TKP4140 Process Control

Department of Chemical Engineering NTNU

Autumn 2018 - Exam

Solution

Problem 1: Feedforward Control

a) The block diagram for feedforward control is shown in Figure 1. G_{md} is the transfer function for the measured disturbance, and C_{ff} is the ~~ideal~~ controller.

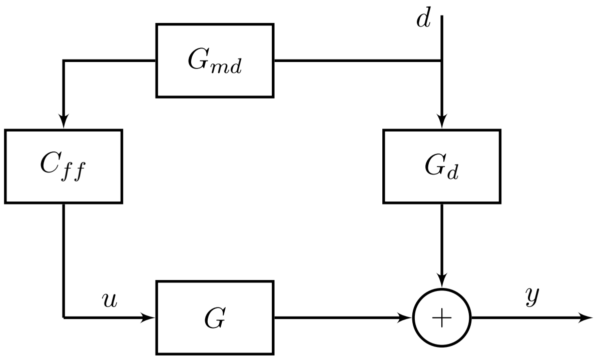

Figure 1: Block diagram for feedforward control

b) Feedforward is recommended for processes with delays in measuring y or with a large delay from the disturbance d to the process output y. If there is a delay in measuring d, then this is a disturbance for feedforward.

c) The transfer function for a perfect controller is given in Eq. 1.

$$C_{ff,ideal}(s) = -\frac{G_d(s)}{G(s)G_{md}(s)} \tag{1}$$

The conditions for a perfect realizable feedforward controller is that G_d has a large time delay and at least as many than G, which may not always be the case. *Comment* Eq. 1 is derived starting from:

$$y = Gu + G_d d \tag{2}$$

where u is given by

$$u = C_{ff}G_{md}d \tag{3}$$

By substituting Eq. 3 into Eq. 2, and assuming perfect control (i.e. $y = 0$), we obtain

$$y = 0 = GC_{ff}d + G_d d \tag{4a}$$

$$0 = (GC_{ff} + G_d)d \tag{4b}$$

$$C_{ff} = -\frac{G_d}{G} \tag{4c}$$

Note that all variables and transfer function are in Laplace domain, and s is omitted for simplicity.

d) We assume perfect measurement, $G_{md} = 1$. $C_{ff,ideal}$ is obtained by substituting G and G_d into Eq. 1:

$$C_{ff,ideal} = -\frac{3}{5(5s+1)} \tag{5}$$

In this case, $C_{ff,ideal}$ is realizable so we have

$$C_{ff} = C_{ff,ideal} = -\frac{0.6}{5s+1} \tag{6}$$

e) The response in the output y to a step in d for the three cases is illustrated in Figure 2. We can observe that in the case of no model error (orange) the process is perfectly controlled and the output y is 0. With a model error (purple), we get $y = -\frac{2.8}{5s+1}d$ (see below), and the output is almost the same as without control ($u = 0$), but with the opposite sign.

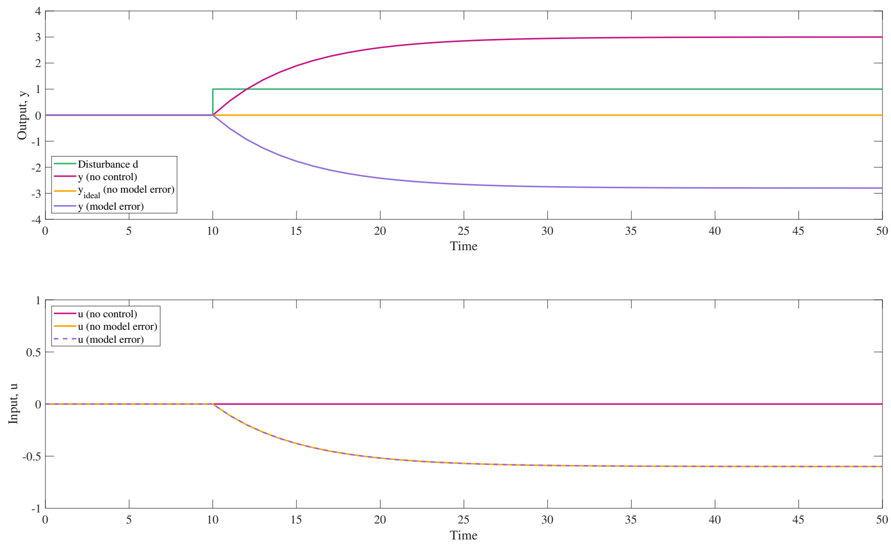

Figure 2: Comparison of step response for open loop, feedforward with no model error and feedforward with model error.

Mode details. We have:

i) with no control,

$$y = G \cdot 0 + G_d \cdot = G_d = \frac{3}{5s+1}d \tag{7}$$

ii) with no model error

$$y = (GC_{ff} + G_d)d \tag{8a}$$

$$= (-G\frac{G_d(s)}{G(s)} + G_d)d \tag{8b}$$

$$= 0 \tag{8c}$$

iii) with model error

$$
\begin{aligned}
y &= (G_{error}C_{ff} + G_{d,error})d && (9a)\\
&= (-8\frac{0.6}{5s+1} + \frac{2}{5s+1})d && (9b)\\
&= -\frac{2.8}{5s+1}d && (9c)
\end{aligned}
$$

Problem 2. Size of mixing tank for disturbance rejection

a) Assumptions:

- perfect level control, i.e. the mass in the tank is constant, and the mass flow int is equal to the mass flow out, $F_{in} = F_{out} = F$
- constant density (it follows that the volumetric flows are also equal, i.e. $q_{in} = q_{out} = q$)
- $c_v \approx c_p \approx$ constant
- the reference temperature is $T_{ref} = 0\,K$.

The dynamic energy balance for the tank is written as:

$$\frac{dH}{dt} = h_{in} - h_{out} \tag{10a}$$

$$\frac{d(mc_pT)}{dt} = Fc_p(T_F - \underbrace{T_{ref}}_{=0}) - Fc_p(T - \underbrace{T_{ref}}_{=0}) \tag{10b}$$

$$\frac{d(mc_pT)}{dt} = Fc_p(T_F - T) \tag{10c}$$

$$mc_p\frac{dT}{dt} = Fc_p(T_F - T) \tag{10d}$$

$$V\rho c_p\frac{dT}{dt} = q\rho c_p(T_F - T) \quad | : V\rho c_p \tag{10e}$$

$$\frac{dT}{dt} = \frac{q}{V}(T_F - T) \tag{10f}$$

$$\frac{dT}{dt} = \frac{1}{\tau}(T_F - T) \tag{10g}$$

where, $\tau = \frac{V}{q}$ is the residence time of the tank.

This equation is already linear. Introducing deviation variables and taking the Laplace transform of Eq. 10g gives Eq. 11. Note that all Laplace variables (T and T_F) are in deviation from the nominal point, but we drop the Δ notation.

$$sT(s) = \frac{1}{\tau}(T_F(s) - T(s)) \tag{11a}$$

$$T(s) = \frac{1}{\tau s + 1}T_F \tag{11b}$$

Conclusion. The transfer function from T_F to T is $g(s) = \frac{k}{\tau s+1}$ with $k = 1$.

b) The period of oscillations P is given by

$$P = \frac{2\pi}{\omega} = \frac{2\pi}{4} = 1.57\text{min} \tag{12}$$

c) The block diagram for the process with input u and output y is shown in Figure 3. We have $T_F(t) = A_0 \sin(\omega t)$ with $A_0 = 5$ and $\omega = 4$ rad/min. We want $T(t) = A(\omega)\sin(\omega t + \phi)$, with $A(\omega) = 1$.

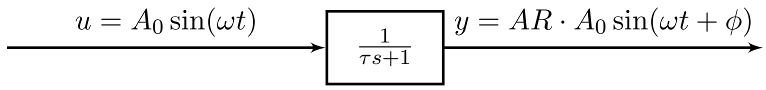

Figure 3: Block diagram for feedforward control

Here, $A(\omega) = AR(\omega)A_0$, where $AR(\omega)$ is the frequency dependent gain of $\frac{1}{\tau s+1}$. We have

$$AR(\omega) = \frac{1}{\sqrt{\omega^2\tau^2 + 1^2}} \tag{13a}$$

At frequency $\quad \omega = 4 \quad \text{rad/s} \quad$ we have$AR \cdot A_0 = 1,$ so

$$\frac{5}{4^2\tau^2 + 1} = 1 \tag{13b}$$

$$16\tau^2 + 1 = 5^2 = 25 \tag{13c}$$

$$\tau = \sqrt{\frac{25-1}{16}} = 1.225 \tag{13d}$$

$$V = \tau q = 1.225 \quad m^3 \tag{13e}$$

This result may be tested by doing a simulation of the process, shown in Figure 4.

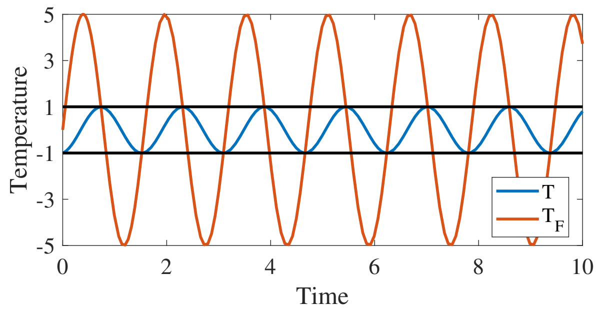

Figure 4: Simulation

Problem 3. SIMC and disturbance rejection

a) The closed-loop transfer functions:

i From the disturbance d to the output y:

$$M(s) = \frac{y(s)}{d(s)} \tag{14a}$$

$$M(s) = \frac{\text{"direct loop"}}{1 + \text{"}closedloop\text{"}} \tag{14b}$$

$$M(s) = \frac{g_d}{1 + cg} \tag{14c}$$

ii From the disturbance d to the input u is:

$$N(s) = \frac{u(s)}{d(s)} \tag{15a}$$

$$N(s) = \frac{\text{"direct loop"}}{1 + \text{"}closedloop\text{"}} \tag{15b}$$

$$N(s) = -\frac{cg_d}{1 + cg} \tag{15c}$$

b) The half rule approximation of $g(s)$ to get a first-order model is:

$$k = 10 \tag{16a}$$

$$\tau = 6 + 6/2 = 9 \tag{16b}$$

$$\theta = -0.3 + 6/2 = 3.3 \tag{16c}$$

$$g_{app}(s) \approx \frac{10e^{-3.3s}}{9s + 1} \tag{16d}$$

The PI-controller tuning are found by applying the SIMC tuning method to $g_{app}(s)$. For "tight control", we select $\tau_C = \theta = 3.3$.

$$K_c = \frac{1}{k}\frac{\tau}{\tau_C + \theta} \tag{17a}$$

$$K_c = \frac{1}{10}\frac{9}{2 \cdot 3.3} \tag{17b}$$

$$K_c = 0.136 \tag{17c}$$

$$\tau_I = \min(\tau, 4(\tau_C + \theta)) = \tau_I = \min(9, 4(2 \cdot 3.3)) = 9 \tag{17d}$$

$$c(s) = K_c(\frac{\tau_I s + 1}{\tau_I s}) \tag{17e}$$

$$c(s) = 0.136(\frac{9s + 1}{9s}) \quad \text{PI-controller} \tag{17f}$$

c) i To plot $|g_d|$ we identify that there is a break frequnecy at $\omega = 16 = 0.0167\, rad/s$, here the slope changes from 0 to -2 (on a log-log scale). The low gain frequency is $|g_d(0)| = 10$. The time delay has no effect on the gain.

$$|g_d| = \frac{10}{(\sqrt{(6\omega)^2 + 1})^2} = \frac{10}{36\omega^2 + 1} \tag{18}$$

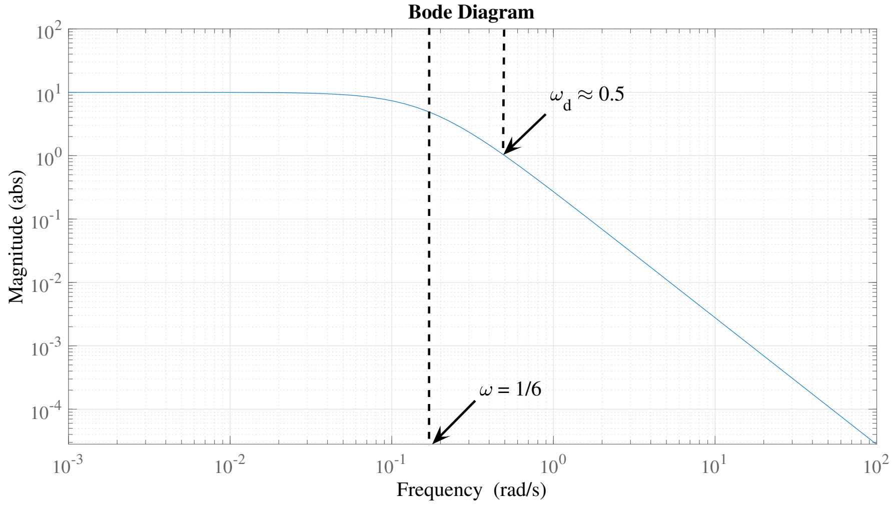

Figure 5: Magnitude plot of g_d

The magnitude of g_d is shown in Figure 5.

ii Let $g(s) = g_d(s)$. Quick solution. Note that $N(s) = -T(s)$, where $T(s)$ is the closed loop setpoint response, $T(s) = \frac{g\,C}{1+gc}$. The steady-state gain of $N(s)$ is -1 (N(s=0) = -1), beause T(0) = 1 with integral action.
With SIMC, we design $T(s) = \frac{e^{(-\theta s}}{\tau_C s+1}$, so with $\tau_C = \theta = 3.3$, we get a first-order response with a delay of 3.3 and time constant of 3.3.

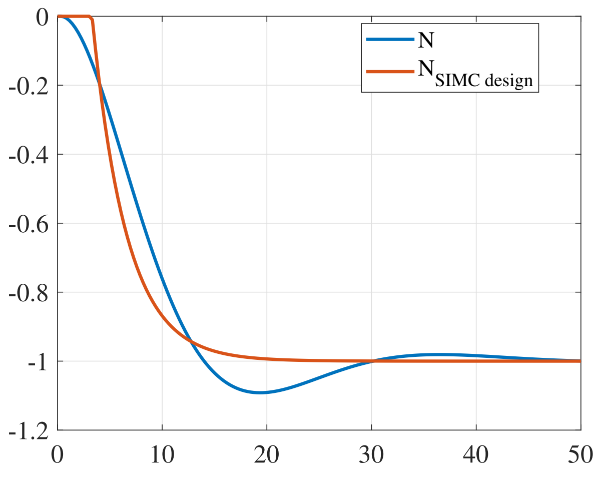

Figure 6: ~~Magnitude plot of g_d~~ Response of u to d=1 with PI-control (the red line is the approximate solution that you are expected to do; the blue line is showing the actual u(t), but this not required).

d) The speed of response is limited by the frequency ω_d (i.e. where $|g_d| = 1$ for a scaled model), but also by the effective time delay, according to:

$$\omega_d \leq \omega_C \leq 1/\theta \tag{19}$$

where, $\omega_C = 1/\theta = 1/3.3 = 0.3$

and, $\omega_d \approx 0.5$, read from the Bode plot in Figure 5. which leads to

$$0.5 \leq \omega_C \leq 0.3 \tag{20}$$

which does not have a feasible solution. We conclude that it is not possible to design a PI-controller to make y(t) acceptable.
For a PID-controller, $\tau_C = 0.3$ and $\omega_C = 1/0.3 = 3.3$, and it is possible to find τ_C such that:

$$\omega_d \leq \omega_C \leq 1/\theta \tag{21a}$$

$$0.5 \leq \omega_C = 1/\tau_C \leq 3.3 \tag{21b}$$

For "tight control", $\tau_C = 0.3$ is a solution of Eq. 21a.

The rest is not required:

The output response to a step disturbance with magnitude 1 both for PI and PID controllers is shown in Figure 7. For the PI-controller, $\max(y(t) \approx 4$, while for the PID controller, $\max(y(t) \approx 0.5$, which is more acceptable. Note. For the PID controller implemented in series form, the filter time constant is $\tau_F = 0.01$.

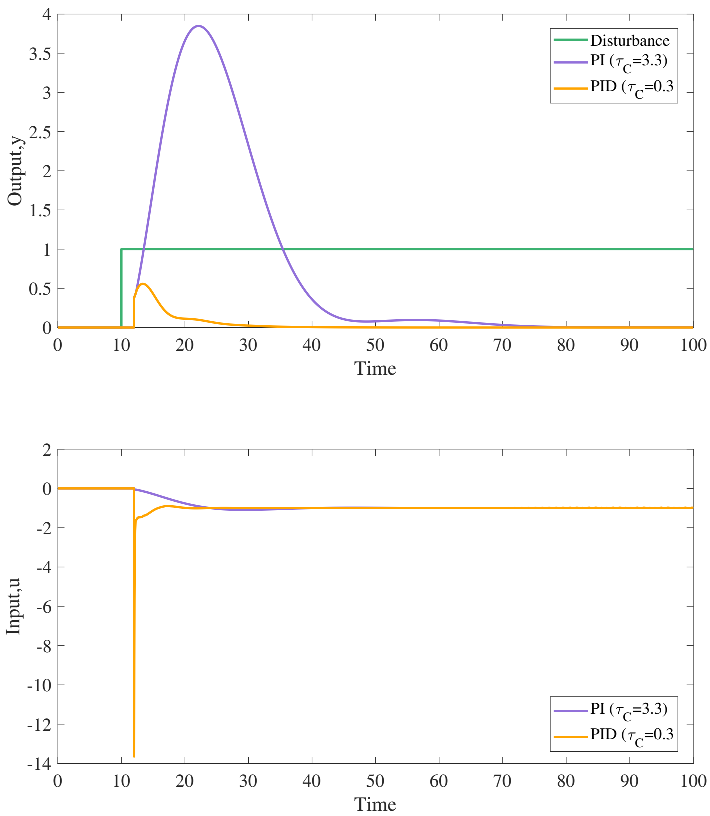

Figure 7: Output response to a step disturbance of magnitude 1 for PI (purple) and PID (orange)

Problem 4. Mixing tank with changing control objectives

a) Total mass balance (at steady-state):

$$F_1 + F_2 = F_3 \tag{22}$$

Component S mass balance at steady-state

$$F_1 x_{S1} + F_2 x_{S2} = F_3 x_{S3} \tag{23}$$

F_2 is pure water $\rightarrow x_{S2} = 0$, so we get:

$$F_1 x_{S1} = F_3 x_{S3} \tag{24}$$

At the nominal point $x_{S1} = 0.1$. Substituting in Eq. a), and solving for F_3 gives:

$$F_3 = \frac{F_1 x_{S1}}{x_{S3}} = \frac{1 \cdot 0.5}{0.1} = 5\, kg/s \tag{25}$$

F_2 is calculated from the total mass balance:

$$F_2 = F_3 - F_1 = 5 - 1 = 4\, kg/s \tag{26}$$

x_{E3} is calculated from the component E mass balance:

$$F_1 x_{E1} + F_2 x_{E2} = F_3 x_{E3} \tag{27a}$$
$$F_1 x_{E1} = F_3 x_{E3} \tag{27b}$$
$$x_{E3} = \frac{F_1 x_{E1}}{F_3} \tag{27c}$$
$$x_{E3} = \frac{1 \cdot 0.002}{5} \tag{27d}$$
$$x_{E3} = 0.0004 \tag{27e}$$

At steady-state, $x_{E3} = 0.0004$, and both requirements for sugar ($x_{S3} = 0.1$) and E concentration ($x_{E3} =\leq 0.001$) in the product stream are fulfilled.

b) We consider the response from $u = F_2$ and $d = F_1$, so we can assume that Note that x_{S1} and x_{E1} are constant. eq. 22 is always linear, and becomes in terms of deviation variables:

$$\Delta F_1 + \Delta F_2 = \Delta F_3 \Rightarrow \Delta F_3 = u + d \tag{28}$$

Linearizing Eq. 24 yields (noting that x_{S1} is constant and $x_{S2} = 0$).

$$\Delta F_1 x_{S1}^* = F_3^* \Delta x_{S3} + \Delta F_3 x_{S3}^* \tag{29a}$$
$$\Delta x_{S3} = \frac{\Delta F_1 x_{S1}^* - \Delta F_3 x_{S3}^*}{F_3^*} \tag{29b}$$
$$\text{Substituting the nominal values gives:} \quad y_1 = \frac{0.5 \cdot d - 0.1 \cdot (u + d)}{5} \tag{29c}$$
$$\text{The linear model becomes:} \quad y_1 = -0.02u + 0.08d \tag{29d}$$

The steady-state gain from u to y_1 is $k_{u,y_1} = -0.02$.

Similarly, linearizing for component E mass balance gives:

$$\Delta x_{E3} = \frac{\Delta F_1 x^*_{E1} - \Delta F_3 x^*_{E3}}{F^*_3} \tag{30a}$$

$$y_2 = \frac{0.002 \cdot d - 0.0004 \cdot (u + d)}{5} \tag{30b}$$

$$y_2 = -0.00008u + 0.00032d \tag{30c}$$

The steady-state gain from u to y_2 is $k_{u,y_2} = -0.00008$.

c) At the nominal point, $y_2 = x_{E1}$ is well below the maximum concentration allowed, and we only need to control $y_1 = x_{S3}$ at its specification. We use u F_2 to control the sugar content in the product stream, as shown in the block diagram in Figure 8.

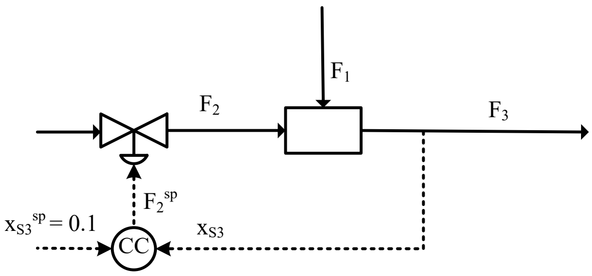

Figure 8: Control structure for the nominal point

d) For a pure I-controller, the setting are calculated by applying the SIMC tunings rules to the transfer function from u to y_1 given Eq. 29d. There are no dynamics, that is $\tau \approx 0$, but there is a measurement delay of 8 seconds which has to be accounted for in designing the controller, that is $\theta = 8\,s$. We get

$$K_C = \frac{1}{k}\frac{\tau}{\tau_C + \theta} = 0 \tag{31a}$$

$$\tau_I = \min(\tau, 4(\tau_C + \theta)) = \tau = 0 \tag{31b}$$

$$\tag{31c}$$

Since $\tau_I = 0$, we get

$$c(s) = \frac{K_c(\tau_I s + 1)}{\tau_I s} = \frac{K_c}{\tau_I s} = \frac{K_I}{s} \tag{32}$$

which is a pure I-controller. The value of K_I using the SIMC rules is:

$$K_I = \frac{1}{k}\frac{\tau}{\tau_C + \theta}\frac{1}{\tau} = \frac{1}{k}\frac{1}{\tau_C + \theta} \tag{33}$$

Assuming "tight control" ($\tau_C = \theta = 0$) gives,

$$K_I = -\frac{1}{0.02}\frac{1}{8+8} = -3.125 \tag{34}$$

e) If $x_{E1} = 0.006$, and $F_2 = 4\,kg/s$ (at the nominal point), then using the steady state component E mass balance (Eq. 29b gives,

$$x_{E3} = \frac{0.0006}{5} = 0.0012 > 0.001$$

. This exceeds the allowed values, so we need to add extra water and give up controlling $y_1 = x_{S3} = 0.1$.

Recalculating the stream F_2 needed to keep the requirement $x_{E3} \leq 0.001$:

$$F_3 = \frac{x_{E1F1}}{x_{E3}} = \frac{0.006}{0.001} = 6\,kg/s = 6 - 1 = 5\,kg/s \tag{35}$$

The corresponding steady-state concentration of sugar in the product stream is

$$x_{S3} = \frac{x_{S1F1}}{F_3} = \frac{0.5}{6} = 0.0833 \tag{36}$$

which as expected is not at setpoint, but this is the closest we an get.

f) In the extreme case of a high disturbance x_{E1}, x_{E3} goes beyond its specification. Note that it is **required** to keep $x_{E3} \leq 0.001$, but it is **desired** $x_{S3} = 0.1$, meaning that it is more important to keep x_{E3} at its specification than x_{S3}.

Thus, in such “extreme case“, we need to give up controlling x_{S3}, and instead use $u = F_2$ to control $y_2 = x_{E3}$ at its specification. Since there is a single manipulated variables and two control variables, we need to use a selector to decide which controller is active (i.e. which concentration is controlled at its specification).

Yes, we always need to use anti-windup for all controllers when using a selector. The controller that is not selected keeps on integrating the error, because the input u it is calculating is not applied to the plant, and therefore there will be an error between the setpoint (e.g. x_{E3}^{sp}) and the uncontrolled process value (e.g.x_{E3}).

The control structure for this case is shown in Figure 9. We use a max selector because in the extreme cases of $x_{E3} = 0.006$, F_2 has to be increase from its normal value to dilute the product, and reach the required specification for x_{E3}.

Extra (not required).
The simulation results for a step disturbance $x_{E1} = 0.006$ given at $t = 0\,s$ using the control structure is Figure 9 are shown in Figure 10. Note that the I-controller for E was tuned based on the extreme case.

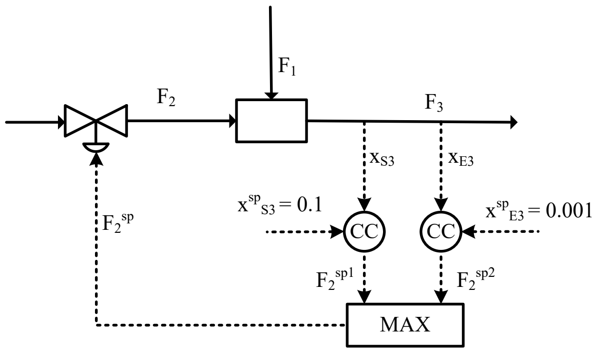

Figure 9: Control structure that handles both the nominal and the extreme cases

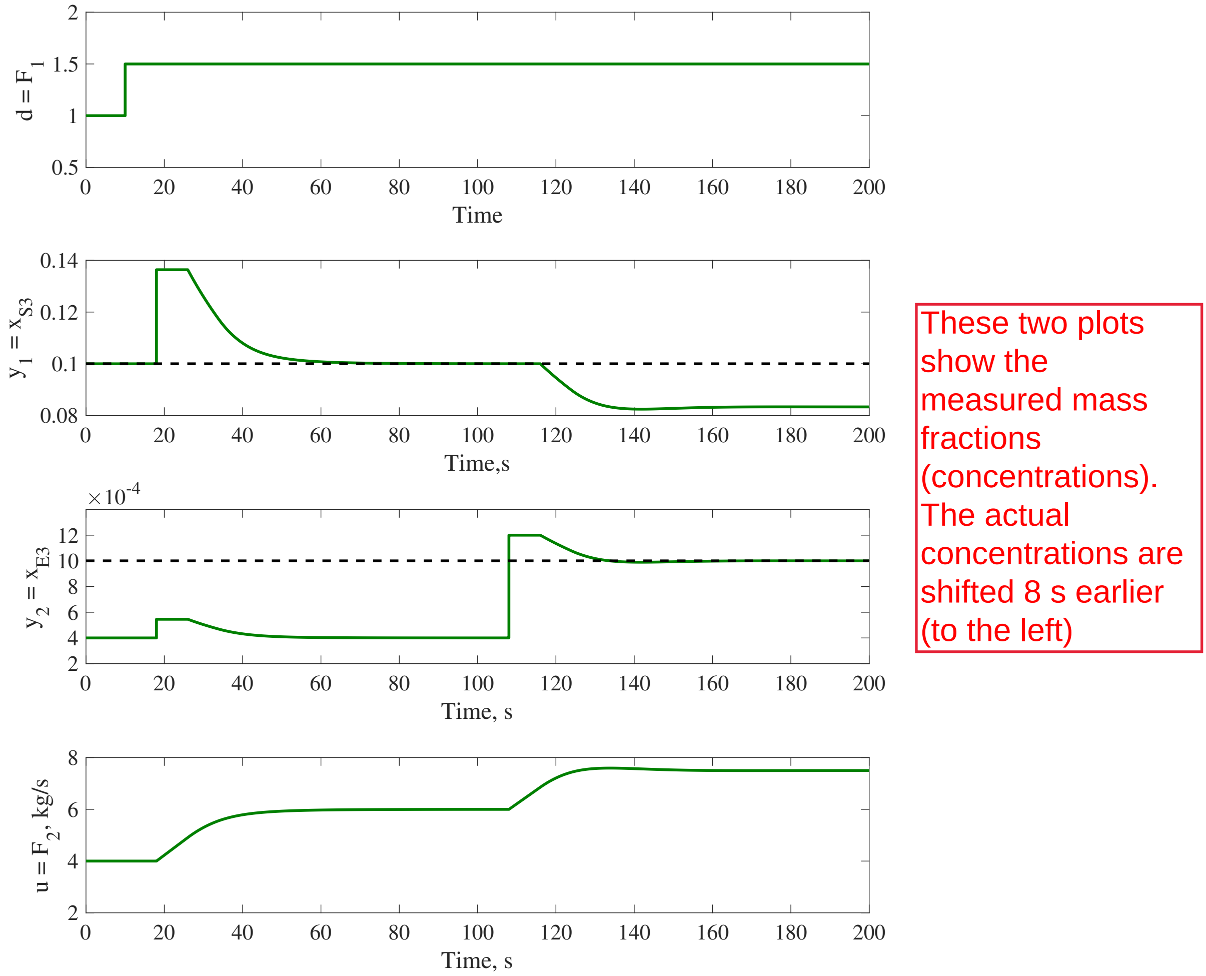

Figure 10: Simulation results for a step disturbance $F_1 = 1.5\,kg/s$ at $t = 10\,s$ and $x_{E1} = 0.006$ at $t = 100\,s$ (extreme case). The black dotted lines show the concentration specification for x_{S3} and x_{E3} respectively. In the normal case, the controller is controlling $y_1 = x_{S3}$ at $y_{1s} = 0.1$, while in the extreme case, the controller is controlling $y_2 = x_{E3}$ at $y_{2s} = 0.001$.