2-\tau_1}e^{-t/\tau_2}$ $(\tau_1 \neq \tau_2)$	$\frac{\tau_3 s+1}{s(\tau_1 s+1)(\tau_2 s+1)}$
$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta t/\tau}\sin[\sqrt{1-\zeta^2}\,t/\tau+\psi]$ $\psi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 \leq \lvert\zeta\rvert < 1)$	$\frac{1}{s(\tau^2 s^2+2\zeta\tau s+1)}$
$\frac{b}{a}(1-e^{-at})$	$\frac{b}{s(s+a)}$

Modellbasert design av regulator (Direkte syntese; Dahlin)

1. Faktoriser $g(s) = g_-(s)g_+(s)$ hvor $g_-(s)$ kan og $g_+(s)$ kan ikke inverteres.
2. Velg $g_+(0) = 1$ for å få integralvirkning.
3. Spesifiser ønsket setpunktsrespons. For 1.ordens respons med tidskonstant τ_c sett:

$$\frac{gc}{1+gc} = \frac{1}{\tau_c s+1}g_+(s)$$

4. Løsning av ligningen over gir:

$$c(s) = \frac{g_-^{-1}}{\tau_c s+1-g_+}$$

Kommentar: IMC-metoden (Internal Model Control) gir samme resultat.

Invertering av matrise.

$$A^{-1} = \frac{\mathrm{adj}A}{\det A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} = \frac{1}{a_{11}a_{22}-a_{12}a_{21}}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

RGA-analyse.

$$RGA(G) = \Lambda = G \times (G^{-1})^T$$

hvor $\times$ angir element-for-element multiplikasjon. For en 2×2-matrise fås:

$$G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}$$

$$RGA(G) = \begin{pmatrix} \lambda_{11} & 1-\lambda_{11} \\ 1-\lambda_{11} & \lambda_{11} \end{pmatrix}; \quad \lambda_{11} = \left(1-\frac{g_{12}g_{21}}{g_{11}g_{22}}\right)^{-1}$$

LAPLACE TRANSFORM OPERATIONS AND PAIRS

OPERATIONS

No.	Name	$f(t)$	$F(s)$
1	Linearity Multiplication by a constant Addition	 $Kf(t)$ $f_1(t)+f_2(t)$	 $KF(s)$ $F_1(s)+F_2(s)$
2	Differentiation	$\frac{d}{dt}f(t)$	$sF(s)-f(0^+)$
3	Integration	$\int f(t)\,dt$	$\frac{F(s)}{s}+\frac{f^{(-1)}(0^+)}{s}$
4	Delayed function	$f(t-a)$ where $f(t-a)=0$ for $0<t<a$	$e^{-as}F(s)$ where $F(s)=\mathcal{L}f(t)$
5	Time scale change	$f(at)$	$\frac{F(s/a)}{a}$
6	Initial value	$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$	
7	Final value	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$	

Routh–Hurwitz stabilitetskriterie

Expand the characteristic equation into the following polynomial form:

$$1 + G_p G_f G_c G_m = a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n = 0$$

Let a_0 be positive. If it is negative, multiply both sides of the equation above by −1.

First test. If any of the coefficients $a_1, a_2, \ldots, a_{n-1}, a_n$ is negative, there is at least one root of the characteristic equation which has positive real part and the corresponding system is unstable. No further analysis is needed.

Second test. If all coefficients $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$ are positive, und n>3 then from the first test we cannot conclude anything about the location of the roots. Form the following array (known as the Routh array):

Row					
1	a_0	a_2	a_4	a_6	...
2	a_1	a_3	a_5	a_7	...
3	A_1	A_2	A_3	.	...
4	B_1	B_2	B_3	.	...
5	C_1	C_2	C_3	.	...
.	. . .	. . .	. . .		
$n+1$	W_1	W_2	.	.	...

where

$$A_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \quad A_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \quad A_3 = \frac{a_1 a_6 - a_0 a_7}{a_1} \quad \ldots$$

$$B_1 = \frac{A_1 a_3 - a_1 A_2}{A_1} \quad B_2 = \frac{A_1 a_5 - a_1 A_3}{A_1} \ldots$$

$$C_1 = \frac{B_1 A_2 - A_1 B_2}{B_1} \quad C_2 = \frac{B_1 A_3 - A_1 B_3}{B_1} \ldots$$

etc.

Examine the elements of the first column of the array above:

$$a_0 \quad a_1 \quad A_1 \quad B_1 \quad C_1 \quad \cdots \quad W_1$$

(a) If any of these elements is negative, we have at least one root to the right of the imaginary axis and the system is unstable.
(b) The number of sign changes in the elements of the first column is equal to the number of roots to the right of the imaginary axis.

Therefore, a system is stable if all the elements in the first column of the Routh array are positive.

Frekvensrespons (eksakt)

$$\tilde{h}(s) = K \frac{(1+T_a s)(1+T_b s)\ldots(1+T_k s)}{(1+T_1 s)(1+T_2 s)\ldots(1+T_n s)} e^{-\theta s} \cdot \frac{1}{s}$$

finner vi amplitudeforholdet

$$|\tilde{h}(j\omega)| = K \sqrt{\frac{(1+(\omega T_a)^2)(1+(\omega T_b)^2)\ldots(1+(\omega T_k))^2}{(1+(\omega T_1)^2)(1+(\omega T_2)^2)\ldots(1+(\omega T_n))^2}} \cdot 1 \cdot \frac{1}{\omega}$$

og fasevinkelen

$$\angle\tilde{h}(j\omega) = \operatorname{arctg}\omega T_a + \operatorname{arctg}\omega T_b + \ldots + \operatorname{arctg}\omega T_k - \operatorname{arctg}\omega T_1 - \operatorname{arctg}\omega T_2 - \ldots - \operatorname{arctg}\omega T_n - \omega\theta - \frac{\pi}{2}$$

Bode-plott oppskrift (asymptoter)

Gitt :

$$g(s) = \frac{k}{s} \frac{(\theta_1 s + 1)(\theta_2 s + 1)\ldots}{(\tau_1 s + 1)(\tau_2 s + 1)\ldots} e^{-\theta_d s}$$

Skal plotte $|g(j\omega)|$ (logaritmisk) og $\angle g(j\omega)$ mot ω (logaritmisk).

1) Start ved lav ω. Tegn asymptote for små ω ($\omega \to 0$)

a) med integrator, $1/s$: $\omega \to 0$: $g(j\omega) \approx k/j\omega$
Stigning : −1, Fase : −90°
(NB! Skjærer $|g|=1$ for $\omega = k$)

b) uten integrator : $\omega \to 0$: $g(j\omega) = k$
Stigning : 0, Fase : 0°

2) Gå fra lav til høy frekvens. Asymptoter fås ved å bytte mellom at reell del og kompleks del dominerer leddene ($\theta_i s + 1$) og ($\tau_i s + 1$).

"Kommer til" hva	ved knekkefrekvens	endring i stigning for $\lvert g\rvert$	asymptote endring i fase
pol $1/(\tau s+1)$	$\omega = 1/\tau$	−1	−90°
nullpkt $\theta s+1$	$\omega = 1/\theta$	+1	+90° for θ>0 −90° for θ<0
Dødtid $e^{-\theta_d s}$	$\omega = 1/\theta_d$	0	fase går rett nedover $\angle g = -57°$ for $\omega = 1/\theta_d$

IMC-PID TUNINGREGLER

$$k\frac{e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

Bruk $e^{-\theta s} \approx 1 - \theta s$ og velg $g_+(s) = 1 - \theta s$. Får

$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_c + \theta}, \quad \tau_I = \tau_1 \quad \tau_D = \tau_2$$

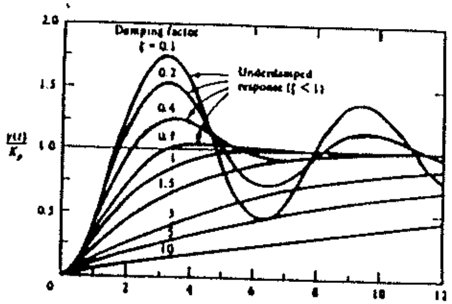
der $\tau_c > 0$ er en tuningsparameter med anbefalt verdi $\tau_c = \theta$. (derivatvirkningen gjelder for PID-regulator på kaskadeform: $c(s) = K_c \frac{\tau_I s+1}{\tau_I s}(\tau_D s + 1)$).

Tuningene er utledet for setpunktsrespons og hvis τ_1 er stor kan innsvingingen bli langsom ved forstyrrelser. Dette kan forbedres ved å redusere integraltiden, men for å unngå svinginger bør man velge $\tau_I \geq \tau_{I2}$ der

$$\tau_{I2} = \frac{1}{K_c k}\tau_1$$

Konklusjon: Anbefalt verdi $\tau_I = \min(\tau_1, \tau_{I2})$.

2.ordens respons ($f(\zeta)$)



$t/\tau \rightarrow$

Dimensionless response $y(t)/K_p$ to unit input step change ($u(t) = 1, t > 0$) for second-order system

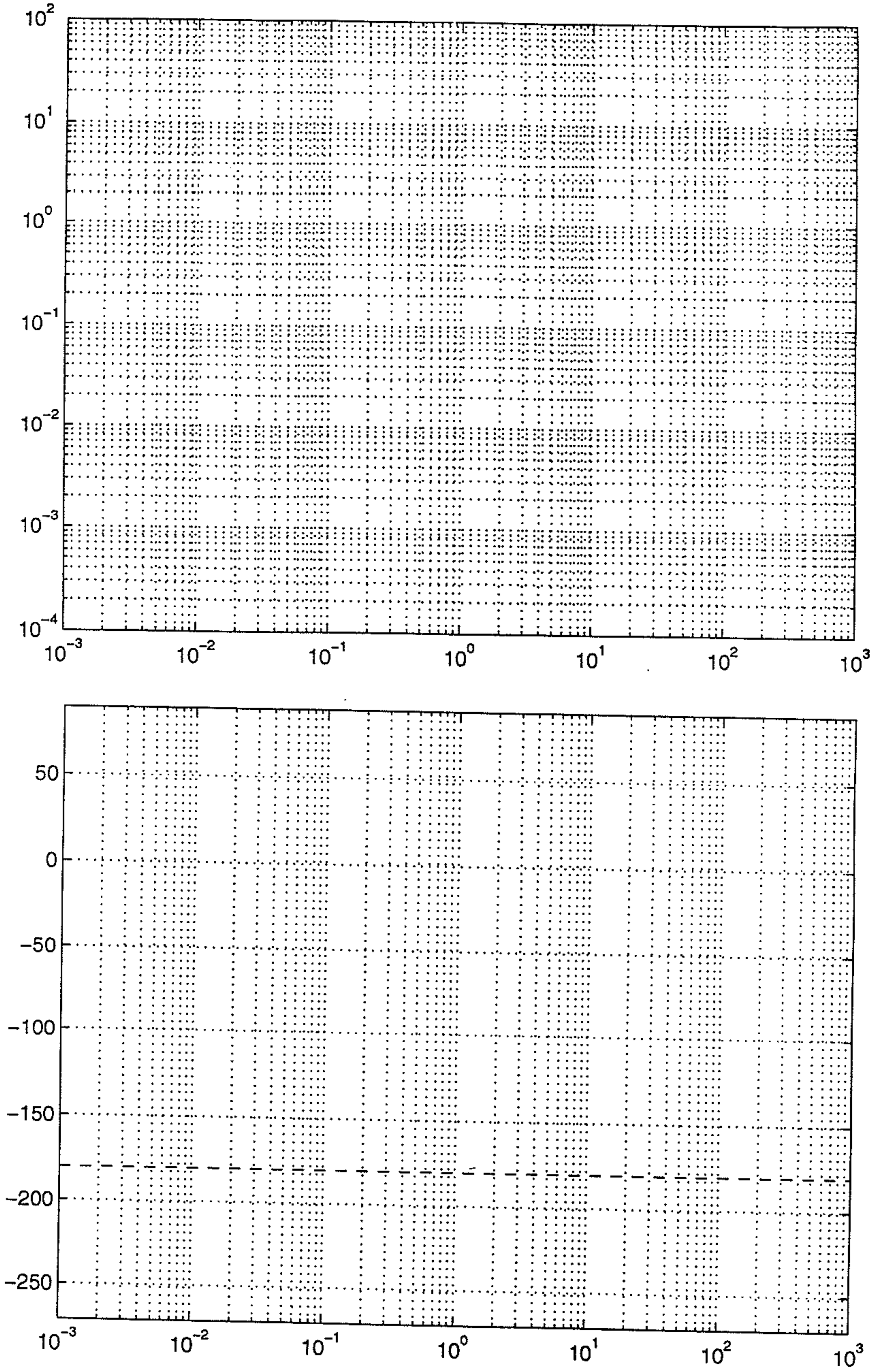
$$h(s) = K_p/(\tau^2 s^2 + 2\tau\zeta s + 1)$$

Period of oscillation $P = 2\pi\tau/\sqrt{1-\zeta^2}$.

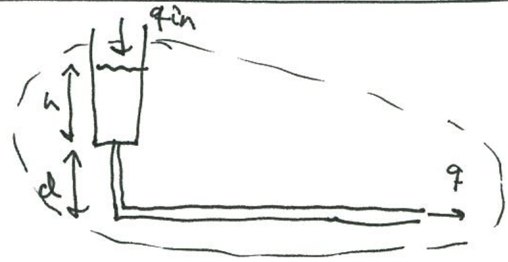
Ziegler–Nichols tuning

Using the values of K_u and P_u, Ziegler and Nichols recommended the following settings for feedback controllers:

	K_c	τ_I (min)	τ_D (min)
Proportional	$K_u/2$	—	—
Proportional-integral	$K_u/2.2$	$P_u/1.2$	—
Proportional-integral-derivative	$K_u/1.7$	$P_u/2$	$P_u/8$


10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-3 10^-2 10^-1 10^0 10^1 10^2 10^3
50
0
-50
-100
-150
-200
-250
10^-3 10^-2 10^-1 10^0 10^1 10^2 10^3

Exam Process Control (52041). 24 NOV. 99
SOLUTION



Problem 1.

(a) Mass balance

$$\frac{dm}{dt} = w_{in} - w \qquad [kg/s]$$

Assume ρ constant $\Rightarrow$ Get "volume balance"

$$\frac{dV}{dt} = q_{in} - q$$

Here $V = \underset{\text{konst}}{\underset{\shortparallel}{V_{pipe}}} + \underbrace{V_{cyl.}}_{A_p \cdot h}$

so we get

$$\frac{dh}{dt} = \frac{1}{A_p}(q_{in} - q)$$

(b) Linearization of the model

$$(1) \quad \frac{d\Delta h}{dt} = \frac{1}{A_p}(\Delta q_{in} - \Delta q)$$

$$(2) \quad \frac{d\Delta q}{dt} = \frac{A_t g}{L}(\overset{0}{\overset{\shortparallel}{\Delta d}} + \Delta h) - \frac{f}{2LA_t} 2q^* \Delta q$$

En del studenter brukte den stasjonære ligningen her. Dette gir en 1. ordens modell $h(s) = \frac{535}{\tau s + 1} \cdot q_{in}(s)$; $\tau = 2.4$ og det stemmer jo ikke....

(c) Laplace

$$(1)' \quad A_p \cdot s\, h(s) = q_{in}(s) - q(s)$$

$$(2)' \quad s \cdot q(s) = \frac{A_t g}{L} h(s) - \frac{f q^*}{L A_t} \cdot q(s) \quad \Rightarrow \left(\frac{L \cdot A_t}{f q^*} s + 1\right) q(s) = \frac{A_t^2 g}{f q^*} h(s)$$

kombinerer:

$$A_p \cdot s \cdot h(s) = q_{in}(s) - \frac{\frac{A_t^2 g}{f q^*}}{\frac{L A_t}{f q^*} s + 1} \cdot h(s)$$

$$A_p s \left(\frac{L A_t}{f q^*} s + 1\right) h(s) + \frac{A_t^2 g}{f q^*} h(s) = q_{in}(s) \left(\frac{L A_t}{f q^*} s + 1\right)$$

$$h(s) = \frac{f q^*}{A_t^2 \cdot g} \cdot \frac{\left(\frac{L A_t}{f q^*} s + 1\right)}{\frac{A_p L}{A_t \cdot g} s^2 + \frac{A_p f q^*}{A_t^2 g} s + 1} \cdot q_{in}(s) = \frac{k(\tau_L s + 1)}{\tau^2 s^2 + 2\tau\zeta s + 1} \cdot q_{in}(s)$$

$$= \frac{535(1.51s + 1)}{3.69s^2 + 2.44s + 1}$$

(d) Innsatt tallverdier

$$k = \frac{f q^2}{A_t^2 g} = 534.56 \left[\frac{m}{m^3/s}\right]$$ ← (e.g. increasing q by a factor of 2, i.e. $\Delta q = 1.31 \cdot 10^{-3}\ m^3/s$, gives $\Delta h = 0.7$ m (700% increase))

$$\tau_L = \frac{L A_t}{f \cdot q} = 1.51 s$$

$$\tau = \sqrt{\frac{A_p L}{A_t g}} = 1.92 s$$

$$\zeta = \frac{1}{2\tau} \frac{A_p f q^*}{g A_t^2} = \frac{1}{2} \frac{A_p^{1/2} f \cdot q^*}{g^{1/2} A_t^{3/2} \cdot L^{1/2}} = 0.636$$

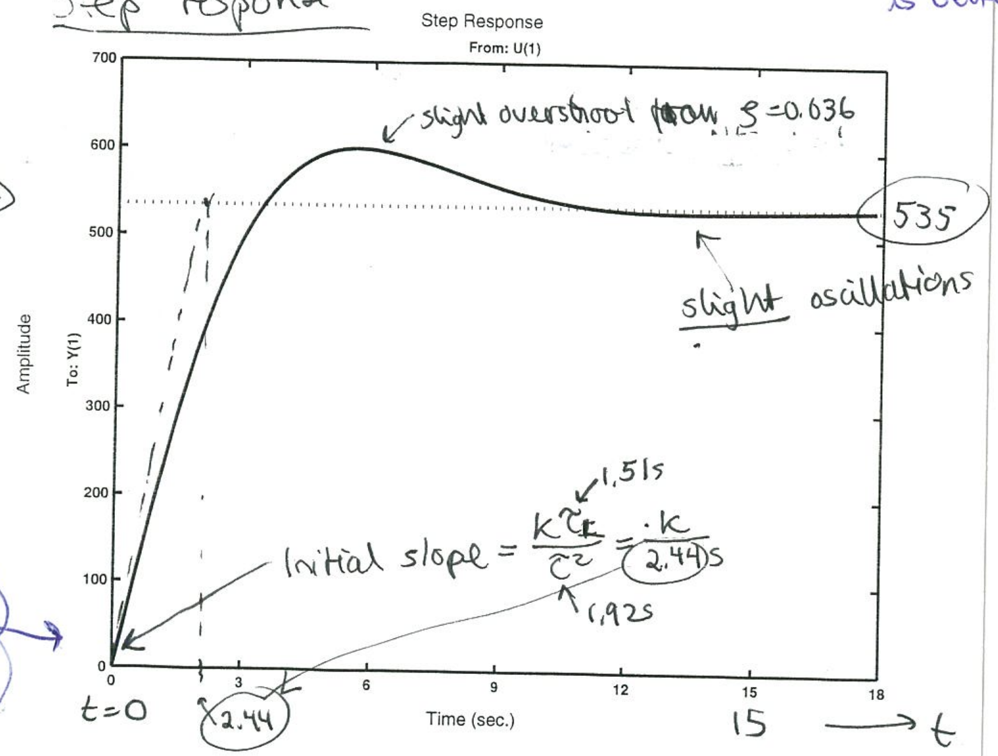
Note: Since $\zeta < 1$ the system is underdamped and we have oscillations (but they will hardly be visible)

(e) To avoid oscillations we need to <u>increase</u> ζ

i.e.
increase A_p
or decrease A_t ← most significant effect since $\sim A_t^{-3/2}$
or decrease L

<u>Conclusion</u>: Decrease the area of the outlet tube to avoid oscillations (we then have less mass to accelerate). But, on the other hand a small A_t means h must increase to maintain q, so possibly to reduce L is better.

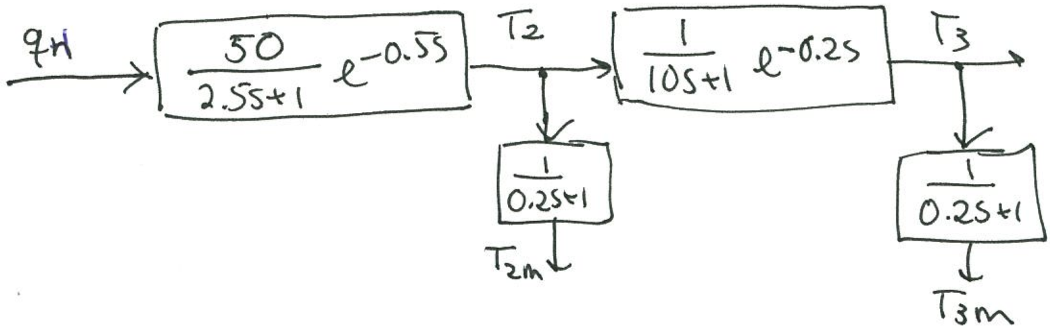
<u>Step response</u>



Note: Initial response ~~starts~~ increasing immediately since "pole excess" is 1

Problem 2.

(a)



(b) Use q_H to control T_3.

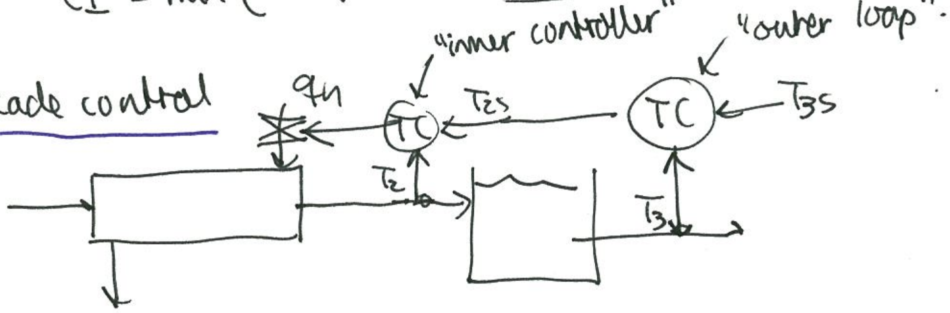
$$\frac{T_{3m}(s)}{q_H(s)} = \frac{50\, e^{-0.7s}}{(10s+1)(2.5s+1)(0.2s+1)} \approx \frac{\overset{K}{50}}{(\underset{\tau}{11.25}s+1)} e^{-\overset{\theta}{\cancel{1.95}\, 2.15}\, s}$$

PI-tunings (Skogestad) ($\tau_c = \theta = 1.95\,\text{min}$)

$$K_c = \frac{0.5}{K}\frac{\tau}{\theta} = \frac{0.5}{50}\cdot\frac{11.25}{\cancel{1.95}\; 2.15} = \underline{\cancel{0.0577}\; 0.052} \;\frac{\text{kg/s}}{\text{K}}$$

$$\tau_I = \min(11.25,\, 8\cdot\theta) = \underline{11.25\,\text{min}}$$

(c) Cascade control



- Integral action is only required in the "outer" loop

- Tuning of inner PI-controller

$$\frac{T_{2m}}{q_H} = \frac{50\; e^{-0.5s}}{(2.5s+1)(0.2s+1)} \approx \frac{50}{2.6s+1} e^{-0.6s}$$

PI-tunings ($\tau_c = \theta = 0.6\,\text{min}$)

$$K_c = \frac{0.5}{K}\frac{\tau}{\theta} = \frac{0.5}{50}\cdot\frac{2.6}{0.6} = \underline{0.0433}\;\frac{\text{kg/s}}{\text{K}}$$

$$\tau_I = \min(\;2.6,\, 8\cdot 0.6) = \underline{2.6\,\text{min}}$$

With the inner controller closed we have approximately

$$\frac{T_2}{T_{2s}} \approx \frac{e^{-0.6s}}{0.6s+1}$$

Desired closed-loop response time

Furthermore

$$T_{3m} = \frac{e^{-0.2s}}{(10s+1)(0.2s+1)} \cdot T_2$$

$$T_{3m} = \underbrace{\frac{e^{-0.8s}}{(10s+1)(0.6s+1)(0.2s+1)}}_{\text{"Effective process as seen from outer loop"} \;\approx\; \frac{1}{10.3s+1} e^{-1.3s}} \cdot T_{2s}$$

Tuning of outer loop then becomes ($\tau_c = \theta = 0.8s$)

$$K_c = \frac{0.5}{k}\frac{\tau}{\theta} = \frac{0.5}{1}\,\frac{10.3}{1.3} = \underline{3.96}\ [K/K]$$

$$\tau_I = \min(10.3,\ 8\cdot 1.3) = \underline{10.3\,\text{min}}$$

(d) In case (b) the closed-loop response is approximately.

$$\frac{T_3}{T_{3s}} \approx \frac{e^{-1.95s}}{1.95s+1}$$

In case (c) the closed-loop response is approximately.

$$\frac{T_3}{T_{3s}} \approx \frac{e^{-1.3s}}{1.3s+1}$$

We thus have a somewhat faster closed-loop response for setpoint changes. However, the main advantage in case (c) is that the response time for disturbances (in the feed, q_H, T_H) is only about 0.6 min, whereas it is 1.95 min in case (b).

Problem 3.

Introduce scaled variables

$$y = \frac{T}{0.5}_{\leftarrow |\Delta T_{max}|} \quad (\text{want } |y|<1)$$

$$u = \frac{q_1}{0.7}_{\leftarrow |\Delta q_{1max}|} \quad (\text{require } |u|<1)$$

$$d = \frac{q_2}{2}_{\leftarrow |\Delta q_{2max}|} \quad (\text{expect } |d|<1)$$

The model then becomes

$$y(s) = \underbrace{\underbrace{\frac{20 \cdot 0.7}{0.5}}_{k=28} \frac{(0.1s+1)e^{-1.6s}}{(6s+1)(2s+1)(0.4s+1)}}_{G} u(s) + \underbrace{\underbrace{\frac{6 \cdot 2}{0.5}}_{k_d=24} \frac{e^{-4.1s}}{(6s+1)}}_{G_d} \cdot d(s)$$

General: **Controllability analysis:** Define ω_d as freq. where $|G_d|=1$. Then
1) to avoid input saturation: Need $|G|>|G_d|$ up to freq ω_d
2) To be able to counteract disturbances: Need $\omega_d < \frac{1}{\theta}$ where θ is the effective delay

Need control up to frequency ω_d

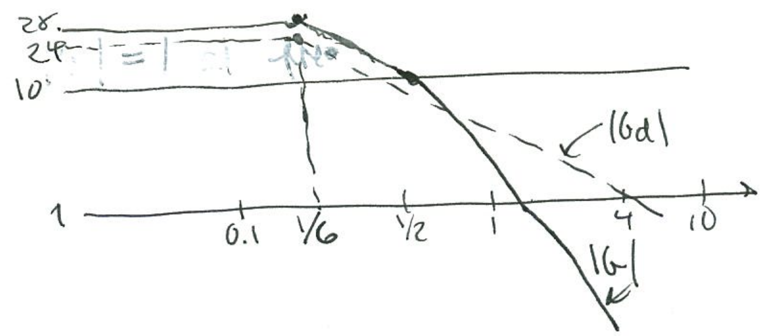
$$G_d = \frac{24}{6s+1} e^{-4.1s} \qquad |G_d|=1 \text{ at } \omega_d \approx \frac{k_d}{\tau_d} = \frac{24}{6} = \underline{\underline{4}} \ \frac{\text{rad}}{s}$$

Effective delay in G

$$\theta = 1.6 - 0.1 + \frac{2}{2} + 0.4 = 2.9\ s$$

Check controllability

1) Constraints. $k=28 > k_d = 24$ (OK), but maybe difficulty at high freq.



Yes, indeed: Se figure $|G|<|G_d|$ for $1.5<\omega<4$

Conclusion: Some problems with input saturation

2) Speed of response.

We have $\frac{1}{\theta} = \frac{1}{2.9} = \underbrace{0.34 < 4}$ rad/s

NO!

Almost one decade where we cannot counteract the disturbance $\Rightarrow$

NOT controllable

3) Feedforward

Yes, will help on the speed of response since delay in disturbance is 4.1

Must require

$$\theta < \overbrace{\frac{1}{\omega_d} + \theta_d}^{4.35}$$

OK!

$\theta = 2.9$, $\frac{1}{\omega_d} = 0.25 = \frac{k_d}{\tau_d}$, $\theta_d = 4.1$

But, will still have some problems with the input saturation.

Problem 4

$$g(s) = k' \frac{e^{-\theta s}}{s}$$

(This is on the form $\frac{k}{\tau_1 s+1} e^{-\theta s}$ with $k' = k/\tau_1$ and $\tau_1 \to \infty$)

(a) (i) PI-tuning Skogestad IMC:

$$\boxed{K_c = \frac{0.5}{k'} \frac{1}{\theta} \quad (\tau_c = \theta). \qquad \tau_I = \min(\underset{\infty}{\tau_1}, 8\cdot\theta) = 8\cdot\theta}$$

(ii) Ziegler-Nichols

Find ω_u: $\angle g(j\omega_u) = -\pi$

$$\underbrace{-\theta\cdot\omega_u}_{\text{from delay}} \underbrace{- \frac{\pi}{2}}_{\text{from integrator}} = -\pi \Rightarrow \underline{\omega_u = \frac{\pi}{2}\frac{1}{\theta}} \Rightarrow P_u = \frac{2\pi}{\omega_u} = \underline{4\theta}$$

Find K_u: $|L(j\omega_u)| = |K_u g(j\omega_u)| = 1$

$$\Rightarrow K_u = \frac{1}{|g(j\omega_u)|} = \frac{\omega_u}{k'} = \underline{\frac{\pi}{2k'}\frac{1}{\theta}}$$

Get: $$\boxed{K_c = K_u/2.2 = \frac{0.71}{k'}\cdot\frac{1}{\theta} \qquad \tau_I = P_u/1.2 = 3.33\,\theta}$$

(b) Frequency ω_{cg} where $|L| = 1$; $L = gc = \frac{k' e^{-\theta s}}{s} \cdot K_c \frac{(\tau_I s+1)}{\tau_I s}$, $|L| = \frac{k' K_c}{\tau_I \omega^2}\sqrt{\tau_I^2\omega^2+1}$

$\angle L = -\pi - \omega\theta + \arctg(\tau_I\omega)$

SIMC: $|L| = \left|\frac{0.5}{\theta\omega} \frac{\sqrt{(8\theta\omega)^2+1}}{8\theta\omega}\right| = 1 \Rightarrow \omega_{cg} = 0.52\cdot\frac{1}{\theta}$ $\Rightarrow \tau_c \approx 1.92\theta$

ZN: $|L| = 1 \Rightarrow \omega_{cg} = 0.76\cdot\frac{1}{\theta}$ $\Rightarrow \tau_c \approx 1.32\theta$ (faster)

(c) 1. Find frequency ω_{180} where $\angle L = \underset{-\pi}{\underline{-180^\circ}}$ (here $\angle L = -\pi - \omega\theta + \arctg(\tau_I\omega)$)

SIMC: $\omega_{180} = 1.49\cdot\frac{1}{\theta}$

ZN: $\omega_{180} = 1.35\cdot\frac{1}{\theta}$

2. Find GM as $1/|L|$ at frequency ω_{180}. Get

SIMC: GM = 2.96

ZN: GM = 1.86

3. Find PM as $\angle L + 180^\circ$ at freqency ω_{cg}. Get

SIMC: PM = 46.8°

ZN: PM = 24.7°

4. Max extra delay is PM(rad)/ω_c. Find 1.58 (158%) for SIMC and 56% for ZN 8

(d) Summary

	$K_c k'$	τ_I	$\omega_c \theta$	$\omega_{180} \theta$	GM	PM	PM/ω_c = $\frac{\Delta\theta_{max}}{\theta}$
SIMC	$0.5/\theta$	8θ	0.52	1.49	2.96	46.8°	1.58
ZN	$0.71/\theta$	3.33θ	0.76	1.35	1.86	24.7°	0.56

- We note that the ZN-tunings give a somewhat faster response (ω_c smaller) and the input disturbance response (not shown) is also better
- However, the robustness in terms of GM, PM and allowed extra delay is much worse for ZN
- In conclusion, ZN-tunings are only recommended if it is critical to have a fast response.