Assumptions (hold for dilute mixtures):
1. Straight operating line (L/V constant)
2. Straight equilibrium line (e.g., y = mx)

From Figure
\[
\begin{align*}
\frac{\Delta y_1}{\Delta x_1} & = \frac{L}{V} \\
\frac{\Delta y_2}{\Delta x_2} & = \frac{L}{m} \\
\frac{\Delta y_3}{\Delta x_3} & = \left(\frac{L}{m}\right)^3
\end{align*}
\]

Similar
\[
\begin{align*}
\frac{\Delta y_2}{\Delta x_1} & = \frac{L}{m} \\
\frac{\Delta y_3}{\Delta x_2} & = \frac{L}{m}
\end{align*}
\]

General
\[
\frac{\Delta y_N}{\Delta y_0} = \left(\frac{L}{m}\right)^N
\]

\[
\frac{y_{N+1} - m x_N}{y_1 - m x_0} = \left(\frac{L}{m}\right)^N
\]

Also note that
\[
A = \frac{L}{m} = \frac{y_{N+1} - y_1}{y_1 - m x_0} \quad \text{(from Henry's)}
\]

\[
N = \ln \frac{y_{N+1} - m x_N}{y_1 - m x_0} \quad \text{(1)}
\]

\[
A = \frac{L}{m} = \frac{y_{N+1} - y_1}{m(x_{N+1} - x_0)} \quad \text{(2)}
\]
Can then write

\[ N = \ln \frac{y_{N+1} - y_o}{y_1 - y_o} \]
\[ \frac{y_{N+1} - y_0}{y_{N-1} - y_o} \]

where

\[ y_{N+1} = \frac{m}{x_{N+1}} \]
\[ y_o = \frac{m}{x_0} \]

Example 10.3-3

\[ y_1 = 0.00101 \]
\[ y_{N+1} = 0.01 \]
\[ x_N = 0.003 \Rightarrow y_N = \frac{2.53}{0.003} = 0.00359 \]
\[ x_0 = 0 \Rightarrow y_o = 0 \]

\[ N = \frac{\ln \frac{0.01 - 0.00359}{0.00101 - 0}}{\ln \frac{0.01 - 0.00359}{0.00101 - 0}} = \frac{\ln 2.386}{\ln 1184} = 5.15 \]

Some comments:
- Some equations apply for absorption and stripping
- Can also use for acid calculations of "corners" in distillation where equilibrium and operability lines are straight, and then McCabe-Thiele is used in middle section.

Cook gives 5.04, but this is because they use another value of \( y/v \) - note that \( y/v \) varies slightly.

Example 10.6-3 (end-1-end-2 notation)

\[ N = \ln \frac{y_{N+1} - y_o}{y_1 - y_o} \]

\( y_1 = 0.022 \)
\( x_0 = 0 \Rightarrow y_o = 0 \)
\( y_2 = 0.02244 \)
\( x_1 = 0.02248 \Rightarrow y_1 = 0.01482 \)
\( m = 0.68 \)

\[ N = \ln \frac{0.022 - 0.01482}{0.002244 - 0} \]
\[ \ln \frac{0.022 - 0.002244}{0.01482 - 0.002244} \]

\[ = \frac{\ln 3.1926}{\ln 1.333} = 4.05 \] (same work)