Problem 1. 
\[ g(s) = \frac{s^2 + 8}{s+2} \quad \left( = \frac{-0.125s + 1}{0.5s + 1} \right) \]

(a) 
\[ g(s) = -1 + \frac{10}{s+2} = -1 + \frac{5}{0.5s + 1} \]
\[ y(s) = g(s) \cdot u(s) \text{ with } u(s) = \frac{1}{s} \]
Inverse transform
\[ y(t) = -1 + 5 \left( 1 - e^{-5t} \right) = -1 + 5 \left( 1 - e^{-\frac{5}{0.5}t} \right) \]

\[ u(t) \]

This is an inverse response as can also be expected because of the RLHP-zero.

(b) Proportional control.
Closed-loop polynomial (characteristic equation):
\[ 1 + \frac{1}{g(s)} = 1 + \frac{k_c}{s+2} = 0 \]
\[ s^2 + 2 + k_c(-5 + 8) = 0 \]
\[ \frac{a_0}{a_1} = \frac{a_0}{(1-k_c)s + 2 + 8k_c} = 0 \]
Stability \( \Rightarrow \) coefficients \( a_0 \) and \( a_1 \) must same sign

\[ \text{sign of } a_0 \begin{cases} > 0 & \text{if } k_c > 0 \\ < 0 & \text{if } k_c < 0 \\ = 0 & \text{if } k_c = 0 \end{cases} \]

Both coefficients have same sign for
\[ -4 < k_c < 1 \]

(Comment: \( k_c > 0 \) is preferred in practice since we want to have negative feedback)
(c) $K_c = 0.5$

Self-potential response: $y = \frac{gc}{gc} \cdot y_s$

\[
\frac{gc}{gc} = \frac{0.5s + 8}{s + 2} = \frac{-0.5s + 4}{s + 2 - 0.5s + 4} = \frac{-0.5s + 4}{0.5s + 6}
\]

\[
= \frac{0.5s + 1}{0.5s + 1} = \frac{0}{0.0833s + 1}
\]

Note: $T(0) = \frac{4}{6} = 0.67$ (steady-state == not 1 because we do not have integral action)

$T(\infty) = \frac{4}{6} \cdot \left(\frac{0.125}{0.0833}\right) = -1$

(d) $K_c = 0.5$

Gain margin is clearly 2 because we have already found that $K_c = 1$ gives instability. Can also find this from Bode plot.

$L(s) = 0.5 \frac{s + 8}{s + 2}$

$|L| = 1$ at $\omega = 4$ (can see directly from asymptote)

$GM = \frac{1}{|L(j\omega)|} \quad \omega_{\infty} = \infty \quad \omega_{15\%} = 2 \quad GM = 0.5$

$PM = 90^\circ$ (at frequency $\omega_c = 4$). Allowed time delay: $\theta = \frac{\theta_m}{\omega_c} = \frac{\pi/2}{4} = 0.39\pi$
(e) SMC PI-controller.

First make first-order plus delay approximation

\[ g(s) = \frac{4 \cdot 0.125s + 1}{0.5s + 1} \approx \frac{4 \cdot e^{-0.125s}}{0.5s + 1} \quad k = 4 \quad c = 0.125s \quad T = 0.5 \]

SMC-controller with \( T_c = \alpha = 0.125s \)

\[ k_c = k \cdot \frac{c}{k_c + \alpha} = \frac{4 \cdot 0.5}{2 \cdot 0.125} = 0.5 \]

\[ \bar{T}_I = \min \left( T_c, 4(\bar{T}_I + \alpha) \right) = \min \left( 0.5, 1 \right) = 0.5s \]

The initial response is similar to the P-controller (same gain \( k_c = 0.5 \)), but the integral action will eliminate the steady-state offset.

![Graph of control system output](image)
Problem 3

(a) Assume Tw = constant

(b) Energy balance for air

\[ \frac{d(mgT)}{dt} = Q - Q_0 - Q_W = (UA)(T - T_0) - (UA)w(T - Tw) \]

where:

\[ Q_0 = UA(T - T_0) \]

\[ Q_W = (UA)w(T - T_W) \]

(c) Steady-state

\[ Q = Q_0 + Q_W \]

\[ = UA(T - T_0) + (UA)w(T - T_W) \]

\[ = 0.8 \text{ kW} \]

(d) The model is already linear. Take deviation variables.

\[ mCV \frac{dT}{dt} = \Delta Q - (UA)(A \Delta w) \Delta T + UA \Delta T_0 + (UA)w \Delta Tw \]

Laplace

\[ (mCV \cdot s) \cdot \mathcal{L}(T) = \mathcal{L}(\Delta Q) - (UA)(A \mathcal{L}(w)) \cdot \mathcal{L}(\Delta T) + UA \mathcal{L}(\Delta T_0) + (UA)w \mathcal{L}(\Delta Tw) \]

To find effect of \( \Delta Q \) on \( \Delta T \), we set \( \Delta T_0 = 0 \).

\[ (CG + 1) \Delta T = \frac{1}{mCV} \Delta Q \]

\[ C = \frac{mCV}{UA + (UA)w} \]

\[ Gel \ k = \frac{1}{UA + (UA)w} = \frac{1}{0.8 + 0.04} = \frac{1}{0.84} \frac{1}{\text{KW}} \]

\[ C = \frac{200 \text{ kJ/k}}{0.84 \text{ kJ/k}} = 238 \text{ s} \ (\approx 3.97 \text{ min}) \]
With wall dynamics.

(i) Air: \[ \frac{d}{dt} (m_r T) = Q - Q_w - Q_o \quad \text{(see above)} \quad (1) \]

Wall: \[ \frac{d}{dt} (m_w (g_w T_w)) = Q_w = (UA)_w (T - T_w) \quad (2) \]

(ii) (1) givener (see above)

\[ (2s + 1) T(s) = k_w T_w(s) + k \cdot Q(s) + k_d \cdot T_o(s) \quad (3) \]

where \[ T = \frac{m_r}{(UA)_w} = \frac{mg}{(UAw)_w} = 238 \quad \text{s} \]

(2) givener upon linearization

\[ m_w(g_w \frac{dT_w}{dt} = (UA)_w (\Delta T - \Delta T_w) \]

Laplace

\[ (2s + 1) \Delta T_w(s) = \Delta T \]

where \[ T_w = \frac{m_r g_w}{(UA)_w} = \frac{5000}{0.8} = 6250 \text{s} (=104 \text{min}) \]

(iii) Insert (4) in (3), assuming \( \Delta T_o = 0 \) (\( T_o(s) = 0 \)):

\[ (2s + 1) T(s) = k_w \cdot \frac{T(s)}{2s + 1} + k \cdot Q(s) \]

\[ T(s) = \frac{k \cdot (2s + 1)}{(2s + 1) (2s + 1) - k_w} \cdot Q(s) \]

\( g(s) \) solution: \( k = 0.84, \quad k_w = 0.84 \)

\[ T = 4 \text{ min}, \quad T_w = 104 \text{ min} \]

(iv) Closed-loop responses

See separate page.
We can further factorize $g(s)$ (not required). Get:

$$g(s) = \frac{k_1 (T_w s + 1)}{(C_i s + 1) (T_e s + 1)}$$

$$k_i = \frac{k}{1 - k_{w}} = \frac{1}{0.84 - 0.08} = 25$$

$$T_w = 104 \text{ min}$$
$$T_i \approx T = 238 \text{ min}$$
$$T_e \approx 250 \text{ min} = 2083 \text{ min} - 35 \text{ hr}$$
Simulations Problem 2b:

Matlab:

```matlab
s=tf('s')
k1=0.8/0.84; k0=1/0.84; tauw = 6250/60; tau = 238/60; % time in min
\( g = \frac{k0\cdot(\tau w\cdot s+1)}{((\tau\cdot s+1)\cdot(\tau w\cdot s+1)-k1)} \)
\( g0 = \frac{k0}{(\tau\cdot s+1)} \)
figure(1), step(g,g0,'--red',3*60)

%Tune based on initial response, g0=k0/(tau*s+1)
%Kc=1/k0*(tau1/tauc+theta), tau1=min(tau1,4*(tauc+theta)
%Have theta=0. Select tauc=tau1 = 4 min. Get Kc=1/k=0.84, tau=tau=4
Kc = 0.84, tau1=4; tau2=0;
sim tuneid4, figure(2), plot(Ti,d,y), figure(3), plot(Ti,u)
```

Open-loop response to step in Q (blue line) (Red is the simplified model)

Closed-loop response for three PI-controllers

```
Output T
Red: tauc=0.25*tau
Green: tauc=tau
Blue: tauc=4*tau
```

Note that the response in y(t) is not much affected by the wall dynamics, but the input Q is reduced as the wall gets hotter and the term Qw approaches zero. The time constant for this reduction in Q (and Qw) is approximately tauw, which is about 100 min.
Problem 3

(a) Assume \( y(t) = \frac{1}{4s+1} \) for \( t = 0 \)

\[ K_c = \frac{1}{K} \frac{k}{0.25z} \quad T_c = \min \{ t, 4(0.25z) \} = 2 \quad (z = 4s) \]

Controller 2 \((C_2), T_c = t\)

\[ K_c = \frac{1}{K} \frac{k}{z} = \frac{1}{k} \quad T_c = \min \{ t, 4t \} = 4t \quad (z = 16s) \]

(b) Closed-loop step response

\[ \frac{y}{Y_s} = \frac{g_c}{1 + g_c} = \frac{1}{2s + 1} = \begin{cases} \frac{1}{0.25z + 1} = \frac{1}{s + 1} & \text{for } C_1 \\ \frac{1}{2s + 1} = \frac{1}{4s + 1} & \text{for } C_2 \end{cases} \]

\[ \frac{u}{Y_s} = \frac{C}{1 + g_c} = g \left( \frac{g_c}{1 + g_c} \right) = 2s + 1 \cdot \frac{1}{2s + 1} = \begin{cases} \frac{8s + 16}{16(k + k)} & \text{for } C_1 (T_c = 0.25z) \\ \frac{1}{k} & \text{for } C_2 (T_c = t) \end{cases} \]

(c) Sketch of response for \( C_1 \) and \( C_2 \)

[Sketch of response graphs for \( C_1 \) and \( C_2 \)]
Problem 4 Answer:

a) Split range control is used when we need to inputs to cover the whole output range (at steady state), for example, we need both heating and cooling in a house to control temperature. The range is split so that only one input is active for control at a time. Input resetting is when we have an extra input for dynamic purposes, for example, a bypass to get tight temperature control (but we want the bypass flow to be minimized, so it should be reset to a small value).

b) Cascade control is used when $y_1$ is the important CV and $y_2$ is an extra measurement located closer to the input $u$. By controlling $y_2$ in a slave loop we can get better control of $y_1$. The setpoint $y_2$s is adjusted by the outer master loop. Override selectors are used when you normally want to keep $y_1$ at a setpoint, but you must make sure that $y_2$ (higher priority) does not exceed a limit. When $y_2$ is controlled one must give up control of $y_1$. Example: Use heat input ($u$) to control temperature($y_1$), but instead control pressure ($y_2$) if pressure is above max limit.

C) Need to follow "radiation" rule

![Diagram]