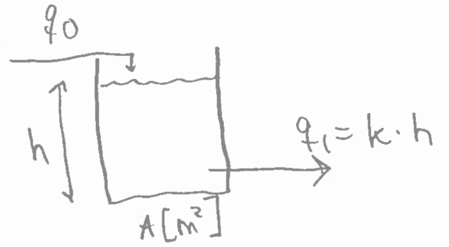
Exam Process Control, Dec. 2019. Solution

Problem 1.



$V = A \cdot h$
$A = 10\,m^2$

(a)
(i) Assume constant density ρ (liquid). Mass balance then gives

(3%) (1) $\frac{dV}{dt} = q_0 - q_1$ $[m^3/s]$

(ii) Steady-state, $\frac{dV^*}{dt} = 0 \Rightarrow q_1^* = q_0^* \Rightarrow \underset{0.02\,m^2/s}{k} \cdot h^* = 0.1\,m^3/s \Rightarrow h^* = \frac{0.1\,m^3/s}{0.02\,m^2/s} = \underline{5m}$

(3%) $V^* = A h^* = 10\,m^2 \cdot 5m = \underline{50\,m^3}$

(So the residence time: $\tau = \frac{V^*}{q_0^*} = \frac{50}{0.1}\,s = 500\,s$)

(iii) Linearize (1) and introduce deviation variables

(3%) $$\frac{d\Delta V}{dt} = \Delta q_0 - \Delta q_1$$

$$A \frac{d\Delta h}{dt} = \Delta q_0 - k \cdot \Delta h$$

(iv) Laplace, $\mathcal{L}(\Delta h(t)) = h(s)$

(6%) $$A s h(s) = q_0(s) - k h(s)$$

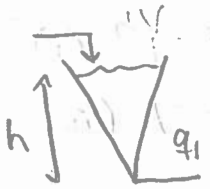
$$\Downarrow$$

$$h(s) = \frac{q_0(s)}{As + k} = \underbrace{\frac{1/k}{\frac{A}{k}s + 1}}_{t(s)} q_0$$

Note:
Gain $= 1/k$
$\tau_{CL} = A/k$

(Comment relevant for Problem 2. Note that
$$q_1(s) = k h(s) = \frac{1}{\frac{A}{k}s + 1} \cdot q_0 \doteq \frac{1}{\tau_{CL} s + 1} \cdot q_0 \;)$$

(5%) (b)



$A(h) = 0.4 h^2$

$V(h) = \int_0^h A(h)\,dh$
$= 0.4 \frac{h^3}{3} = \frac{1}{3} A(h) h$

(i) $\frac{dV}{dt} = q_0 - q_1$

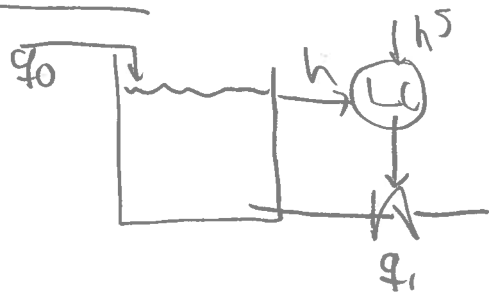
(ii) $h^* = 5m$, $A^* = 0.4 \cdot 25\,m^2 = 10\,m^2$, $V^* = \frac{1}{3} A^* h^* = \frac{50}{3} = 16.7\,m^3$

(iii) $\frac{d\left(\frac{0.4 h^3}{3}\right)}{dt} = q_0 - k \cdot h \Rightarrow \underbrace{0.4 h^2}_{A(h)} \cdot \frac{dh}{dt} = q_0 - k \cdot h$

linearize
$\Rightarrow A(h^*) \frac{d\Delta h}{dt} = \Delta q_0 - k \cdot \Delta h$ (as above)

(iv) Get same $t(s) = \frac{1/k}{\frac{A(h^*)}{k} s + 1}$. But note that

Problem 2.



(a)

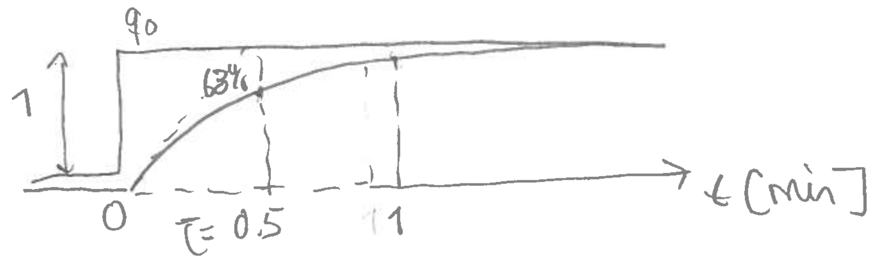
$$q_1 = \frac{1}{0.5s+1}\cdot q_0$$

Why? – The gain is clearly 1 because $q_0 = q_1$ at steady state.

– The time constant is clearly the same as for the level control since we with P-control have $q_1(s) = K_c \cdot h(s)$.

– See also comment on Problem 1 (a)(iv).

(b)



(c)

$$q_1 = g(s)\cdot q_0 \quad \text{where } g(s) = \frac{1}{0.5s+1} = \frac{1}{\tau s+1}$$

Sinusoidal response (frequency response) is given by

$q_0 = 1\cdot \sin(\omega t)$

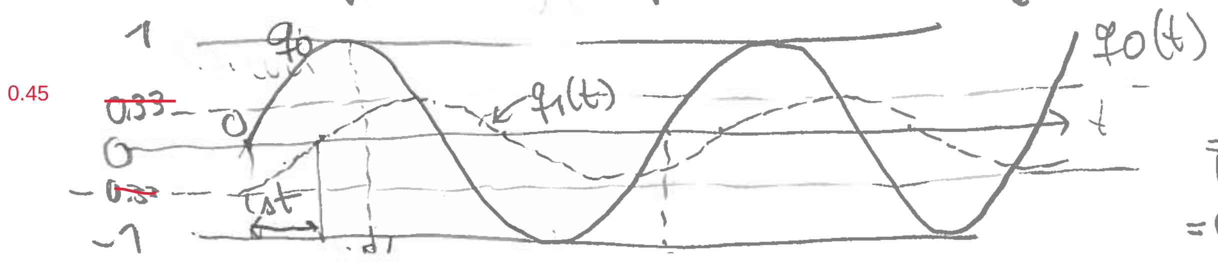
$q_1 = AR \cdot \sin(\omega t + \varphi)$

$\omega = 4$ rad/min ⇒ period $P = \frac{2\pi}{\omega} = 1.57$ min

is given by $g(j\omega)$ and we have

$$AR = |g(j\omega)| = \frac{1}{\sqrt{0.5^2\omega^2+1}} = \frac{1}{\sqrt{0.5^2\cdot 4^2+1}} = \cancel{\frac{1}{3} = 0.33}\ 0.45$$

$$\varphi = \angle g(j\omega) = -\arctg(\omega\tau) = -\arctg(\underbrace{0.5\cdot 4}_{2}) = -63.4°$$

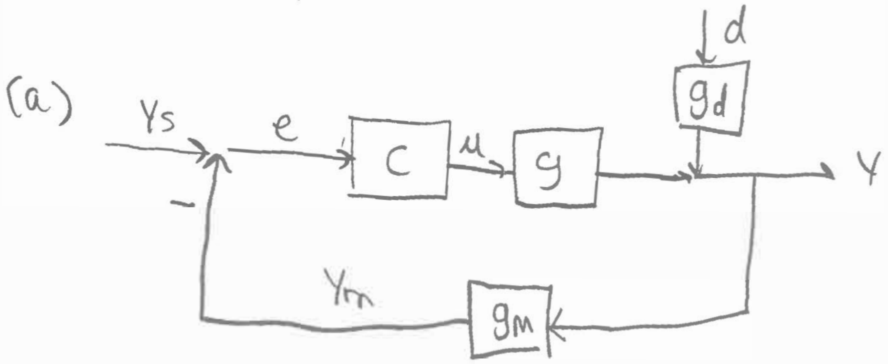


$\Delta t = \frac{-\varphi}{2\pi}\cdot P = \frac{63.4}{360}\cdot 1.57 = 0.18 \cdot 1.57 = 0.27$ min

Problem 3

3

(a)



$$g = \frac{3}{(2s+1)(0.5s+1)^2}$$

$$g_m = \frac{e^{-3s}}{(1.5s+1)}$$

$$g_d = \frac{10}{2s+1}$$

(b) The SIMC-controller is designed based on the transfer function $g g_m$. Using the half rule we get

$$g g_m = \frac{3e^{-3s}}{(2s+1)(1.5s+1)(0.5s+1)^2} \approx \frac{k e^{-\theta s}}{\tau s + 1}$$

with $k = 3$

$$\theta = 3 + \frac{1.5}{2} + 2 \cdot 0.5 = 4.75 \quad \text{(effective delay)}$$

$$\tau = 2 + \frac{1.5}{2} = 2.75$$

"Tight" SIMC-tunings $(\tau_c = \theta = 4.75)$

$$K_c = \frac{1}{k}\frac{\tau}{\tau_c + \theta} = \frac{1}{3}\frac{2.75}{4.75 + 4.75} = 0.096$$

$$\tau_I = \min(\tau, 4(\tau_c + \theta)) = \tau = 2.75$$

(c) Closed-loop response from d to y:

(i)

$$y = T_d(s)\, d$$

$$T_d = \frac{\text{direct}}{1 + \text{loop}} = \frac{g_d}{1 + g g_m c}$$

(ii)

$$c(s) = K_c\left(1 + \frac{1}{\tau_I s}\right) = \frac{K_c(\tau_I s + 1)}{\tau_I s} = \frac{0.035(2.75s+1)}{s}$$

$$T_d(s) = \frac{\frac{10}{2s+1}}{1 + \underbrace{\frac{e^{-3s}}{(1.5s+1)}}_{gm(s)} \underbrace{\cancel{\frac{10}{2s+1}}}_{g(s)} \underbrace{\frac{0.035(2.75s+1)}{s}}_{c(s)}}$$

oops should be g(s)=3/((2*s+1)*(0.5*s+1)^2), not gd(s)=10/(2s+1), in the loop transfer function.
In any case, difficult to simplify this.

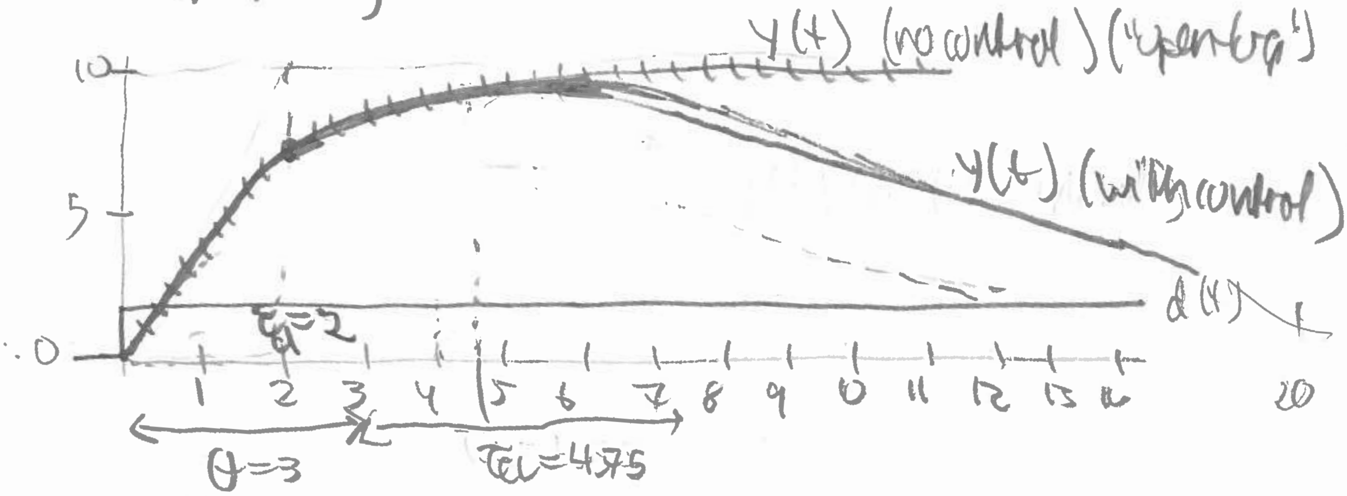
We can also introduce the half-rule approximation of g*gm, but also this does not help much since it's not possible to find an expression that can be used to get the time response ---- so let's try a sketch.

(difficult to make it simpler)

Sketch of response (T_d)

- $T_d(0) = 0 \Rightarrow y(t)$ goes to 0 at steady-state
- Initial response follows $g_d(s)$ (it always does this because it takes some time for the controller to take action, especially since we have a time delay in the measurement)
- It's difficult to predict the peak value for $y(t)$ but since there is a delay of $\theta = 3$ followed by a response time $\tau_{CL} = 4.75$ it will be close to 10 ($= k_d$). It is expected to peak at around $\theta + \tau_{CL} = 7.75$

- Well, let's try



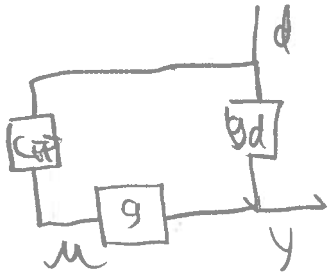
- This is indeed close to the correct as can be confirmed using Matlab. It crosses 0 at about $t = 20$!

(d) Yes, here feedforward control can clearly be beneficial because most of the effective time delay is in the measurement of y.

Not required

The ideal feed-forward-controller is

$$G_{FF}^{id} = -\frac{g_d}{g} = -\frac{10/(2s+1)}{3/\left((2s+1)(0.5s+1)^2\right)} .$$



$$= -3.33\,(0.5s+1)^2$$

So if we try just

$$G_{FF} = -3.33$$

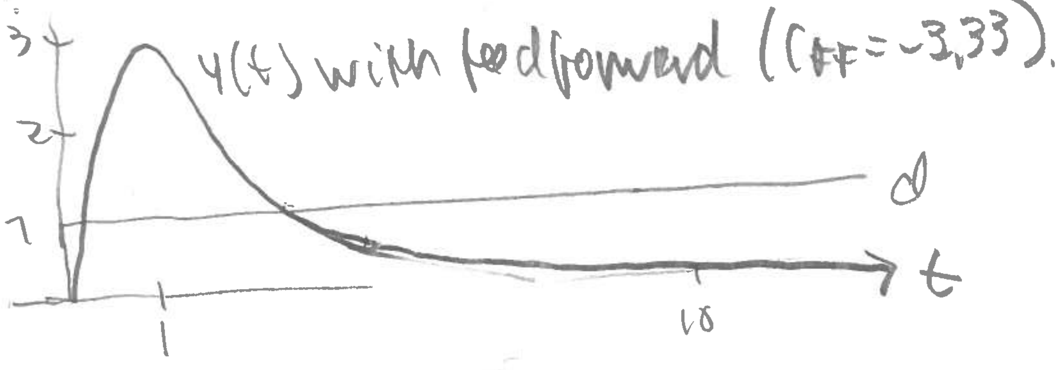
Then the response is

$$y = g_d \cdot d + g \cdot G_{FF} \cdot d$$

$$= \left(\frac{10}{2s+1} - \frac{10}{2s+1} \cdot \frac{1}{(0.5s+1)^2} \right) d$$

etc ... $(0.5s+1)^2 - 1$

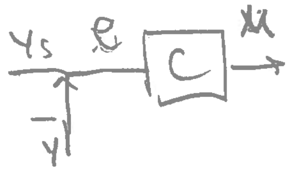
It get's a bit complicated, but the response looks quite good as you can test with Matlab. It has a peak of $y \approx 2.8$ around $t = 1.5$.



Comment: Comparing the dynamics of g and g_d we see that g contains the term $\frac{1}{(0.5s+1)^2} \approx e^{-1s}$ so one would expect with $G_{FF} = -3.33$ that there is "delay" of about 1 before feedforward becomes effective!

Problem 4. (b)

$$C(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

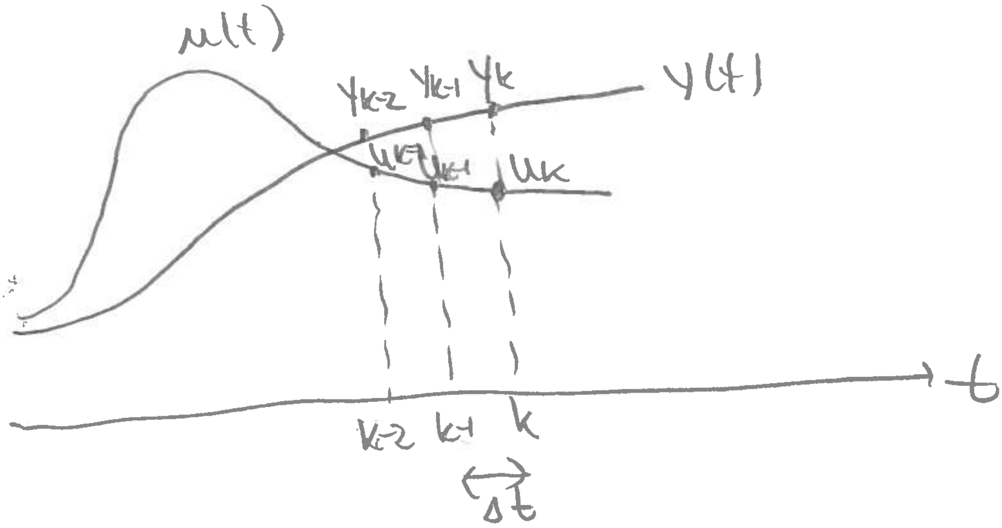


(a) Time domain

$$(*)\quad u(t) = u_0 + K_c \cdot e(t) + \frac{K_c}{\tau_I}\int_0^t e(t)\,dt$$

0 ← start-up of controller

(b)



k = current time (t)

$k-1$: previous time of sample $(t-\Delta t)$.

(c) Discretize $(*)$

$$e_k = e(t) = y_{s,k} - y_k$$

Define:

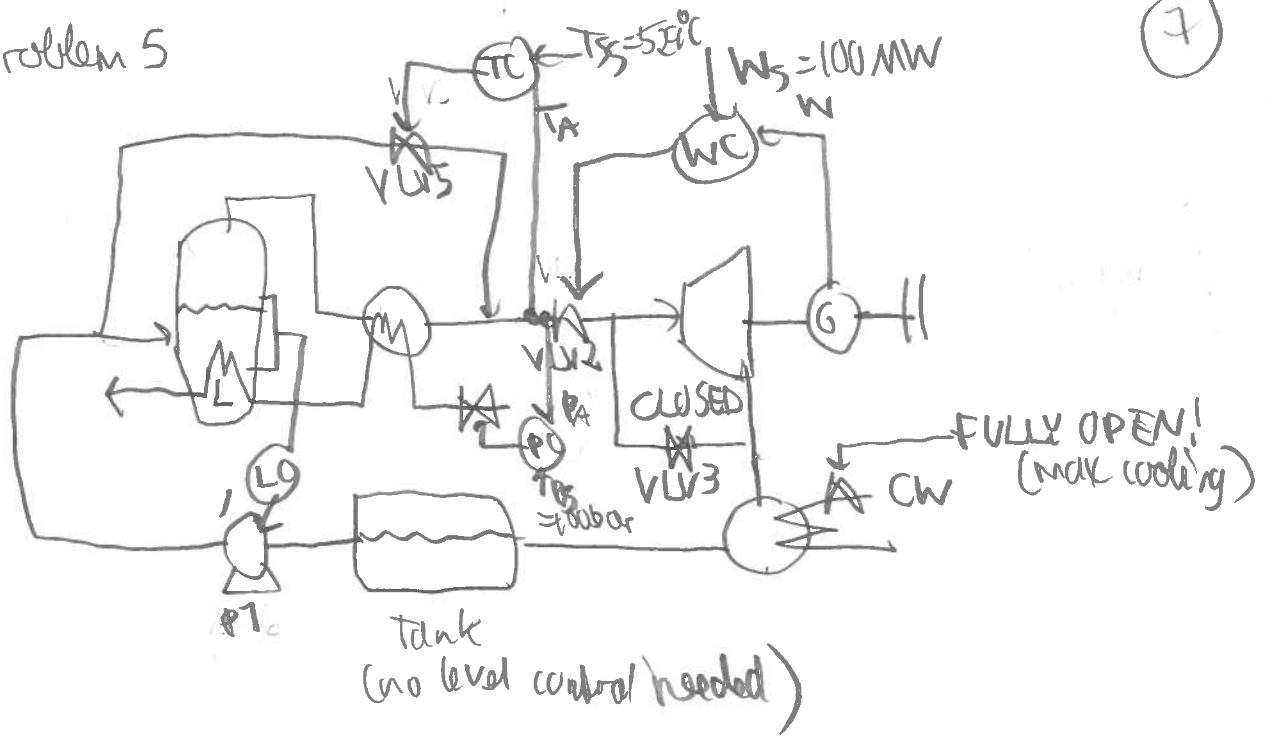
$$\bar{u}_k = u_0 + \frac{K_c}{\tau_I}\int_0^t e(t)\,dt = \bar{u}_{k-1} + \frac{K_c}{\tau_I}\overbrace{\int_{t-\Delta t}^{t} e(t)\,dt}^{\approx\, e_k \cdot \Delta t \text{ (Backward Euler approx.)}}$$

$$\approx \bar{u}_{k-1} + \underbrace{\frac{K_c}{\tau_I} e_k \cdot \Delta t}_{f_1(e_k)}$$

Overall PI-controller is then

$$u_k = \bar{u}_k + \underbrace{K_c \cdot e_k}_{f_2(e_k)}$$

Problem 5



(a) $MV = \begin{bmatrix} VLV1 \\ VLV2 \\ VLV3 \\ VLV4 \\ VLV5 \\ P1 \end{bmatrix} = \begin{bmatrix} \text{Hot gas feed} \\ \text{Steam valve} \\ \text{Bypass steam} \\ \text{CW} \\ \text{Bypass water} \\ \text{Pump} \end{bmatrix}$

should be zero! (closed)

should be max (so not used for control)

$CV = \begin{bmatrix} W \text{ (power generator)} \\ L \text{ (drum level)} \\ T_A \text{ (superheat temperature)} \\ P_A \text{ (steam pressure)} \end{bmatrix}$

Main disturbance: Load W_S and heating value of fuel.

With the bypass of steam (VLV3) closed and max CW, we have four MVs and four CVs. The pairing is based on the "pair close" rule (there are also other possibilities):

VLV2 ⇔ W

P1 ⇔ L

VLV5 ⇔ T_A

VLV1 ⇔ P_A (this is a quite long loop, but pressure is not a critical

Note: Condenser pressure P_C is minimized by max cooling, so it's uncontrolled

Note: We cannot keep VLV2 open if we want to control steam pressure, P_A