TKP 4140 - Process Control
2017 Solution (preliminary)

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Problem 1

(a) We assume constant density.
Overall mass balances and component mass balances (acid) for the two tanks then given

Tank 1:

$$(1)\quad \frac{dV_1}{dt} = q_A + q_{W1} - q_1 \qquad [\text{mol/s}]$$

$$(2)\quad \frac{d(c_1V_1)}{dt} = q_A c_A + q_{W1} \overset{0}{c_{W1}} - q_1 c_1 \qquad [\text{mol acid/s}]$$

Tank 2:

$$(3)\quad \frac{dV_2}{dt} = q_1 + q_{W2} - q_2 \qquad [\text{mol/s}]$$

$$(4)\quad \frac{d(c_2V_2)}{dt} = q_1 c_1 + q_{W2} \overset{0}{c_{W2}} - q_2 c_2 \qquad [\text{mol acid/s}]$$

(b) Transfer matrix G_1

$$\begin{array}{c|ccc} & q_A & q_{W1} & q_1 \\ \hline V_1 & \frac{1}{s} & \frac{1}{s} & -\frac{1}{s} \\ c_1 & \frac{k_A}{\tau_1 s+1} & \frac{k_{W1}}{\tau_1 s+1} & 0 \end{array}$$

$$\tau_1 = \frac{V_1^*}{q_A^* + q_{W1}^*} = \frac{V_1^*}{q_1^*} = \text{residence time}$$ (could be derived by linearizing (2)) but not required.

$k_A > 0$
$k_{W1} < 0$

(c) (1) Feedback

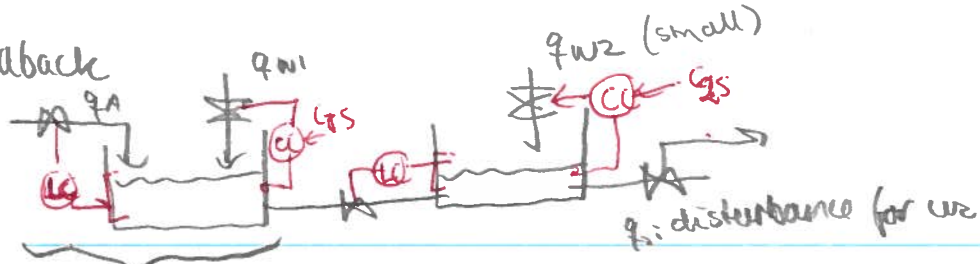


Fig. 1

Could interchange LC and CC, the largest flow should be used for level control to reduce interactions, so here we have assumed $q_A > q_{W1}$, but it could be opposite. Actually, this is probably more likely since it is given that q_{W1} is large!

2) No measurement of C_1

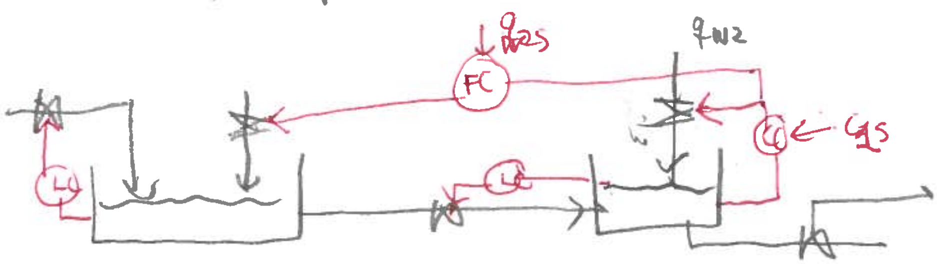


Fig. 2

- Also here the LC and FC could be interchanged (tank 1).
- The FC may alternatively be called VPC (valve position controller) as it resets the position of the valve for q_{w2} to some middle position (say 50% open)
- Comment on structure 1 (feedback): The setpoint for C_1 needs to be decided. It could be set by a FC (or VPC) similar to Fig. 2.

Problem 2.

Overall process:

(a) $$G(s) = \frac{3e^{-3s}}{(100s+1)(10s+1)} \approx \begin{cases} \frac{3}{105s+1} e^{-1\ldots} & \text{(for PI)} \\ \text{unchanged} & \text{(for PID)} \end{cases}$$

Here I would go for the PID, since in the original model $\tau_2 = 10 > \theta = 3$. But let us design both.

PI-controller:

$$K_c = \frac{1}{k}\cdot\frac{\tau}{\tau_c+\theta} = \frac{1}{3}\,\frac{105}{18+18} = 0.972$$

$$\tau_I = \min(\tau_1, 4(\tau_c+\theta)) = \min\left(105, \underset{144}{\underline{4\cdot 2\cdot 18}}\right) = 105$$

Cascade-PID:

$$K_c = \frac{1}{3}\,\frac{100}{3+3} = 5.6$$

$$\tau_I = \min(\tau_1, 4(\tau_c+\theta)) = \min(100, 4\cdot 2\cdot 3) = 24$$

$$\tau_D = \tau_2 = 10$$

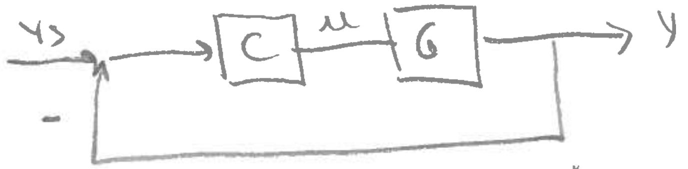
(b) $$G(s) = \frac{3e^{-22s}}{(100s+1)(10s+1)} \approx \frac{3}{105s+1}e^{-27s}$$

PI: ($\tau_c = \theta$) $$K_c = \frac{1}{k}\,\frac{\tau}{\tau_c+\theta} = \frac{1}{3}\,\frac{105}{2\cdot 27} = 0.65$$

$$\tau_I = \tau = 105$$

Here PI is recommended since $\tau_2 = 10 < \theta = 22$.

Problem 3.



(a) $y = T_1 \, y_s \qquad T_1 = \dfrac{\text{"direct"}}{1+\text{loop}} = \dfrac{GC}{1+GC}$

$u = T_2 \, y_s \qquad T_2 = \dfrac{C}{1+GC}$

(b) $G = \dfrac{k}{\tau_1 s+1} \qquad (\theta = 0)$

SIMC-rule with $\tau_c = \tau_1$

$K_c = \dfrac{1}{k}\dfrac{\tau_1}{\tau_c+\theta} = \dfrac{1}{k}\dfrac{\tau_1}{\tau_1} = \dfrac{1}{k}$

$\tau_I = \min\left(\tau_1, 4(\tau_c+\theta)\right) = \min\left(\tau_1, 4\tau_1\right) = \tau_1$

Get $GC = \dfrac{k}{\tau_1 s+1} \cdot K_c \dfrac{\tau_I s+1}{\tau_I s} = \cancel{\dfrac{k}{\tau_1 s+1}} \cdot \dfrac{1}{\cancel{k}} \dfrac{\cancel{\tau_1 s+1}}{\tau_1 s} = \underline{\underline{\dfrac{1}{\tau_1 s}}}$

(very simple!)

$T_1 = \dfrac{GC}{1+GC} = \dfrac{1/\tau_1 s}{1+1/\tau_1 s} = \dfrac{1}{\tau_1 s+1}$ (as expected when we use SIMC!!)

$T_2 = \dfrac{C}{1+GC} = \dfrac{\frac{1}{k}\frac{(\tau_1 s+1)}{\tau_1 s}}{1+1/\tau_1 s} = \dfrac{\frac{1}{k}(\tau_1 s+1)}{\tau_1 s+1} = \underline{\underline{\dfrac{1}{k}}}$

(very simple again!!)

Comment:

This makes sense, we with this controller make the closed-loop response have the same dynamics as the open loop response. So just stepping u gives the desired $T = \dfrac{1}{\tau_1 s+1}$.

Problem 4.

We want to avoid pairing on negative RGA.

(a) There are two possible pairings on positive values

$$
RGA(0) = \begin{array}{cccc} u_1 & u_2 & u_3 & \\ \left[\begin{array}{ccc} 0.45 & 0.22 & 0.33 \\ -0.82 & 2.45 & -0.64 \\ 1.36 & -1.67 & 1.31 \end{array}\right] & & & \begin{array}{c} y_1 \\ y_2 \\ y_3 \end{array} \end{array}
$$

Alt-1

Alt-2

Probably, alternative 2 is probably the best because of the delay on the 11-element in $G(s)$. Conclusion (Alt-2):

$u_1 \leftrightarrow y_3$

$u_2 \leftrightarrow y_2$

$u_3 \leftrightarrow y_1$

(b) The RGA is defined as $\lambda_{ij} = \frac{g_{ij}}{\hat{g}_{ij}}$

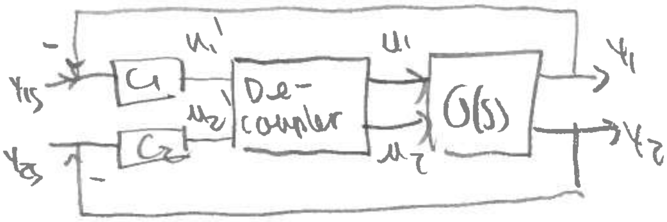
where g_{ij} is the open-loop gain from u_j to y_i (with the inputs u_k constant) and $\hat{g}_{ij}$ is the closed-loop gain with the other outputs controlled. We here consider steady-state RGA. Ideal case is $\lambda_{ij} = 1$ for pairing.

1. $\lambda_{ij} < 0$: Gain reversal, that is, gain from u_j to y_i will change sign when the other loops are closed. This may obviously lead to instability, at least with integral action in the controller.

2. $0 < \lambda_{ij} < 1$: We then have $|\hat{g}_{ij}| > |g_{ij}|$ which means that the interactions increase the gain. This may give stability problems, especially with λ_{ij} close to 0.

3. $\lambda_{ij} > 1$: The interactions reduce the gain, that is, the loops "fight each other". This may yield poor performance (slow response) when λ_{ij} is large.

c) Decoupling is used to get acceptable performance when using single-loop controllers.



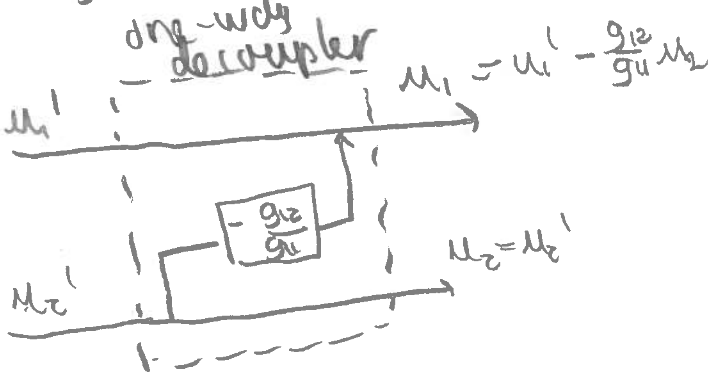
A one-way decoupler may be designed by considering u_2 as a disturbance on y_1.

$$y_1 = g_{11} u_1 + g_{12} u_2$$

Design a "feedforward" controller based on measuring u_2. Ideal case is to have $y_1 = 0$ which gives

$$u_1 = -\frac{g_{12}}{g_{11}} u_2$$

so the one-way decoupler is



To be realizable we need $-\frac{g_{12}}{g_{11}}$ to be realizable. This means that the delay in g_{11} should be smaller than in g_{12} and g_{11} should be lower order (# poles - # zeros) than g_{12}, that is, g_{11} should have a more direct effect, that is, "pair close"!

Problem 5.

(a) $G(s) = \frac{1-5s}{8s+1}$ $\quad C(s) = K_c\left(\frac{\tau_I s+1}{\tau_I s}\right)$ with $\tau_I = 8$

loop transfer function

$$L(s) = G(s)C(s) = \frac{(1-5s)}{(8s+1)} \cdot \frac{K_c(8s+1)}{8s} = K_c\frac{(1-5s)}{8s}$$

Closed-loop poles
$1 + L(s) = 0 \Rightarrow 1 + \frac{K_c(1-5s)}{8s} = 0 \Rightarrow 8s + K_c - 5K_c s = 0$

$\Rightarrow (8-5K_c)s + K_c = 0$

Want both coefficients to have same sign to be stable. This gives

$K_c > 0$

$8 - 5K_c > 0 \Leftrightarrow K_c < \frac{8}{5} = \underline{\underline{1.6}}$ $K_{c,max}$

(b) SIMC. We first have to approximate as a first-order plus delay process

$$G(s) = \frac{1-5s}{8s+1} \approx \frac{e^{-5s}}{8s+1}$$

SIMC-rules with $\tau_c = \theta = 5$

$$K_c = \frac{1}{k}\frac{\tau}{\tau_c+\theta} = \frac{1}{1}\cdot\frac{8}{5+5} = 0.8$$

$$\tau_I = \min(\tau, 4(\tau_c+\theta)) = \min(8, 40) = 8$$

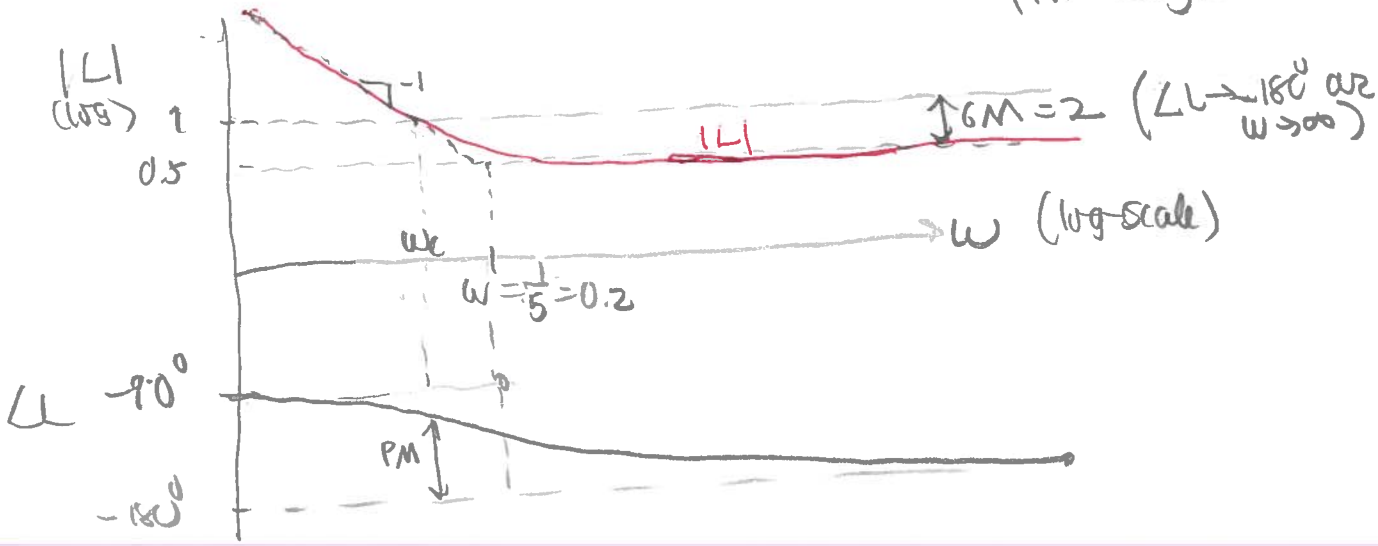
This is the same as the controller from part a) but with $K_c = 0.8$. Since $K_{c,max} = 1.6$, we have that $GM = 2$.

This can also be found from the Bode-plot:

$$L(s) = \frac{1-5s}{8s+1} \cdot 0.8 \, \frac{8s+1}{8s} = 0.1 \, \frac{1-5s}{s} = \begin{cases} \to \frac{\ldots}{5} \ \text{as } s \to 0 \\ \to 0.5 \ \text{as } s \to \infty \end{cases}$$

Exact values: $|L(j\omega)| = \dfrac{0.1\sqrt{1+(5\omega)^2}}{\omega}$

$$\angle L(j\omega) = \arctan(-5\omega) \underbrace{- \frac{\pi}{2}}_{\text{from integrator}}$$



From the $\overbrace{\text{plot}}^{\text{asymptotic}}$ we see that $\omega_c \approx \frac{0.2}{2} = 0.1$ rad/s, but we make a small table to be more accurate

ω	$\lvert L(j\omega)\rvert$
0.1	$0.1\sqrt{1+(5\cdot 0.1)^2}/0.1 = 1.11$
0.11	1.038
0.115	1.003

Conclusion: $\omega_c = 0.115$ rad/s

This gives $\angle L(j\omega_c) = -\arctan(5 \cdot 0.115) - \frac{\pi}{2} =$

$= -0.522 - 1.571 = -2.091$ rad

So $PM = \pi + \angle L(j\omega_c) = 3.14 - 2.091 = 1.049$ rad
$(= 60.1^\circ)$

Delay margin $= \Delta\theta = \dfrac{PM}{\omega_c} = \dfrac{1.049 \text{ rad}}{0.115 \text{ rad/s}} = \underline{\underline{9.12 \text{ s}}}$

Problem 6.

(a) F = 30% (it is meant F = +30%) is what is given in the figure, so the red curve corresponds to d = 1 (step). Since no scaling is needed for y (=T), the red curve directly gives

G_d:

$$G_d = \frac{3}{\tau_d s + 1}$$

τ_d is where y is 63%, that is when y = T = 3 · 0.63 = 1.89 and we find $\tau_d \approx 3$

Q = 50% (it is meant Q = +50%) corresponds to u = 1 so the response in the figure is with u = 0.2 (Q = +10%). So the steady-state gain from u to y is 5 times larger than for the blue curve. Otherwise, the dynamics are the same and we derive

$$G = \frac{5\, e^{-s}}{3\, s+1}$$

since it reaches 0.63 at $t \approx 4 = \tau + \theta$

(b) - We have $|G| > |G_d|$ at all frequencies so there is no problem with input constraints.

- But what about disturbances? Can we react fast enough? We have a delay $\theta = 1$, so we need $\omega_c < \frac{1}{\theta} = 1$

But for acceptable disturbance rejection we need $\omega_c > \omega_d \approx \frac{k_d}{\tau_d} = \frac{3}{3} = 1$. Hmm... this is a borderline case, so we may not be OK. From the guessed response, I think we will not be OK with PI since y>1.