Modeling and optimizing petroleum wells

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Petroleum production

Important operational question: How to maximize oil production?
Optimizing the production

Today

- Large models of the system is optimized
- Very time consuming
- Often done only once or twice per day
**Problem**

- Disturbances are not accounted for until the next optimization

**Leads to . . .**

- Non-optimal operation of the wells in-between the optimizations
Possible solution

To ensure optimal operation:

- Design a control structure
- Avoid re-optimization
- Control variables
- Measurements
The system

Two-well manifold

- Oil, gas and water is produced
- Maximize oil production
- Maximum production $\rightarrow$ constraints become active
- Active gas constraint
  $$m_{\text{well}}^a + m_{\text{well}}^b = m_{\text{max}}^g$$
Optimal solution

For a manifold operation under constrained conditions
We already know the optimal way to operate the wells:

\[
\frac{\partial m_g}{\partial m_o} \bigg|_{\text{well } a} = \frac{\partial m_g}{\partial m_o} \bigg|_{\text{well } b}
\]

**Problem:** We have no way to measure this relation . . .
Developing a model

Overall mass balance:
\[ m_{\text{tot}} = m_g + m_o + m_w \]

Valve:
\[ m_{\text{tot}} = \frac{C_v \sqrt{\rho_{\text{mix}} \Delta P}}{C_v \sqrt{\rho_{\text{mix}} \Delta P}} \]

Pipeline:
\[ \Delta P = \rho g \Delta L \]

Reservoir:
\[ m_o = k_o (P_{wf} - P_R) \]
\[ m_w = k_w (P_{wf} - P_R) \]
\[ m_g = k_g (P_{wf}^2 - P_R^2) \]
\[ \text{or} \]
\[ m_g = \text{GOR} \times m_o \]

Gas-Oil-Ratios used in the models:
\[ \text{GOR}_D = \frac{k_g}{k_o} (P_{wf} + P_R) \]
\[ \text{GOR}_1 = \frac{k_g}{k_o} (P_{wf}^2 - P_R^2) \]
\[ \text{GOR}_2 = \frac{k_g}{k_o} (P_{wf} - P_R) \]
\[ \text{GOR}_3 = \frac{k_g}{k_o} (P_{wf} - P_R)^2 \]
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Optimal solution for the the well models

The optimal solution is known to be:

\[
\left( \frac{\partial m_g}{\partial m_o} \right)_{\text{well } a} = \left( \frac{\partial m_g}{\partial m_o} \right)_{\text{well } b}
\]

Optimal solution for the the well models

\[
\left( \frac{\delta m_g}{\delta m_o} \right)_{\text{GOR}_D} = \text{GOR}_D + \frac{k_g}{k_o} (P_{wf} - P_R)
\]

\[
\left( \frac{\delta m_g}{\delta m_o} \right)_{\text{GOR}_1} = \text{GOR}_1 + 2 \frac{k_g}{k_o} P_{wf} (P_{wf} - P_R)
\]

\[
\left( \frac{\delta m_g}{\delta m_o} \right)_{\text{GOR}_2} = 2 \text{GOR}_2
\]

\[
\left( \frac{\delta m_g}{\delta m_o} \right)_{\text{GOR}_3} = 3 \text{GOR}_3
\]
Testing the optimal solution

The change in GOR with increasing production rate:

\[ \delta m_g \frac{\delta m}{\delta m_o} \]

GOR\(_D\) = \( \frac{k_g}{k_o} (P_{wf} + P_R) \)

GOR\(_1\) = \( \frac{k_g}{k_o} (P_{wf}^2 - P_R^2) \)

GOR\(_2\) = \( \frac{k_g}{k_o} (P_{wf} - P_R) \)

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\[ m_g^{\text{max}} \] [t d\(^{-1}\)]

A start...
Questions?