Feedback:

The simple and best solution.
Applications to self-optimizing control and stabilization of new operating regimes

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Abstract

- Feedback: The simple and best solution
- Applications to self-optimizing control and stabilization of new operating regimes
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- Most chemical engineers are (indirectly) trained to be “feedforward thinkers” and they immediately think of “model inversion” when it comes doing control. Thus, they prefer to rely on models instead of data, although simple feedback solutions in many cases are much simpler and certainly more robust.

The seminar starts with a simple comparison of feedback and feedforward control and their sensitivity to uncertainty. Then two nice applications of feedback are considered:

1. Implementation of optimal operation by "self-optimizing control". The idea is to turn optimization into a setpoint control problem, and the trick is to find the right variable to control. Applications include process control, pizza baking, marathon running, biology and the central bank of a country.

2. Stabilization of desired operating regimes. Here feedback control can lead to completely new and simple solutions. One example would be stabilization of laminar flow at conditions where we normally have turbulent flow. In the seminar a nice application to anti-slug control in multiphase pipeline flow is discussed.
Outline

- About Trondheim
- I. Why feedback (and not feedforward)?
- II. Self-optimizing feedback control: What should we control?
- III. Stabilizing feedback control: Anti-slug control
- Conclusion

- More information:
Trondheim, Norway
NTNU,
Trondheim
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Example

Plant (uncontrolled system)

State space:
\[
\begin{align*}
\frac{dx_1}{dt} &= -\frac{1}{\tau}x_1(t) + k \cdot u(t - \theta), & \frac{dx_2}{dt} &= -0.1x_2(t) + 10 \cdot d(t) \\
y(t) &= x_1(t) + x_2(t)
\end{align*}
\]

Laplace:
\[
\begin{align*}
G(s) &= \frac{k}{\tau s + 1} e^{-\theta s}, & G_d(s) &= \frac{10}{10s + 1}
\end{align*}
\]

Nominal parameters \((G = G_d)\):
\[
\begin{align*}
k &= 10; & \tau &= 10[s]; & \theta &= 0
\end{align*}
\]

Consider feedforward and feedback control:
\[
\begin{align*}
\text{with changes in } G: & \text{ gain } (k), \text{ time constant } (\tau) \text{ and input delay } (\theta)
\end{align*}
\]
Model-based control = 
Feedforward (FF) control

"Perfect" feedforward control: $u = -G^{-1} G_d d$

Our case: $G = G_d \rightarrow$ Use $u = -d$
Feedforward control: Nominal (perfect model)
Feedforward: sensitive to gain error
Feedforward: sensitive to time constant error
Feedforward: Moderate sensitive to delay
(in G or G_d)
Measurement-based correction = Feedback (FB) control

- Most common in industry: $C =$ PID controller. Two adjustable tuning parameters: $K_c$ and $\tau_I$
- Exists many tuning rules including Skogestad’s (SIMC) tuning rules
  
  $$K_c = \frac{1}{k} \cdot \frac{\tau}{\tau_c + \theta}; \quad \tau_I = \min\{\tau, 4(\tau_c + \theta)\}$$

  “Probably the best simple PID tuning rules in the world”

- Example ($k = 10, \tau = 10, \theta = 0$ and $\tau_c = 1$):
  
  $$K_c = \frac{1}{10} \cdot \frac{10}{1} = 1 \quad \tau_I = \min\{10, 4 \cdot 1\} = 4 \ [s]$$
Feedback PI-control: Nominal case

\[ y_s \rightarrow e \rightarrow C \rightarrow u \rightarrow G \rightarrow y \]

Feedback generates inverse!

Input \( u \)  
Output \( y \)

Feedback PI-control (nominal)
Feedback PI control: insensitive to gain error
Feedback: insensitive to time constant error
Feedback control: sensitive to time delay
Comment

- Time delay error in disturbance model ($G_d$): No effect (!) with feedback (except time shift)
- Feedforward: Similar effect as time delay error in $G$
Conclusion: Why feedback? (and not feedforward control)

- Simple: High gain feedback!
- Counteract unmeasured disturbances
- Reduce effect of changes / uncertainty (robustness)
- Change system dynamics (including stabilization)
- Linearize the behavior
- No explicit model required

**MAIN PROBLEM**
- Potential instability (may occur “suddenly”) with time delay/RHP-zero
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Optimal operation (economics)

- Define scalar cost function $J(u_0,d)$
  - $u_0$: degrees of freedom
  - $d$: disturbances

- Optimal operation for given $d$:

$$\min_{u_0} J(u_0,x,d)$$

subject to:

$f(u_0,x,d) = 0$

$g(u_0,x,d) < 0$
"Obvious" solution: Optimizing control = "Feedforward"

Estimate $d$ and compute new $u_{\text{opt}}(d)$

*Problem*: Complicated and sensitive to uncertainty
Engineering systems

- Most (all?) large-scale engineering systems are controlled using hierarchies of quite simple single-loop controllers
  - Commercial aircraft
  - Large-scale chemical plant (refinery)
- 1000’s of loops
- Simple components:
  - on-off + P-control + PI-control + nonlinear fixes + some feedforward

Same in biological systems
In Practice: Feedback implementation

Issue: What should we control?
Further layers: Process control hierarchy

\[ y_1 = c \oplus (\text{economics}) \]
Implementation of optimal operation

- Optimal solution is usually at constraints, that is, most of the degrees of freedom are used to satisfy “active constraints”, $g(u_0,d) = 0$

- **CONTROL ACTIVE CONSTRAINTS!**
  - Implementation of active constraints is usually simple.

- **WHAT MORE SHOULD WE CONTROL?**
  - We here concentrate on the remaining unconstrained degrees of freedom.
Optimal operation

Cost $J$

$J_{\text{opt}}$

$c_{\text{opt}}$

Controlled variable $c$
Optimal operation

Two problems:
- 1. Optimum moves because of disturbances $d$: $c_{opt}(d)$
- 2. Implementation error, $c = c_{opt} + n$
Effect of implementation error

(a) Active constraint control: Implementation easy
(b) Flat optimum: Implementation easy
(c) Sharp optimum: Sensitive to implementation errors
Self-optimizing Control

- Define loss:
  \[ L(u, d) = J(c_s + u, d) - J_{opt}(d) \]

- Self-optimizing Control
  - Self-optimizing control is when acceptable operation (=acceptable loss) can be achieved using constant set points \( c_s \) for the controlled variables \( c \) (without the need for re-optimizing when disturbances occur).
Self-optimizing Control – Marathon

• Optimal operation of Marathon runner, J=T
  – Any self-optimizing variable c (to control at constant setpoint)?
Self-optimizing Control – Marathon

- Optimal operation of Marathon runner, J=T
  - Any self-optimizing variable c (to control at constant setpoint)?
    - $c_1 = \text{distance to leader of race}$
    - $c_2 = \text{speed}$
    - $c_3 = \text{heart rate}$
    - $c_4 = \text{level of lactate in muscles}$
Self-optimizing Control – Marathon

- Optimal operation of Marathon runner, J=T
  - Any self-optimizing variable c (to control at constant setpoint)?
    - $c_1 =$ distance to leader of race (Problem: Feasibility for d)
    - $c_2 =$ speed (Problem: Feasibility for d)
    - $c_3 =$ heart rate (Problem: Impl. Error n)
    - $c_4 =$ level of lactate in muscles (Problem: Impl.error n)
Self-optimizing Control – Sprinter

• Optimal operation of Sprinter (100 m), J=T
  – Active constraint control:
    • Maximum speed (”no thinking required”)


Further examples

- **Central bank.** $J = \text{welfare. } u = \text{interest rate. } c = \text{inflation rate (2.5\%)}$
- **Cake baking.** $J = \text{nice taste, } u = \text{heat input. } c = \text{Temperature (200C)}$
- **Business,** $J = \text{profit. } c = \text{"Key performance indicator (KPI), e.g. }
  - \text{Response time to order}
  - \text{Energy consumption pr. kg or unit}
  - \text{Number of employees}
  - \text{Research spending}
\text{Optimal values obtained by "benchmarking"}$
- **Investment** (portfolio management). $J = \text{profit. } c = \text{Fraction of investment in shares (50\%)}$
- **Biological systems:**
  - "Self-optimizing" controlled variables $c$ have been found by natural selection
  - Need to do "reverse engineering":
    - Find the controlled variables used in nature
    - From this possibly identify what overall objective $J$ the biological system has been attempting to optimize
Candidate controlled variables \( c \) for self-optimizing control

**Intuitive**

1. The *optimal value* of \( c \) should be *insensitive* to disturbances (*avoid problem 1*).

2. Optimum should be flat (*avoid problem 2 – implementation error*). Equivalently: *Value of \( c \) should be sensitive* to degrees of freedom \( u \).
   “Want large gain”

Charlie Moore (1980’s): Maximize minimum singular value when selecting temperature locations for distillation
Mathematical: Local analysis

\[
\text{Loss} = J(u, d) - J_{\text{opt}}(d) = \left( \frac{\partial J}{\partial u} \right)_{\text{opt}}^T (u - u_{\text{opt}}(d)) = 0
\]

\[
+ \frac{1}{2} (u - u_{\text{opt}}(d))^T \left( \frac{\partial^2 J}{\partial u^2} \right)_{\text{opt}} (u - u_{\text{opt}}(d)) + \cdots
\]

\[
\approx \frac{1}{2} \left( c - c_{\text{opt}}(d) \right)^T G^{-T} J_{uu} G^{-1} (c - c_{\text{opt}}(d))
\]

\[
\approx \frac{1}{2} G_s^{-T} J_{uu} G_s^{-1} \quad \text{where } G_s = \text{scaled gain}
\]
Minimum singular value of scaled gain

**Maximum gain rule** (Skogestad and Postlethwaite, 1996): Look for variables that maximize the scaled gain \( \sigma(G_s) \) (minimum singular value of the appropriately scaled steady-state gain matrix \( G_s \) from \( u \) to \( c \))

\[
\text{Loss} \approx \frac{\bar{\sigma}(J_{uu})}{2} \cdot \frac{1}{\sigma(G)^2}
\]
Self-optimizing control: Recycle process

\[ J = V \text{ (minimize energy)} \]

Given feedrate \( F_0 \) and column pressure:

\[ N_m = 5 \]

3 economic (steady-state) DOFs

Constraints: \( M_r < M_{r\text{max}}, \quad x_B > x_{B\text{min}} = 0.98 \)

DOF = degree of freedom
Recycle process: Control active constraints

One unconstrained DOF left for optimization:
What more should we control?
Maximum gain rule: Steady-state gain

| Rank | $c$   | $|G(0)| \cdot 10^3$ |
|------|-------|---------------------|
| 1    | $x_D$ | 13.1                |
| 2    | $L/F$ | 8.9                 |
| 3    | $D/L$ | 7.7                 |
| 4    | $D/V$ | 5.8                 |
| 5    | $V/L$ | 4.5                 |
| 6    | $B/L$ | 4.1                 |
| 7    | $V/F$ | 4.0                 |
| 8    | $B/D$ | 3.3                 |
| 9    | $L$   | 3.0                 |
| 10   | $B/F$ | 2.6                 |
| 11   | $D$   | 2.6                 |
| 12   | $F/F_0$ | 2.5              |
| 13   | $D/F$ | 2.5                 |
| 14   | $F$   | 1.9                 |
| 15   | $B/V$ | 0                   |
| 15   | $V$   | 0                   |
| 15   | $x_r$ | 0                   |
| 15   | $B$   | 0                   |

Gain from $u = L$ to $c$ with active constraints ($M_r$ and $x_B$) constant. Scaling: $\Delta c_i = \max_d |c_{opt,i}(d) - c_{opt,i}(d^*)| + \text{implementation error}.$ $d = F_0, z_F.$

Conventional: Looks good

Luyben snow-ball rule: Not promising economically
Recycle process: Loss with constant setpoint, $c_s$

Large loss with $c = F$ (Luyben rule)

Negligible loss with $c = L/F$ or $c = \text{temperature}$

$C_{1,s} = F_s = 996$

$C_{2,s} = (L/F)_s = 0.82$

Reoptimized $J_{\text{opt}}(d)$
Recycle process: Proposed control structure for case with $J = V$ (minimize energy)

Active constraint $M_r = M_{r\text{max}}$

Self-optimizing loop:
Adjust $L$ such that $L/F$ is constant

Active constraint $x_B = x_{B\text{min}}$
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Application stabilizing feedback control:

Anti-slug control

Two-phase pipe flow (liquid and vapor)

Slug (liquid) buildup
Slug cycle (stable limit cycle)

Experiments performed by the Multiphase Laboratory, NTNU
Flow map with open valve

- Steady flow
- Steady/Pulsing
- Pulsing flow
- Pulsing/Slugging
- Riser slugging

Flow regions:
- Steady flow
- Streaking/Pulsing
- Pulsing flow
- Pulsing/Slugging
- Riser slugging

Diagram shows the relationship between $U_{sg}$ [m/s] and $U_{so}$ [m/s] with markers indicating different flow regimes.
Experimental mini-loop
Experimental mini-loop
Valve opening \((z) = 100\%\)
Experimental mini-loop
Valve opening \( (z) = 25\% \)
Experimental mini-loop
Valve opening \((z) = 15\%\)
Experimental mini-loop: Bifurcation diagram

No slug

Slugging
Avoid slugging?

- Design changes
- Feedforward control?
- Feedback control?
Avoid slugging:
1. Close valve (but increases pressure)

No slugging when valve is closed
Avoid slugging:
2. Other design changes to avoid slugging
Minimize effect of slugging:
3. Build large slug-catcher

- Most common strategy in practice
Avoid slugging: 4. Feedback control?

Comparison with simple 3-state model:

Predicted smooth flow: Desirable but open-loop unstable
Avoid slugging:
4. "Active" feedback control

Simple PI-controller
Anti slug control: Mini-loop experiments

Controller ON

Controller OFF
Anti slug control: Full-scale offshore experiments at Hod-Vallhall field (Havre, 1999)
## Analysis: Poles and zeros

### Operation points:

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<th>z</th>
<th>$P_1$</th>
<th>DP</th>
<th>Poles</th>
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<td>0.175</td>
<td>70.05</td>
<td>1.94</td>
<td>-6.11 0.0008±0.0067i</td>
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<tr>
<td>0.25</td>
<td>69</td>
<td>0.96</td>
<td>-6.21 0.0027±0.0092i</td>
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### Zeros:

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<tr>
<th>y</th>
<th>$P_1$ [Bar]</th>
<th>DP [Bar]</th>
<th>$\rho_T$ [kg/m$^3$]</th>
<th>$F_Q$ [m$^3$/s]</th>
<th>$F_W$ [kg/s]</th>
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<td>-0.0032</td>
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</tbody>
</table>

Topside measurements: Ooops.... RHP-zeros or zeros close to origin
Stabilization with topside measurements:
Avoid “RHP-zeros by using 2 measurements

- Model based control (LQG) with 2 top measurements: DP and density $\rho_T$
Summary anti slug control

- Stabilization of smooth flow regime = $$$$$!
- Stabilization using downhole pressure simple
- Stabilization using topside measurements possible
- Control can make a difference!

Thanks to: Espen Storkaas + Heidi Sivertsen and Ingvald Bårdsen
Conclusions

- Feedback is an extremely powerful tool
- Complex systems can be controlled by hierarchies (cascades) of single-input-single-output (SISO) control loops
- Control the right variables (primary outputs) to achieve "self-optimizing control"
- Feedback can make new things possible (anti-slug)