NTNU
Kunnskap for en bedre verden

Kandidat nr. / *Candidate no.:* 10040
Emnekode / *Subject:* TKP4140
Studie progr. / *Programme of study:* Erasmus
ENGINEERING
Dato / *Date:* 11/12/23
Antall ark / *Number of pages:* 14 + BODE PLOT
15

OMSLAG FOR EKSAMENSBESVARELSE

COVER FOR EXAMINATION ANSWER PAPERS

Problem 1: 4.5 + 10 + 7 + 3 + 35 + 5 + 3 = 35/35
Problem 2: 8 + 5 + 10 + 5 + 3 + 3 = 34/35
Problem 3: 5 + 10 + 15 = 30/30
= 99/100

The student didn't do everything right but got 99% with bonus points!

For invigilator:

Det oppførte antall ark er kontrollert.

Inspektør: Leif Lie[?]

PROBLEM 1) CONTROLLER TUNING

4.515

a) $y = G_d d_2$ where $G_d = \dfrac{2\text{''}\,6\text{''}}{1 + 2G\,c/G_m}$

$$G_d = \frac{\dfrac{20e^{-0,3s}}{(8s+1)(3s+1)}}{1 + \dfrac{20e^{-0,3s}}{(8s+1)(3s+1)} \cdot C \cdot \dfrac{e^{-0,5s}}{0,8s+1}}$$

$$= \frac{(0,8s+1)\,20e^{-0,3s}}{(8s+1)(3s+1)(0,8s+1) + 20Ce^{-0,8s}}$$

if $C = 0$

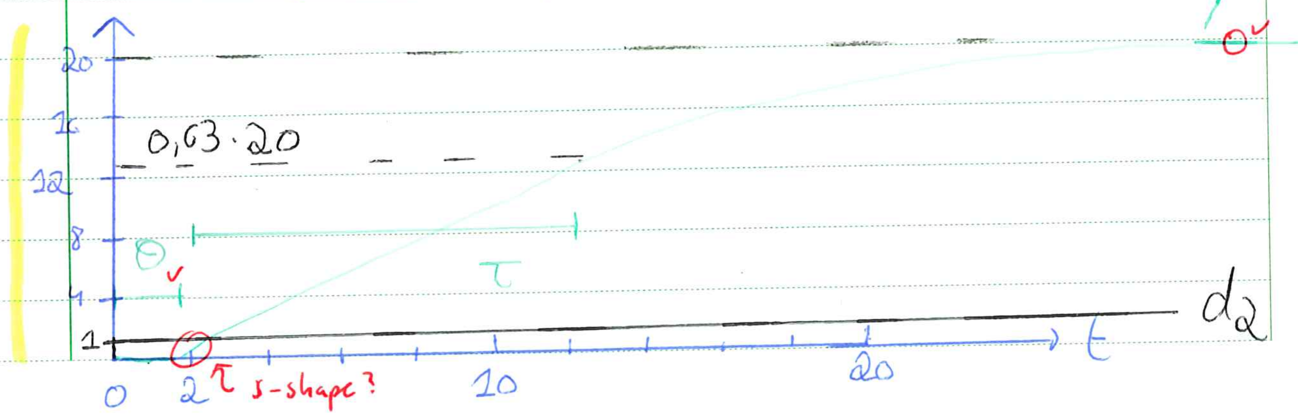
$$G_d(s) = 2 \cdot \frac{10e^{-0,3s}}{(8s+1)(3s+1)}$$ ✓

We approximate $G_d(s)$ with half-rule

ok!

$$G_d(s) \approx 20\frac{e^{-1,8}}{9,5s+1}$$ ✓

$$\tilde{\theta} = 0,3 + 1,5 = 1,8$$
$$\tilde{\tau} = 8 + 1,5 = 9,5$$



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a) continued

yes, we need control, for $d_2(t) = 1$ the steady state value of y is 20 which is too large, we want $|y(t)| < 1$ ✓

10/10 b) G_c = transfer function used for controller

$$G_c(s) = 2 \frac{10 e^{-0,3s}}{(8s+1)(3s+1)} \cdot \frac{e^{-0,5s}}{0,8s+1}$$

$$= 20 \frac{e^{-0,8s}}{(8s+1)(3s+1)(0,8s+1)}$$

half rule too first order:

$$\tilde{\theta} = 0,8 + 0,8 + \frac{3}{2} = 3,1$$ ✓

$$\tilde{\tau} = 8 + 1,5 = 9,5$$ ✓

$$\tilde{G}_c(s) = \frac{20 e^{-3,1s}}{9,5s+1}$$

tight $\tau_c = \theta = 3,1$

$$K_c = \frac{\tau}{k(\tau_c + \theta)} = \frac{9,5}{20(3,1+3,1)} = 0,0766$$ ✓

$$\tau_I = \min\{\tau; 4(\tau_c + \theta)\} = \min\{9,5; 24,8\} = 9,5$$ ✓

$$C(s) = K_c \frac{\tau_I s + 1}{\tau_I s} = 0,0766 \frac{9,5s+1}{9,5s}$$ ✓

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7/7

c) PID

Half rule for G_c to second order

$$G_c = \frac{20 e^{-0,8s}}{(8s+1)(3s+1)(0,8s+1)}$$

$\tilde{\theta} = 0,8 + 0,4 = 1,2$ ✓ $\tilde{\tau}_2 = 3 + 0,4 = 3,4$ ✓

since $\tilde{\tau}_2 > \tilde{\theta}$ PID improves performance

tight control $\tau_c = \tilde{\theta}$

$$\tilde{G}_c(s) = \frac{20 e^{-1,2s}}{(8s+1)(3,4s+1)}$$

$$K_c = \frac{\tau_1}{k(\theta + \tau_c)} = \frac{8}{20(1,2+1,2)} = 0,1667 ✓$$

$$\tau_I = \min\{\tau_1, 4(\tau_c + \theta)\} = 8 ✓$$

$$\tau_D = \tau_2 = 3,4 ✓$$

$$C(s) = 0,1667 \frac{8s+1}{8s} \cdot (3,4s+1)$$

3/3

d) Good disturbance rejection if $\omega_d < \omega_c < \frac{1}{\theta}$

$$\omega_d : |G_d(j\omega_d)| = \left|\frac{\Delta y_{max}}{\Delta d_2}\right| = 1$$

$$G_d(s) = \frac{20 e^{-0,3s}}{(8s+1)(3s+1)}$$

$$\Rightarrow 20 = \sqrt{64\omega_d^2 + 1}\sqrt{9\omega_d^2 + 1}$$

$$\Rightarrow \omega_d = 0,87825$$

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d) continued

PI : $\theta = 3{,}1$ where $\omega_c = \frac{1}{\tau_c} = \frac{1}{3{,}1}$ tight

$0{,}878 < \omega_c < 0{,}32$ ↯ ✓

PID : $\theta = 1{,}2$ $\omega_c = \frac{1}{\tau_c} = 0{,}833$ tight

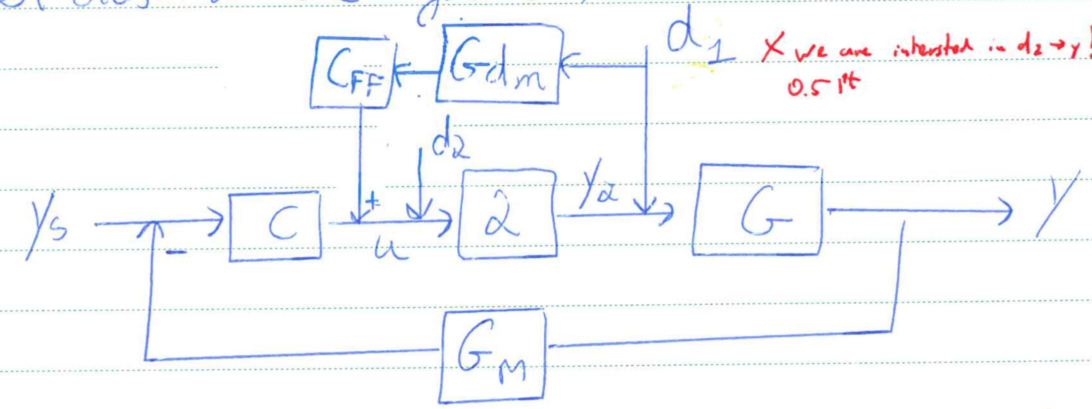
$0{,}878 < \omega_c < 0{,}8333$ ↯ ✓

In both PI & PID control $\omega_c < \omega_d$ which is not good for disturbance rejection. Therefore another solution for example a better controller is needed. However we used the estimated time delays for this. In reality Gd has a delay of only $\theta = 0{,}3$. Than Wc can be a bit larger for a better disturbance rejection

Ok!

3.515

e)



X we are interested in $d_2 \to y$!
0.5 pt

We use d_1 to calculate a control signal using the controller C_{FF}. G_{dm} are the delay measurement dynamics, in this case assume $G_{d_m} = 1$

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e) continued

$$Y = 2uG + d_1 G$$

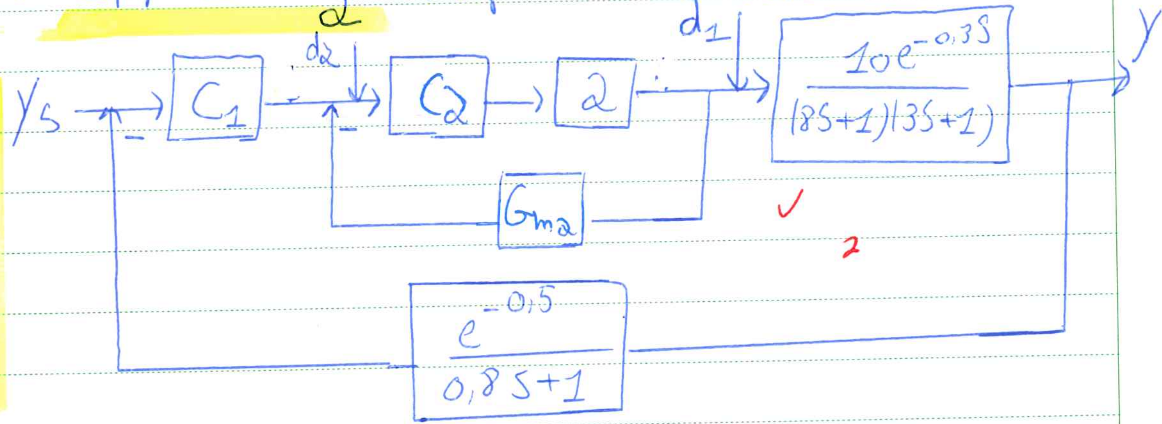
where $u = d_2 + C(y_s - y_m) + C_{FF} d_1$

goal is that y is not dependent on d_1

(✓) ok with sketch

$$\Rightarrow C_{FF} = -\frac{1}{2} \quad \text{Proportional controller}$$

5/5

f) 

assumed same measurement dynamics in inner cascade as in outer cascade

$\tau_{c2} = 0,6 > \theta_2 = 0,5$. τ_{c2} is alright when the inner loop has a delay in the measurement dynamics. If there is no or a smaller delay the closed loop time constant τ_{c2} can be chosen smaller so that the slave controller and inner loop are faster (more tight control) ✓ 1

$$C_2 \text{ based on } 2G_{m2} = \frac{2e^{-0,5}}{0,8S+1}$$

"1!

$K_c = 0,364$ (✓)

$$C_2 = 0,364 \frac{0,8S+1}{0,8S}$$

$\tau_I = 0,8$ (✓) 1

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The inner cascade can be approximated as: $T_2 \approx \frac{e^{-\theta s}}{\tau_{c2} s+1} = \frac{e^{-0,5s}}{0,6s+1}$ (✓)

The new $C = C_1$ must be tuned of $T_2 G G_m$

$$T_2 G G_m = \frac{10 e^{-1,3}}{(8s+1)(3s+1)(0,8s+1)(0,6s+1)}$$

half rule

$\tilde{\theta} = 1,3 + 0,6 + 0,8 + 1,5 = 4,2$ (✓)

$\hat{\tau}_1 = 8 + 1,5 = 9,5$ (✓)

assume tight control $\tau_{c_1} = \theta$

$K_{c_1} = 0,113$ (✓)

$\tau_{I_1} = 9,5$ (✓)

$$\Rightarrow C_1 = 0,113 \frac{9,5s+1}{9,5s}$$

C must be retuned because of the extra delay and dynamics in the inner cascade.

$\frac{\tau_{c1}}{\tau_{c2}} \approx 7 \Rightarrow$ cascade works

Cascade is very useful when there is a significant d_2 and also for high measurement delays for y. For PI: $\tau_c = 3,1$ and PID: $\tau_c = 1,2$ and cascade $\tau_{c_1} = 4,2$ so although the system is slower the d_2 disturbance rejection is a lot better than with only single loop feedback $\Rightarrow$ I would recommend using cascade ✓

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3/3

g) Already done in f: $\boxed{PI}$

Also on previous page

$$T_2 G G_m \approx 10 \frac{e^{-4{,}2s}}{9{,}5s+1}$$

PI: $K_{c_1} = 0{,}113$ tight $\tau_c = \theta$ control

$\tau_{I_1} = 9{,}5$

$$\Rightarrow C_1(s) = 0{,}113 \frac{9{,}5s+1}{9{,}5}$$

$\boxed{PID}$ $$T_2 G G_m \approx 10 \frac{e^{-2{,}3}}{(8s+1)(3{,}4s+1)}$$

$\tau_2 > \theta \Rightarrow \tau_D = \tau_2 = 3{,}4$

$$K_{c_1} = \frac{\tau_1}{k(\theta + \tau_c)} = 0{,}174 \text{ tight control}$$

$\tau_{I_1} = 8$

$$\Rightarrow C_1(s) = 0{,}174 \frac{8s+1}{8s}(3{,}4s+1)$$

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PROBLEM 2: closed loop stability

a)

8/8

$$G(s) = \frac{4(-2s+1)}{(20s+1)(6s+1)} \qquad C(s) = K_c \frac{20s+1}{20s}$$

$$T = \frac{L}{1+L} \quad \text{where } L = GC = \frac{4(-2s+1)K_c}{20s(6s+1)}$$

$$= \frac{4(-2s+1)K_c}{20s(6s+1) + 4K_c(-2s+1)}$$

for $K_c = 1$: $d(s) = 20s(6s+1) + 4(-2s+1)$

$d(s) = 120s^2 + 12s + 4$ ✓

Since all the coeffisients have the same sign, the closed loop is stable. This condition is necessary and sufficient for 2nd order (or lower) polynomials $d(s)$. ✓

For higher orders, check that all the poles are in the left halve plane

5/5

b) $d(s) = 120s^2 + (20 - 8K_c)s + 4K_c$.

system is stable for $0 < K_c < 2{,}5$

$\Rightarrow$ gain margin is $\underline{2{,}5}$ ✓

We check this via bode stability

$$|L(j\omega_{180})| < 1$$

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b) continued

$\angle L(j\omega_{180}) = -180$

$-180 = \text{atan}(-2\omega_{180}) - 90 - \text{atan}(6\omega_{180})$

$\Rightarrow \omega_{180} = 0{,}288675$

$GM = \frac{1}{|L(j\omega_{180})|} = \frac{1}{0{,}4} = 2{,}5$ ✓

good, closed loop system is stable

1010 c) $L = \frac{4(-2s+1)}{20s(6s+1)}$

→ see Bode paper

Pole in $0, -\frac{1}{6}$ zero (RHP ∇) in $0{,}5$ ✓

GAIN: $-1, -1$ $+1$

PHASE: $-90°, -90°$ $-90°$

• with rules • smooth approximations

$\omega_c = 0{,}15378 \rightarrow PM \approx 30°$ ✓

515 d) $DM = \Delta\theta = \frac{PM}{\omega_c} = \frac{30{,}2065 \cdot \frac{\pi}{180}}{0{,}15378}$

$= 3{,}4283$ seconds ✓

assuming ω in rad/s

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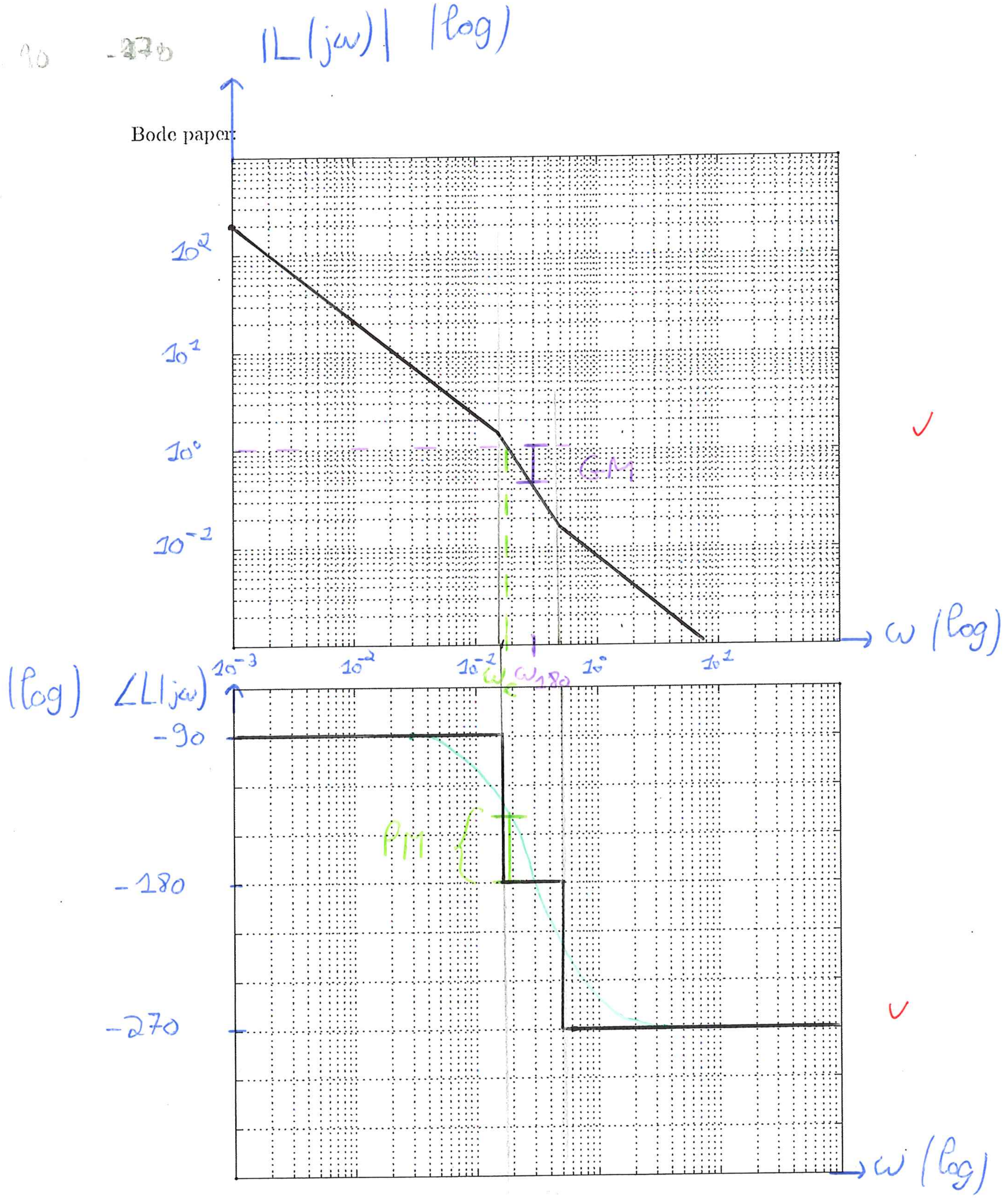

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3/7

e) system becomes unstable at

$K_u = GM\, K_c = 2{,}5 \cdot 1 = 2{,}5$ ×

This was for a PI!

$\Rightarrow K_{c,ZN} = 0{,}45 K_u = 1{,}125$ (✓)

$P_u = \frac{2\pi}{\omega_{180}} = 21{,}7656$ × see above

$\Rightarrow \tau_I = \frac{P_u}{1{,}2} = 18{,}138$ (✓)

$\Rightarrow$ Ziegler - Nichols PI - controller

$$C(s) = 1{,}125 \frac{18{,}138s+1}{18{,}138s}$$

This tuning is quite close to the PI tuning we started with, which was a good stable tuning, see GM & PM

3/3

f) half rule $\tilde{\theta} = 2 + 3 = 5$ ✓

$\tilde{\tau} = 20 + 3$ ✓ $= 21$ 23!

$$\tilde{G} = \frac{4e^{-5s}}{21s+1}$$ assume tight control

$K_c = \frac{\tau}{k(\theta+\tau_c)} = 0{,}525$

$\Rightarrow C(s) = 0{,}525 \frac{21s+1}{21s}$

$\tau_I = \min\{\tau, 4(\tau_c+\theta)\} = 21$

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PROBLEM 3 : MIXING PROCESS

5/5

a) $$F_1 = C_{v_1} z_1 \sqrt{\frac{P_1 - P_2}{\rho}}$$ ✓

linear valve $f_1(z) = z_1$

$$K_1 = \frac{F_1}{z_1} = C_{v_1} \sqrt{\frac{P_1 - P_2}{\rho}}$$ ✓

where C_{v_1} is the valve coeffient $[m^2]$

$C_v = A C_d$ where A is the surface and C_d a coeffient often close to 1

10/10

b) i) assuming ρ = constant

$$\frac{dV}{dt} = F_1 + F_d - F_2 = 0 \quad \text{mass}$$ ✓

$$\frac{d C_2 V}{dt} = \cancel{C_1 F_1} + C_d F_d - C_2 F_2 \quad \text{component}$$ ✓

ii) F_1 = pure $\rightarrow C_1 = 0$

$C_2^* = 20 \ kgA/m^3$, $C_d^* = 700 \ kgA/m^3$, $F_2^* = 2 m^3/min$

$V = 0,36 m^3$

mass $F_d^* = F_2^* - F_1^*$ $\quad F_1^* = \frac{68}{35} \approx 1,94 \frac{kg}{m^3}$

$\Rightarrow$

component $C_d^* F_d^* = C_2^* F_2^*$ $\quad F_d^* = \frac{2}{35} \approx 0,057 \frac{kg}{m^3}$

iii) V & P_2 are constant $C_2 = G\,F_1$

linearise:

$$V\frac{d\Delta C_2}{dt} = F_d^*\Delta C_d + C_d^*\Delta F_d - F_2^*\Delta C_2 - C_2^*\Delta F_2$$

$$\Delta F_2 = \Delta F_1 + \Delta F_d$$

$$\Rightarrow V^*\frac{d\Delta C_2}{dt} = F_d^*\Delta C_d + (C_d^* - C_2^*)\Delta F_d - F_2^*\Delta C_2 - C_2^*\Delta F_1$$ ✓

LAPLACE

$$\rightarrow V^* s\, C_2 = F_d^* C_d + (C_d^* - C_2^*) F_d - F_2^* C_2 - C_2^* F_1$$ ✓

$$\Rightarrow C_2 = \frac{-C_2^*}{V^* s + F_2^*} = \frac{-20}{0{,}36s + 2} = \frac{-10}{0{,}18s + 1}$$

$$G(s) = \frac{-10}{0{,}18s + 1}$$ ✓

negative gain is logic, when F_1 goes up the concentration C_2 will lower (more water compared to solid A). ✓

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c) 1) main objective is to keep C_a constant and Z_a does not influence C_a. We can add a simple level controller to Z_a so that the volume stays constant. Since Fd is the TPM this LC follows the radiation rule and is therefore possible. Since there is no level measurement, we use a PC to keep p_a constant

2) delay in measurement C_a becomes outer cascade resulting in a ratio of setpoints that is used for ratio control. Since P_1 is close to F_1 and it is possible to determine F_1, measurement, this is the inner cascade! Assuming P_1 can be measured really fast (possible since close to valve). The P_1 measurement (want to keep constant) control can be used for the "speed control" of the pump. Feedforward using P_0 is not possible since that PC would then control the flow, two TPM's in 1 system is not possible according the radiation rule

↳ No, it is possible to have multiple TPMs

3) Note that the FC for F_1 is not a TPM since it is controlled by Fd, therefore this FC sets the flow adjusted to Fd

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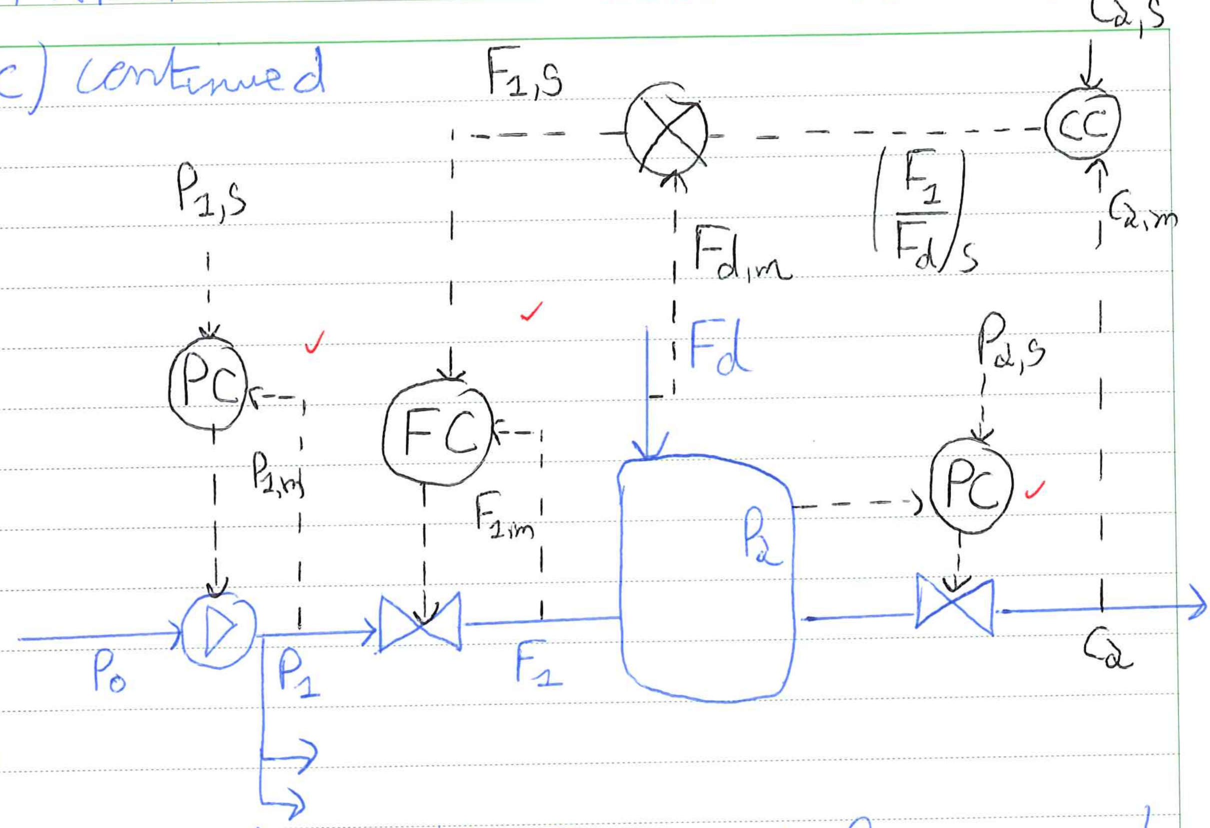
c) continued



We can choose the $P_{1,S}$ and $P_{2,S}$ we want to keep constant.

We first add and tune the ratio & cascade control since keeping C_2 constant is the main control objective.

Next we added two PC to keep P_1 & P_2 constant while following the radiation rule. Since keeping P_1 & P_2 constant is desirable but not the main control objective, these controllers are less important and tuned later.

And FC?

Very good!