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Modeling of Cold Thermal Storage in Buildings

By

Emma M. Johansson

Supervisor: Sigurd Skogestad
Co-supervisor: Chriss Grimholt

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Author
Emma M. Johansson

Supervisors
Prof. Sigurd Skogestad
Chriss Grimholt
Abstract

The goal of this project has been to study the cooling of buildings/rooms with the use of cold thermal storage. The report presents a mathematical model of a system that uses cold thermal storage to regulate the temperature inside a room. The system has been tested under different conditions in order to see if it the dynamics of the system are working according to the requirements.

The results from the different cases tested on the system showed that the system works as expected. As a result of ventilation the the indoor temperature changes with outdoor temperature. The room gets cooled down when the radiators are turned on. The radiator temperature changes as cold water is circulating through the radiator and the cooling load changes as the amount of circulating water increases or decreases. Ice is consumed during cooling and generated when the chiller is turned on.

In order to test the system further, recommendations for further work is to include an air condition unit and optimize and regulate the system with respect to cost and energy consumptions.

All simulations in the report were done using MATLAB and the built-in function sfuntmpl.
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## List of symbols and abbreviations

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<th>Description</th>
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<tr>
<td>TES</td>
<td>Thermal energy storage</td>
</tr>
<tr>
<td>CTS</td>
<td>Cold thermal storage</td>
</tr>
<tr>
<td>CFC</td>
<td>Cholorofluorocarbon</td>
</tr>
<tr>
<td>CWS</td>
<td>Chilled water system</td>
</tr>
<tr>
<td>CSTR</td>
<td>Continuous stir tank reactor</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$Q$</td>
<td>Heat</td>
</tr>
<tr>
<td>$U$</td>
<td>Overall heat transfer coefficient</td>
</tr>
<tr>
<td>$A$</td>
<td>Heat exchange area</td>
</tr>
<tr>
<td>$m$</td>
<td>mass</td>
</tr>
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<td>$H$</td>
<td>Enthalpy</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Heat capacity</td>
</tr>
<tr>
<td>$\Delta H_{fus}$</td>
<td>Enthalpy of fusion</td>
</tr>
<tr>
<td>$M_{freeze}$</td>
<td>Change of ice</td>
</tr>
<tr>
<td>$T_w$</td>
<td>Freezing temp of water</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Split fraction</td>
</tr>
<tr>
<td>$V_{vent}$</td>
<td>Air ventilation</td>
</tr>
<tr>
<td>$mass_{sys}$</td>
<td>Mass circulating in the system</td>
</tr>
<tr>
<td>$A_{rad}$</td>
<td>Radiator volume</td>
</tr>
<tr>
<td>$V_{tank}$</td>
<td>Volume of storage tank</td>
</tr>
</tbody>
</table>
1 | Introduction

In a modern world like today, energy is one of the most essential needs used in daily life. Therefore a sustainable use of energy is important such that future generations can profit from the energy resources of the world. To run a sustainable energy consumption energy sources should be fully utilized, unnecessary losses should be eliminated and the exploration of new energy sources investigated. One of the areas where energy utilization has been investigated is in heat exchange where thermal energy storage (TES) has been used. In TES the energy pattern can be reshaped by storing coldness or heat in mediums when the electricity rates changes or the energy consumption can be reduced by utilizing energy that normally would be wasted. One of the areas where this technology is applied is in the heating and cooling of buildings where energy needs are significant.

The energy demand of heating and cooling of buildings depends on both the geographical location and the season of the year. Such as in the summertime temperature outside can be quite high, and the need for cooling is high, requiring energy. To be able to control the inside environment whether it is a hot or cold day a heat exchanger is necessary. Today there are several systems that can regulate the inside temperature. One sample is an air condition unit that can both control the temperature and humidity inside or a cold thermal storage.

As already mentioned earlier the outside temperature can vary between hot and cold. This fluctuation in temperature will lead to a significant energy need in order to keep the indoor temperature at a comfortable level, especially if the building is poorly isolated. To be able to control the inside air to the recommended temperatures [5], the system has to be able to meet the cooling or heating requirements at the peak temperatures, which are the highest and coldest outdoor temperature during the day. This will require a system that has a high maximum capacity. At the off-peak periods the system is working at a lower efficiency since the temperature is lower and the need for cooling is decreased. This will lead to an inefficient system, and a loss of money. To increase the efficiency of the system a thermal energy storage can be used in addition to an chiller unit. A thermal energy stor-
Figure 1.1: Simplified model of a cooling system with, a) no storage and b) with storage

For new facilities, cold storage should be integrated carefully into the overall building and its energy systems, to a temperature differential in a storage medium [6], e.g. a domestic hot water heater. A TES can be divided into hot and cold storage systems depending on the requirement.

In this project the aim is to cool a room with a cold thermal storage (CTS) when the temperature outside is high and fluctuating. The CTS can provide the room with cooling during peak-hours, while the an additional cooling unit can take care of the average cooling. The CTS can also be run with different operational strategies depending on what the optimal strategy is for the case where it is used. Using a CTS will lead to a decrease in the size and capacity of the chiller while the cooling need is still achieved [6]. Figure 1.1 show a simplified model of how an TES is used in buildings,
2 | Thermal Energy Storage

In the introduction a cold thermal storage was suggested as an energy utilization when cooling rooms and buildings. TES systems have been used since the 1950s to store excess heat that would normally be wasted. TES examples are the storage of ice for a space cooling in the summer and storage of heat or coolness generated electrically during the off peak hours for use during the following peak hours [3]. TES can be applied to buildings where the electricity rated allow for TES to compete with other forms of cooling. The most economical benefit is found in the buildings that have a significant cooling demand that contributes to high demand charges [3].

CTS is a type of TES where coldness can be stored and shift electrical peak loads as a way to manage the energy in the building. By shifting the load the energy can be more utilized and less dependent on peak-units which have higher operating costs [3]. The general benefits of using CTS for the energy users, energy supply companies and the environment are summarized below [7]:

- reduce chiller sizes
- more efficient and effective system operations
- less capital and maintenance costs
- energy and financial savings
- reduction in carbon dioxide and CFC emissions

When implementing such a system there are several options regarding which type of CTS should be used. In the following section some of them are presented.
2.1 Chilled water system

The chilled water system (CWS) contains tanks charged with cold water at temperatures of 4-6 °C. The cooled water is used for the heat exchange and cooling of the air. The water will go from a storage tank throughout a radiator cycle while cooling the room. As the water passes through the radiator the room heats the water, while the cold water cools the room. The heated water then returns to the storage tank. In ideal systems the water consists of perfect layers in the tank with decreasing temperature towards the bottom [3]. In reality the heated water running back from the heat exchanger loop will mix with the cold storage water. To minimize the mixing the heated water is supplied with low flow rates. Still this will decrease the cooling capacity of the CWS due to increased temperature [3]. CWS also require large tank volume since each cubic meter only can provide 5.8 kWh of cooling when a temperature rise of 5 °C is used [3].

2.2 Ice storage system

In ice storage systems ice is used as a phase changing medium in addition to water, and will provide the system with more cooling capacity then chilled water systems due to the latent heat of fusion. In this type of system ice is generated to charge the thermal storage. When cooling is required production of ice stops and the melting of ice provides the cooling system with cold water. In an ice storage system a chiller is required for the ice building process. A chiller is an electrical unit that produces ice in the storage tank. The chiller is located outside of the storage tank and can either generate the ice directly from a radiator in the tank or by pipes filled with glycol inside the tank. In the case of a glycol solution the chiller will cool the glycol to a temperature of about -6.7 °C - 5.6 °C. The cold glycol will then be pumped through the pipes and the water will freeze to ice around the coils [6].

The main benefit of thermal ice storage systems compared to conventional chilled water systems is that an ice storage system has the ability to provide energy needs based on time rather than cooling requirements [6].

2.3 Eutectic salt storage system

Eutectic salts are mixtures of inorganic or organic salts, water and nucleating and stabilizing agents [3]. Eutectic mixtures have the ability to tune the freezing
temperature, making them lower than the individual freezing temperatures for the components in the mixture [8]. The eutectic salt system is similar to ice storage in the aspect that the cooling capacity depends on the latent heat of fusion of the mixture [8].
In order to cool or heat a room a heat exchange has to take place in order to keep the temperature stable and at a comfortable level. The inside temperature is affected by the natural heating or cooling of the walls from the outdoor as well as the ventilation. Ventilation involves air exchange between the room and the outdoor which can create substantial airspeed in the room which that can cool people and provide comfort, specially in humid areas [9]. The ventilation air entering the room often contributes to a temperature change inside the room, in addition to the walls, which leads to the need to heat or cool the air inside to satisfy the indoor temperature requirements. In order to do so a heat exchanger is required. A heat exchanger transports energy in the form of heat between media. The driving force for the heat transfer is the temperature difference between the media [4]. The heat flows from the high temperature side to the low temperature side to equal out the temperature difference. Figure 3.1 shows a counter current heat exchanger with air and water on the hot and cold side, respectively.

![Figure 3.1: A counter current heat exchanger with air and water.](image)

When heat gets transferred from one medium to another there are three basic heat transfer mechanisms [4]:

- Conduction
- Convection
• Radiation

Of these mechanisms most industrial processes use conduction [10]. In this project the heat transfer will also be according to this mechanism. In this system the cold water will flow through pipelines to a heat exchanger, radiator, where heat transfers trough the metallic element of the radiator which is made of steel. The air will be on the other side of the heat exchanger, shown in Figure 3.1. The radiator will thus cool the air in the room in addition to a conventional air condition system.

The energy transferred from the hot to the cold side in Figure 3.1 is $Q$. When assuming constant temperatures on the hot and cold sides, the heat transfer can be calculated by Equation (3.1) [10].

$$ Q = UA\Delta T $$

(3.1)

$\Delta T = T_h - T_c$, $A$ is the heat exchange area and $U$ is the overall heat transfer coefficient.

Usually the temperature will not be constant throughout the heat exchanger, and the $\Delta T$ in Equation (3.1) can be replaced by the mean temperature difference [10]. The mean temperature difference is presented in Equation (3.2) where $\Delta T_1$ and $\Delta T_2$ are the temperature differences for the two inlet and outlets of the heat exchanger, respectively [10].

$$ \Delta T = \frac{\Delta T_1 - \Delta T_2}{2} $$

(3.2)
4 | Model

4.1 Operational system

In this project the system is defined as a room that needs to be cooled down due to high outdoor temperatures. The outdoor temperatures vary during the day as the sun rises and sets, and the cooling load has to be continuously adjusted in order to keep the inside temperature at a comfortable level. To regulate the temperature of the room a TES is used. Of the TESs available an ice storage system is chosen because it is easier to control than an eutectic salt system and requires less storage volume than a CWS.

A simplified process sheet for the system is presented in Figure 4.1. The room is assumed to be a perfectly mixed CSTR with an inlet and outlet stream (ventilation). The inlet air cannot be controlled and will be a disturbance as well as an input to the system. The outlet temperature will have the same temperature as the room, assuming perfect mixing. The size of the room is set to be 100 m$^3$ and well isolated. In the room there are radiators that will cool the room down if the temperature reaches set level. The radiators are steel radiators with a cold circulating water stream inside. The radiators are provided with water from a CTS and the amount of cold water entering the radiator is controlled either by a split or by the pump from the storage tank, see Figure 4.1. The water from the storage tank that enters the radiator first goes through a pump that sends the water into the cooling system. The water enters a split that controls the fraction of water circulating through the radiator. If the split fraction, $\alpha$, is less than one some of the water moves through the radiators, whereas the rest circulates back to the tank. If $\alpha$ is equal to zero all of the water is circulated in the chiller loop and no cooling will be applied to the room. When the water has circulated through the radiator it returns to the storage tank. Before entering the tank the stream from the radiators gets mixed with the recycle stream. The temperature will now be a mixed temperature of the recycled stream and the stream from the radiators. This stream then continues to the tank. When entering the tank the
stream is carefully poured into the tank in order to prevent mixing of the fluid in the tank. The pipelines the water flows through and the storage tank are assumed to be well isolated. The radiators in the room are also assumed perfectly mixed. With both the room and radiator perfectly mixed the system has constant cold and hot temperatures and Equation (3.1) can be used for the system.

![Figure 4.1: The cooling system](image)

When the room is not in need of cooling ice will be generated in the ice storage by the $Q_{\text{chiller}}$. $Q_{\text{chiller}}$ is an electrical cooling unit (reservoir) for the ice production. Often this is a glycol solution circuit that has a temperature below the freezing point of water (0 °C) [6] which will flow through pipelines in the tank and freeze the water. In this system it is assumed to be an electrical reservoir that gets turned on and off depending on the operating hours of the ice storage system.
4.2 Mass and energy balances

The model is based on the general mass balance, Equation (4.2), and energy balance, Equation (4.3), for the states of the system. The four states are the primary variables and are presented in Equation (4.1).

\[
x = \begin{bmatrix}
    T_{\text{room}} \\
    T_{\text{radiator}} \\
    \text{mass}_{\text{ice}} \\
    \text{mass}_{\text{water}}
\end{bmatrix}
\] (4.1)

The general mass equation for an open system, Equation (4.2), includes mass into and out of the system \( \text{mass}_{\text{in}} \) and \( \text{mass}_{\text{out}} \), generated mass in the system, \( \text{mass}_{\text{generated}} \), and consumed mass, \( \text{mass}_{\text{consumed}} \) [10].

\[
\frac{dm_{\text{room}}}{dt} = m_{\text{in}} - m_{\text{out}} + m_{\text{generated}} - m_{\text{consumed}}
\] (4.2)

The general energy balance for an open system, Equation 4.3, includes enthalpy, \( H \) and heat [10]. The kinetic and potential energy are assumed to be low and are therefore neglected in the equation. These two general mass and energy equation are used for the respective states from Equation (4.4) through (4.20).

\[
\frac{dH}{dt} = H_{\text{in}} - H_{\text{out}} + Q
\] (4.3)

The mass and energy balances for the room i presented below by Equations (4.4) through (4.7).

\[
\frac{dm_{\text{room}}}{dt} = m_{\text{room}_{\text{in}}} - m_{\text{room}_{\text{out}}}
\] (4.4)

The ventilation is assumed constant so there will not be any accumulation of air in the room.

\[
m_{\text{room}_{\text{in}}} = m_{\text{room}_{\text{out}}}
\] (4.5)
\[ \Rightarrow \frac{dm_{\text{room}}}{dt} = 0 \]  \hspace{1cm} (4.6)

\[ m_{\text{room}} C_{P,\text{air}} \frac{dT_{\text{room}}}{dt} = m_{\text{room, in}} C_{P,\text{air}} T_{\text{room, in}} - m_{\text{room, out}} C_{P,\text{air}} T_{\text{room, out}} - Q_{\text{room}} \]  \hspace{1cm} (4.7)

The balances for the radiator are presented below in Equations (4.8) through (4.11).

\[ \frac{dm_{\text{rad}}}{dt} = m_{\text{rad, in}} - m_{\text{rad, out}} \]  \hspace{1cm} (4.8)

As in the case of the room, the inlet and outlet stream of the radiator are equal in order to avoid accumulation in the radiator. The inlet stream can be controlled either by a pump or the split from the cold storage as shown in Figure 4.1.

\[ m_{\text{rad, in}} = m_{\text{rad, out}} \]  \hspace{1cm} (4.9)

\[ \Rightarrow \frac{dm_{\text{rad}}}{dt} = 0 \]  \hspace{1cm} (4.10)

\[ m_{\text{rad}} C_{P,w} \frac{dT_{\text{rad}}}{dt} = m_{\text{rad, in}} C_{P,w} T_{\text{rad, in}} - m_{\text{rad, out}} C_{P,w} T_{\text{rad, out}} + Q_{\text{room}} \]  \hspace{1cm} (4.11)

\[ Q_{\text{room}} \] is heat transferred from the room to the radiator and is calculated by Equation (4.12).

\[ Q_{\text{room}} = UA\Delta T \]  \hspace{1cm} (4.12)

Here \( \Delta T \) is the temperature difference between the hot and cold sides of the heat exchanger, \( \Delta T = T_h - T_c \). A [m²] is the area of the heat transfer surface and U [W/(m² K)] is the overall heat transfer coefficient.
In a heat exchanger the temperature varies with the position throughout the heat exchanger [4]. Equation (4.12) refers to a perfectly mixed solution where there is no temperature difference within the volume. The temperature gradient is the same throughout the whole heat exchanger. In reality this is not the case, but as a simplification in this model both the room and radiator operate as two perfectly mixed CSTRs. With this assumption the heat transfer can be simplified to the expression in Equation (4.12).

The balances for the ice and storage tank are presented in Equations (4.13) through (4.20). For the mass balance of ice Figure 4.2 shows the dynamic model of the melting and generation of ice.

![Dynamic model of the melting and generation of ice in the storage tank](image)

**Figure 4.2:** Dynamic model of the melting and generation of ice in the storage tank

\[
\frac{dm_{\text{ice}}}{dt} = m_{\text{ice}_{\text{in}}} - m_{\text{ice}_{\text{out}}} \tag{4.13}
\]

\[
\frac{dm_w}{dt} = m_{w_{\text{in}}} - m_{w_{\text{out}}} - M_{\text{freeze}} \tag{4.14}
\]

Here \(m_w\) is the mass water, and \(m_{w_{\text{in}}}\) and \(m_{w_{\text{out}}}\) are assumed equal.

\[
\Rightarrow \quad \frac{dm_w}{dt} = -M_{\text{freeze}} \tag{4.15}
\]

\[
M_{\text{freeze}} = \frac{dm_{\text{ice}}}{dt} \begin{cases} \text{negative when melting} \\ \text{positive when freezing} \end{cases} \tag{4.16}
\]
The energy balance for the storage tank will then be as shown in Equation (4.17).

\[ m_w C_{P,w} \frac{dT_w}{dt} = m_{w\text{in}} C_{P,w\text{in}} T_{w\text{in}} - m_{w\text{out}} C_{P,w\text{out}} T_{w\text{out}} - M_{\text{freeze}} \Delta H_{fus} \]  

(4.17)

As the ice is melting or being generated the temperature in the tank, \( T_w \), will be constant and Equation (4.17) will be simplified to Equation (4.18).

\[ 0 = m_{w\text{in}} C_{P,w\text{in}} T_{w\text{in}} - m_{w\text{out}} C_{P,w\text{out}} T_{w\text{out}} - M_{\text{freeze}} \Delta H_{fus} \]  

(4.18)

\[ M_{\text{freeze}} \Delta H_{fus} = m_{w\text{in}} C_{P,w\text{in}} T_{w\text{in}} - m_{w\text{out}} C_{P,w\text{out}} T_{w\text{out}} \]  

(4.19)

This gives a change of \( m_{\text{ice}} \), given in Equation (4.20).

\[ \frac{dm_{\text{ice}}}{dt} = M_{\text{freeze}} = \frac{m_{w\text{in}} C_{P,w\text{in}} T_{w\text{in}} - m_{w\text{out}} C_{P,w\text{out}} T_{w\text{out}}}{\Delta H_{fus}} \]  

(4.20)

The recycle stream and the heated stream from the radiator get mixed on the way back to the storage tank, as shown in Figure 4.3.

![Figure 4.3: The mixing point of the recycle flow and the return flow from the radiator.](image-url)
The stream $T_{mix}$ will then enter the storage tank as $T_{w_in}$ in Equation (4.20). The steady state equations for the mixing are presented in Equation (4.21) and (4.22).

\[ T_{mix} m_{sys} = T_r \alpha m_{sys} - (1 - \alpha) T_w m_{sys} \]  
\[ T_{mix} = T_r \alpha + (1 - \alpha) T_w \]  
(4.21)  
(4.22)
5 | Operational Strategy

When operating a cooling storage system the strategy of how to use it is important. In conventional systems (only one cooling unit) the cooling load is managed by satisfying the cooling requirements, while thermal ice storage systems allow for the management of energy consuming components as well [6]. This means that it is possible to control whether the system should use the ice storage or the additional cooling unit for cooling.

When developing an operating strategy there are several factors that should be evaluated. The main strategy should be to use the system at it’s optimal. From Skogestad [11] the goal of optimization is to minimize an objective function, Equation (5.1), to its given constraints, Equation (5.2). In this project the object function is not jet chosen, but some alternatives to look at will be the total cost, total energy consumption or time. The equality constraints are presented in Chapter 6 from the mass balances in Chapter 4.2 while the inequality constraints for the system are presented in to Table 5.1.

\[
\text{minimize } J(x, u_t, d) \\
\text{Subject to equality constrains: } g(x, u, d, t) = 0 \\
\text{Subject to inequality constrains: } h(x, u, d, t) \geq 0
\]

Where \( J \) is the objective function, which is a function of the states, \( x \) Equation (4.1), the manipulated variables, \( u_t \), and the disturbances, \( d \). The manipulated variables also denotes the systems degrees of freedom, DOFs. In order to optimize the system the DOFs, \( u_t \), must be assigned to specific variables. If the states, \( x \), are eliminated by using the model equations, \( g \), the remaining problem will be
Equation (5.3).

\[ \min_u J(u, d) = J(u_{opt}, d) = J_{opt}(d) \]  \hspace{1cm} (5.3)

Here \( u_{opt} \) is to be found and \( J_{opt} \) is the optimal value of the object function \( J \).

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{room} )</td>
<td>Temperature of the room</td>
<td>( 20 &lt; T_{room} &lt; 26 ) [°C] [5]</td>
</tr>
<tr>
<td>( T_{radiator} )</td>
<td>Temperature in the radiator</td>
<td>( 0 \leq T_{radiator} \leq T_{room} ) [°C]</td>
</tr>
<tr>
<td>( mass_{ice} )</td>
<td>Mass of ice</td>
<td>( 0 \leq mass_{ice} \leq 140 ) [kg]</td>
</tr>
<tr>
<td>( mass_{water} )</td>
<td>Mass of water</td>
<td>( 0 \leq mass_{water} \leq 140 ) [kg]</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Split fraction</td>
<td>( 0 \leq \alpha \leq 1 )</td>
</tr>
<tr>
<td>( Q_{chiller} )</td>
<td>Chiller unit</td>
<td>( Q_{min} \leq Q_{chiller} \leq Q_{max} )</td>
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</tbody>
</table>

In order to minimize the cost function, \( J_{opt}(d) \) in Equation (5.3), two main factors are important \[6\]. One is the electricity prices, Section 5.1, and the other is storage strategy, Section 5.2.

### 5.1 Electric cost and rate schedules

Many electrical companies have rate schedules of electricity prices that change with season of the year and time of the day. This will cause a fluctuation in the cost of energy and lead to the inequality such that a kW \( \neq \) a kW. The energy units are the same, but the time-of-use will change the value of the energy \[6\]. Generally low prices of electricity occur during the night time where the energy usage is low. Times when energy consumption is low is refereed to as off-peak periods. In the day time the energy consumption is high and the generating capacity and distribution of energy might be limited and will lead to higher prices. This time interval is referred to as on-peak.

Figures 5.1 and 5.2 show the average energy use during the day (and weekend) for a typical household and primary-, secondary- and tertiary industry, respectively. Figure 5.1 shows the household consumption for the whole week where the green line represents week days while the red line represents weekend.
Figure 5.1: The average energy consumption for a household during a week [2]

Figure 5.1 shows that regular households have two peaks during the day. One in the morning around 7-12 a.m. and one in the evening between 17-24 p.m.. During weekends the morning peak is slightly later due to a later rise in the morning and the consumption during the day is higher probably since people stay at home.

Figure 5.2 shows the industry sector where primary industries are represented by the green line, the secondary by the yellow, and tertiary industries by the blue.

Figure 5.2: The average energy consumption by the industry during a week [2].
For the industry the secondary and tertiary industries have similar energy consumption levels. There is a peak during the day which starts around 7 am and ends at 17 pm. The primary industry has aberrant consumption pattern. More energy is used during the night than in the day, and there is an opposite pattern relative to the secondary and tertiary industries.

Both households and secondary and tertiary industries have peak periods in the morning. This will cause a large load on the power system. To avoid strained situations like this measures should be taken in order to move the morning peaks by using time differenced prices for the sector that can move their peak hours [2]. The energy price can be lowered at off-peak periods which will lead to a decrease in costs if the industries move their energy peak away from peak time periods. Another way to do it is to increase the price when the energy consumption increases to a specific limit [2]. In this way the peaks should disappear and the energy consumption should be more spread out during the day.

5.2 Storage

When using a cold thermal storage there are two main types of storage strategies, full storage and partial storage [3]. These storages refer to the amount of on-peak cooling load that is shifted to off-peak cooling load. Figure 5.3 shows these basic storage operating strategies and in the preceding sections the systems are briefly explained.

![Figure 5.3: T basic thermal storage operating strategies ,from the left, full storage, partial storage load leveling and partial storage demand limiting[3].](image-url)
5.2.1 Full storage

Full storage or load shifting strategy divides the process in an on and off strategy. Full storage uses maximum load shifting by charging the entire daily cooling requirement at night and using the storage capacities during the day. While using the storage the refrigerating system is turned off, so the energy demand during the day will decrease. This strategy provides the highest operating cost savings, but requires relatively large refrigeration and storage capacities. Figure 5.3 shows how the full storage is used [3].

5.2.2 Partial storage

Partial storage will unlike full storage not cover the whole cooling requirement with storage only. The cooling demand will be divided between the storage and chilling equipment. Partial storage can also be divided into two categories; load leveling and demand limiting operation [3]. A load leveling system runs with full capacity for 24 h. When the cooling load is lower than the chiller output the remaining cooling will be stored, while when the cooling load is higher than chiller output the requirement will be met by discharging the storage [3]. Figure 5.3 shows load leveling storage in the middle picture. A demand limiting system has a chiller that runs all the day except for hours of maximum non-cooling demand. In this storage strategy the refrigeration equipment operates at reduced capacity (or demand level) during the on-peak period [3]. To control this system is complicated since the peak demand must be met by the storage. The strategy is applicable to buildings and rooms with significant demands for rapid charging and short occupancy periods that allow for longer storage time [3].

The benefits with using partial storage in comparison with full storage is that the costs will be lower, and the size of the chiller can be reduced [3]. In partial storage the load leveling strategy is less expensive then the demand limiting strategy.
The system presented in Chapter 4.1 has been implemented in MATLAB and Simulink. The build-in function `sfuntmpl` has been used in all the simulations. The purpose of implementing the model in MATLAB was to see if the mathematical model was correct. First several simulations were done in order to get reasonable parameters for the ventilation of the system and the circulation mass of the cooling circuit. The results for testing these parameters are found in the Appendix A, the values of the parameters are presented in Table 6.1 together with estimated parameters. The constants that were used are presented in Table A.1 in Appendix A.

The system was tested by changing the different manipulated variables in the MATLAB file, and measuring the states in order to see if the system fulfilled its purpose. The conditions and results for the different test are presented in Sections 6.1 - 6.4. MATLAB scripts and Simulink model are to be found in Appendix B.

The equality constraints, \( g \), of the system are presented in Equation (6.1) and are designed from the mass and energy balances in Chapter 4.2. The equality constraints are the same in all the case studies. The inputs of the system are denoted as \( u \), and Equation (6.2) show the three inputs of the system. The states are denoted with \( x \) and are presented in Equation (4.1).

\[
g = \begin{pmatrix}
\frac{V_{\text{vent}}(u(1)-x(1))}{V_{\text{room}}} - \frac{UA(x(1)-x(2))}{V_{\text{room}}C_{p_{\text{air}}}} \\
\frac{\text{mass}_{\text{sys}}u(2)(T_w-x(2))}{V_{\text{rad}}p_{\text{water}}} + \frac{UA(x(1)-x(2))}{V_{\text{rad}}C_{p_{\text{water}}}} p_{\text{water}} \\
\frac{u(3)}{\Delta H_{fus}} - \frac{\text{mass}_{\text{sys}}C_{p_{\text{water}}} u(2)(x(2)-T_w)}{\Delta H_{fus}} \\
\frac{\text{mass}_{\text{sys}}C_{p_{\text{water}}} u(2)(x(2)-T_w)}{\Delta H_{fus}} - \frac{u(3)}{\Delta H_{fus}}
\end{pmatrix}
\]  

(6.1)
\[ u = \begin{pmatrix} T_{\text{outside}} \\ \alpha \\ Q_{\text{chiller}} \end{pmatrix} \]  

(6.2)

**Table 6.1: Standard simulation parameters**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U )</td>
<td>Overall heat transfer coefficient</td>
<td>0.03 [kW/(m(^2),K)]</td>
</tr>
<tr>
<td>( A )</td>
<td>Heat exchange area</td>
<td>1.8 [m(^2)]</td>
</tr>
<tr>
<td>( V_{\text{vent}} )</td>
<td>Air ventilation</td>
<td>0.05 [m(^3)/s]</td>
</tr>
<tr>
<td>( \text{mass}_{\text{sys}} )</td>
<td>Mass circulating in the system</td>
<td>0.07 [m(^3)/s]</td>
</tr>
<tr>
<td>( A_{\text{rad}} )</td>
<td>Radiator volume</td>
<td>0.06 [m(^3)]</td>
</tr>
<tr>
<td>( V_{\text{tank}} )</td>
<td>Storage tank volume</td>
<td>140 [dm(^3)]</td>
</tr>
</tbody>
</table>
6.1 Case 1: Inlet room and radiator temperature

In the first case the room and radiator temperature were tested in order to check if the room temperature corresponded correctly with the outdoor temperature since the room is ventilated. This simulation was done with ventilation only, no cooling. The results are presented in Figure 6.1.

![Figure 6.1: Changes in, from the top, outdoor temperature, room temperature and radiator temperature with varying outside temperature.](image)

The outside temperature is around 20 °C in the night and increases in the morning as the sun rises. Figure 6.1 shows how the temperatures change in both the room and radiator as the outside temperature varies. The room temperature decreases in the beginning when the water in the radiator is still cold (the initial radiator
temperature is 15 °C). After approximately an hour the temperature in the radiator is the same as in the room then the radiator will not provide the room with cooling and the temperature in the room will increase since the outdoor temperature is higher than the room temperature. The temperature profiles for the room and radiator are similar, but the radiator has not as distinct changes in the temperature as the room. This result shows that the heat capacity of water is higher than for air so the heating of the radiator should be slower than the heating of the air in the room. In addition the radiator is not exposed to the air ventilation in the same degree as the air in the room.

6.2 Case 2: Room and radiator temperature during cooling

In case 2 cooling is applied to see if the room temperature decreases when the radiator starts to cool. The room and the radiator temperature are measured in the simulation and should be affected by the cooling load. The cooling load of the system is controlled by the split fraction, $\alpha$. If the split is 0, as in case 1, then all of the cooling water is circulating in the chiller loop. When the split is higher than zero ($\alpha > 0$) the water in the radiator is circulated, not only in the chiller circuit, but also through the radiator, see Figure 4.1. The results form the simulation are shown in Figure 6.2 and the manipulated variable for case 2 in Table 6.2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.0</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 6.2 displays how the room and radiator temperatures are effected when a split is introduced into the system. From the start the temperatures respond in the same way as in case 1, Figure 6.1. Where the temperature decreases because the radiator temperature is lower then the room temperature. The splits changes from 0 to 1 at time 10 h, the temperature in the radiator decreases. The water in the radiator is replaced with the cold water from the chiller loop that has a temperature of 0 °C. Further when the radiator temperature decreases as the water is replaced, the room temperature also decreases. The room gets cooled down and the temperature in the room falls with 8.8 °C within 3.5 hours. In the radiator the temperature drop is 19.7 °C.
Figure 6.2: Changes in, from the top, the room temperature, the radiator temperatures and split fraction.

6.3 Case 3: \( Mass_{ice} \) and \( mass_{water} \)

In case 3 the goal was to see how the cooling affected the \( mass_{ice} \) and \( mass_{water} \). The same simulation as in case 2 were made but with different values of \( \alpha \). The manipulated variables for the simulation are presented in Table 6.3 and the results in Figure 6.3.
## Table 6.3: Simulation variables case 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ 0.0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\alpha$ 0.5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>$\alpha$ 0.0</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>$Q_{chiller}$ 0.0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

![Mass ice](image1)

![Mass water](image2)

![Split fraction](image3)

**Figure 6.3:** Changes in, from the top, $mass_{ice}$, $mass_{water}$ and $\alpha$.

The results presented in Figure 6.3 show how the cooling of the room changes the amount of ice and water in the storage tank. When no cooling is applied (0-10 h and 15-24 h) both outputs are constant. When cooling ($\alpha > 0$) is applied the amount of ice is decreasing while amount of water is increasing. In the start of the cooling, at time 10 h, the $mass_{ice}$ and $mass_{water}$ decreases and increases, respectively, faster than at the end of the simulation. This is because the radiator temperature is high in the beginning when there has been no circulation in the radiator and the temperature here is the same as in the room. The hot radiator
temperature will increase the $T_{\text{mix}}$ that goes into the storage tank and increase the speed of melting the ice. The reason why ice is melting when cooling is applied is to produce cold water for the radiator when the need for cooling is present. Since the $Q_{\text{chiller}}$ is turned off ($Q_{\text{chiller}} = 0$) there is no generated ice as the system runs.

The system in this project is a closed system, no water or ice enters or leaves the system, the total amount of $\text{mass}_{\text{ice}}$ and $\text{mass}_{\text{water}}$ should be constant. From Figure 6.3 the sum of these two are constant. A more accurate calculation is done represented by Figure A.4 in the Appendix A.3.

### 6.4 Case 4: $Q_{\text{chiller}}$

In case 4 the $\text{mass}_{\text{ice}}$ and $\text{mass}_{\text{water}}$ were studied with both a split and a chiller unit, $Q_{\text{chiller}}$. The manipulated variables used are presented in Table 6.4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.0</td>
<td>10</td>
</tr>
<tr>
<td>$Q_{\text{chiller}}$</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>$Q_{\text{chiller}}$</td>
<td>0.5</td>
<td>15</td>
</tr>
<tr>
<td>$Q_{\text{chiller}}$</td>
<td>0.0</td>
<td>20</td>
</tr>
</tbody>
</table>

Case 4 is a continuation of case 3. The same splits are used as in the previous case. In addition $Q_{\text{chiller}}$ is turned on and off after 15 h and 20 h. The purpose of the simulation is to see whether $Q_{\text{chiller}}$ generates ice.

The results from the simulation are shown in Figure 6.4. When the $Q_{\text{chiller}}$ is turned on the amount of water decreases while the amount of ice increases meaning that $Q_{\text{chiller}}$ is generating ice. In this simplified model of cooling a room the capacity of the $Q_{\text{chiller}}$ is said to be a reservoir. In this case it is set to 1.0 kW. This results in a change of ice from 10.6 kg to 64.4 kg in 5 h. To increase the speed of generating ice the values of $Q_{\text{chiller}}$ can be increased to a higher value. Simulations with higher capacity on the chiller are presented in the Appendix A.5. The results show that with increased capacity on the chiller the amount of generated ice is increased in the same time period.
Figure 6.4: Changes in, from the top, mass $\text{ice}$, mass $\text{water}$, $\alpha$ and $Q_{\text{chiller}}$.

### 6.5 Case 5: All the states, $x$

As a final simulation the system was run with a cooling load and a chiller. The aim was to see how all the states in the system correspond to each other as the manipulated variables were changed. The manipulated variables are presented in Table 6.5. The results are shown in Figure 6.5.

Figure 6.5 shows how all the states of the system changes as the manipulated variables are changed. The results for the indoor and radiator temperature are
Table 6.5: Simulation variables case 5

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.0</td>
<td>10</td>
</tr>
<tr>
<td>$Q_{chiller}$</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>$Q_{chiller}$</td>
<td>0.5</td>
<td>15</td>
</tr>
<tr>
<td>$Q_{chiller}$</td>
<td>0.0</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure 6.5: Changes in, from the top, room temperature, radiator temperature, $mass_{ice}$ and $mass_{water}$ while changing the manipulated variables $\alpha$ and $Q_{chiller}$, similar to the results in case 2, Section 6.2 except for the lower split fraction (0.5
compared to 1.0 in case 2), and shows how they are not affected by the chiller in the storage tank. The $mass_{ice}$ and $mass_{water}$ results are the same as in case 3, and are not effected by the change in outdoor temperature.
The overall results from the simulation show that the system responds properly to the changes applied. In case 1 the indoor temperature changed as a result of ventilation. The ventilation contributed to air entering the room with an other temperature than the inside temperature. In Appendix A.1 the ventilation has been further investigated. These results shows that varying the ventilation volume has a direct effect on the room temperature. As the ventilation increases the temperature in the room is more and more influenced by the outdoor temperature. When the ventilation is zero the room temperature is unaffected. These responses show correct behaviour to what is expected when an air exchange with different temperatures is present. The temperature outside will also affect the material of the building. The building materials will be heated from the outside and contribute to a temperature increase inside because of heat conduction. This affect is not included in the model neither is heat contributions from people inside and interior sources as computers. If this would be included in the model the indoor temperature would be affected by four inputs, the ventilation, the walls of the building, the people and the interior. The ventilation is an input that can be controlled to a certain degree as long as the air quality specifications are achieved. The heating or cooling of the building due to conduction, people and interior are sources that can not be controlled and will be a disturbance to the system. One way to minimize the effect of natural conduction through the walls is to isolate the room or building. The two remaining disturbances, people and interior, are inputs that is hard to control and will be present in different degrees during the 24 h circle.

The radiator temperature is also affected indirectly by the ventilation of the room. This is shown in the mass and energy balances in Chapter 4.2. Therefore, as a result of increasing indoor temperature and no circulation in the radiator, the radiator temperature increases as the room temperature increases. Both the room and radiator temperature profiles follow the outdoor profile of the temperature. The radiator profile is less defined because it is not directly affected by the ventilation,
also water has a higher heat capacity than air and require more energy in order to change temperature, see Table A.1.

In the simulation in case 1 the temperature of the water in the radiator and the air in the room has different values at simulation start. The results show that after approximately an hour the water in the radiator and the air in the room has the same temperature. This was evaluated as a reasonable response. To change the time of the response, the parameters $U$ and $A$ should be evaluated further. In order to get more accurate values for the parameters they should be compared to parameters used in the industry or in household applications. This would give an indication of how correct the values selected in this report are. The parameters can also be changed if the response is too slow or too fast in comparison with what the use of the system is intended for.

In case 2 a split is added to the system, initiating cooling of the room. The split opens for cold water to circulate through the radiator and replaces the still water in the radiator. The results show that the temperature in the radiator changes quickly as the water in the radiator is replaced. The room temperature decreases as the split is applied and the aim to cool the room is reached. The room is still affected by the ventilation and the temperature changes with the outdoor temperature since the cooling load is maximized with a split fraction of 1. In order to keep the temperature inside at a stable level (unaffected by the outdoor) there are two opportunities. Either the split has to be lower than 1 or the mass$_{sys}$ has to be increased or decreased. With a split less than 1, the cooling load can increase as the temperature in the room increases. If the split is set to 1 no more water can circulate through the radiator, if not the mass$_{sys}$ is increased. The mass$_{sys}$ has been tested out in simulations in order to get a reasonable values, Appendix A.2. With a higher mass the cooling load is increased and the cooling is more rapid. It also increases the ice consumption and makes the storage last for a shorter time.

The third case presents the amount of ice and water during cooling and no cooling. As the split is 0 all the water is circulated in the chiller loop and the amount of ice and water remains unchanged. When a split is applied to the system the ice is consumed in order to provide the radiator with cold water. As the ice decreases the amount of water is increasing as a consequence that ice melts to water. The total amount of ice and water should still be conserved as the system is closed. Calculations in Appendix A.3 show that total volume ice and water is constant. The simulations in case 3 show that an ice storage of 50 kg is almost used up during a 5 h period with a split equal to 0.5. In the Appendix A.4 more simulations with different splits are shown, and the results show that with a lower split the ice storage lasts longer.
The fourth case show how the behaviour of $Q_{chiller}$. $Q_{chiller}$, affects both the amount of water and ice in the tank. As $Q_{chiller}$ is turned on $mass_{ice}$ and $mass_{water}$ increases and decreases, respectively. The result show that $Q_{chiller}$ generates ice which is the goal of the chiller. In the Appendix A.5 is also tested with the aim of melting ice, and the results show that this is possible if required.

In case 5 all the states are presented in order to get an overview of all the responses from the states when ventilation, cooling and $Q_{chiller}$ is applied. The results show that the states corresponds the same as in the individual cases. $Mass_{ice}$ and $mass_{water}$ are not affected by the ventilation and the temperature in the room and radiator are not affected by $Q_{chiller}$. 
8 | Further work

In this rapport a mathematical model for a room with a cold ice storage has been developed. The model has been tested with different input parameters and the results show that the system reacts properly to the input changes. Further work will be to regulate and control the model. Boundaries for $mass_{ice}$ and $mass_{water}$ need to be implemented into the Simulink model to make sure that the volumes do not become negative. An air condition unit should be added in order to test out different operating strategies, full or partial storage. In addition the system should be optimized with respect to either energy consumption, total costs or time. Scenarios with varying electricity prices should be optimized and a conclusion of the best possible solution should be presented.

The model of the heat exchange and the radiator needs to be more realistic. In this model both the room and radiator is said to be perfectly mixed CSTRs. In reality there will be temperature gradients in both the radiator and the room.

In addition the temperature in the storage tank is assumed constant due to perfect mixing and constant generating of ice or melting. In reality the water can be used in the radiator even if it does not have 0 °C and there should be made a dynamic model for increasing temperature after all ice is consumed.
9  | Conclusion

In this rapport a mathematical model for a room with a cold ice storage has been developed. Aspects on what to consider when running the model are presented, as well as strategies.

The system has been tested in 5 cases with different values of the manipulated variables. The states in the system has been measured and the results analysed. The results showed that the inlet and radiator temperatures are affected by the ventilation. Increasing ventilation increases the affect of the ventilation. The temperature in the room decreases as the radiator is turned on. The cooling load is adjusted by amount of cold water running through the radiator. Ice is consumed during cooling and increased cooling increases the consumption of ice. Amount of ice and water is constant. Ice is generated when the chiller in the storage tank is turned on.

The system should now be ready for a optimizing and a regulatory layer.
Bibliography


A | General Facts and Calculations

The constants used in the simulations in the main report are presented in the MATLAB script in Appendix B, and in Table A.1. The constants are the same in all the simulations in the project. The heat capacities are assumed constant for both the water and air.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{p_{\text{air}}}$</td>
<td>Heat capacity air</td>
<td>1.005</td>
<td>[kJ/kg,K]</td>
</tr>
<tr>
<td>$C_{p_{\text{water}}}$</td>
<td>Heat capacity water</td>
<td>4.220</td>
<td>[kJ/kg,K]</td>
</tr>
<tr>
<td>$\rho_{\text{water}}$</td>
<td>Density water</td>
<td>999.87</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>$\rho_{\text{air}}$</td>
<td>Density air</td>
<td>1.2929</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>$\Delta H_{\text{fus}}$</td>
<td>Latent heat of fusion for water</td>
<td>344</td>
<td>[kJ/kg]</td>
</tr>
</tbody>
</table>

The mathematical model for the system also required some parameters for the tank, radiator and room. These parameters are presented in Table A.2. The parameters were either tested in MATLAB in order to get reasonable values or estimated from values found in litterature. The size of the room is an estimate of a conference room.
Table A.2: Standard simulation parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{room}$</td>
<td>Volume of the room</td>
<td>100</td>
<td>$m^3$</td>
</tr>
<tr>
<td>$V_{vent}$</td>
<td>Ventilation volume</td>
<td>0.05</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>$U$</td>
<td>Overall heat transfer coefficient</td>
<td>0.03</td>
<td>$kw/(m^2,K)$</td>
</tr>
<tr>
<td>$A$</td>
<td>Heat exchange area</td>
<td>1.8</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$V_{tank}$</td>
<td>Volume of storage tank</td>
<td>140</td>
<td>$dm^3$</td>
</tr>
<tr>
<td>$mass_{sys}$</td>
<td>Circulation mass</td>
<td>0.07</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>$V_{radiator}$</td>
<td>Volume of radiators</td>
<td>0.06</td>
<td>$m^3$</td>
</tr>
</tbody>
</table>

A.1 Varying Ventilation.

In case 1 the room and radiator temperature were measured when no cooling was applied while the outdoor temperature changed during the day. The room and radiator are affected by the outdoor temperature because of the ventilation. The ventilation parameter was then tested in order to see if the affect of outdoor temperature changed as the ventilation volume changed. The specific parameters for this simulation are shown in Table A.3. Figure A.1 displays the results from the simulation.

Table A.3: Simulation variables no ventilation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{vent}$</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure A.1 shows that without ventilation the indoor temperature is not affected by the outdoor temperature. Neither is the radiator temperature. Both the room and radiator temperature changes in the first hour as the temperatures in the room and radiator is not the same and a heat exchange takes place. When the temperatures are equal no more heat exchange occurs.

The model was then tested with different ventilation volumes in order to see if the values of the parameter were reasonable and if the the room and radiator temperatures were affected. The different values for the ventilation are presented in Table A.4. In the main simulations in the project the air ventilation, $V_{vent}$, is 0.05 $m^3/s$. The results from the simulation are shown in Figure A.2.

As the ventilation increases the amount of air that is exchanged in the room
Figure A.1: Changes in, from the top, outdoor temperature, room temperature and radiator temperature with no ventilation.

Table A.4: Simulation variables changing ventilation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value [m³/s]</th>
<th>Colour</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{vent}$</td>
<td>0.02</td>
<td>Blue</td>
</tr>
<tr>
<td>$V_{vent}$</td>
<td>0.05</td>
<td>Green</td>
</tr>
<tr>
<td>$V_{vent}$</td>
<td>0.10</td>
<td>Red</td>
</tr>
</tbody>
</table>

increases. This will influence the temperature inside the room, and indirect the radiator. The blue line in Figure A.2 represents the lowest ventilation of 0.02 m³/s, this line shows that the indoor temperature changes slowly in comparison with the red and green line which have a higher ventilation volume. When the ventilation
Figure A.2: Changes in, from the top, outdoor temperature, room temperature and radiator temperatures with different ventilations.

increases the indoor and radiator temperature changes more distinct as displayed well in Figure A.2. The results with varying ventilation parameters also gives a good indication that the parameter for the ventilation in the test are reasonable.

A.2 Varying $mass_{sys}$

In order to see how the circulation mass, $mass_{sys}$, influences the states in the system simulations were done with different masses. The different values of $mass_{sys}$ used in the simulations are presented in Table A.5 and the results in Figure A.3.
Table A.5: Simulation parameters $mass_{sys}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value [m$^3$/s]</th>
<th>Colour</th>
</tr>
</thead>
<tbody>
<tr>
<td>$mass_{sys}$</td>
<td>0.04</td>
<td>Red</td>
</tr>
<tr>
<td>$mass_{sys}$</td>
<td>0.07</td>
<td>Green</td>
</tr>
<tr>
<td>$mass_{sys}$</td>
<td>0.10</td>
<td>Blue</td>
</tr>
</tbody>
</table>

Figure A.3: Changes in, from the top, room temperature, radiator temperature, $mass_{ice}$ and $mass_{water}$ with varying values of $mass_{sys}$.

Figure A.3 shows how all the states are affected by the change in circulation volume. As the volume increases the affect on all the states increases. The tem-
perature in the room is higher as the $mass_{sys}$ is lower. While the temperature in the radiator is higher as the $mass_{sys}$ is lower. The $mass_{ice}$ is decreasing slower and $mass_{water}$ increases slower. All these parameters respond to an decrease in cooling load as the $mass_{sys}$ decreases. From the results in Figure A.3, $0.07 \text{ m}^3/\text{s}$ was selected as a reasonable value $mass_{sys}$ for the main simulations in the project.

A.3 Total Amount of $mass_{ice}$ and $mass_{water}$

During cooling the amount of ice is decreasing while amount of water is increasing. Case 3 in the main rapport, Section 6.3 show these results. As a result of this the total amount of ice and water should be constant during simulation. The simulation from case 3 was simulated again and the total values of ice and water were measured. The parameters used in the simulation are the same as in case 3 and are presented in Table A.6. Figure A.4 shows the total amount of $mass_{water}$ and $mass_{ice}$ during the simulation.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.0</td>
<td>10</td>
</tr>
<tr>
<td>$Q_{chiller}$</td>
<td>1.0</td>
<td>15</td>
</tr>
<tr>
<td>$Q_{chiller}$</td>
<td>0.0</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure A.4 shows that the sum of $mass_{ice}$ and $mass_{water}$ is constant at a value of 130 kg. This proves that the total amount of mass in the system is conserved.
A.4 Varying Split Fractions

In case tree in the main rapport the split was turned to 1 at time 10 h and the temperatures both in the room and radiator decreased as a result of this. The split is here tested to see if the cooling load decreases as the split decreases. The input parameters used here are presented in Table A.7 and the results in Figure A.5.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Time [h]</th>
<th>Colour</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.2</td>
<td>10</td>
<td>Red</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>10</td>
<td>Green</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.0</td>
<td>10</td>
<td>Blue</td>
</tr>
</tbody>
</table>

When the split is decreased the amount of cooling drops. The temperature in the room is higher as the split is lower. Which is represented by the red line in Figure A.5.
Figure A.5: Changes in, from the top, room temperature, radiator temperature and $\alpha$.

There were also made simulations to see how the different splits affected the $\text{mass}_{\text{ice}}$ and $\text{mass}_{\text{water}}$. The parameters for the simulation are presented in Table A.8 and the results in Figure A.6.

**Table A.8:** Simulations variables split

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>colour</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.2</td>
<td>Blue</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4</td>
<td>Green</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.6</td>
<td>Red</td>
</tr>
</tbody>
</table>

Figure A.6 presents the results of the $\text{mass}_{\text{ice}}$ and $\text{mass}_{\text{water}}$ as the cooling load is varying with different split fractions, $\alpha$. As expected the consumption of ice increases as the cooling load increases. This also implies that $\text{mass}_{\text{water}}$ increases more rapidly.
A.5 Capacity of $Q_{\text{chiller}}$

The capacity of the chiller was tested in order to see if it could generate ice at a higher speed. It was also investigated to see if the $Q_{\text{chiller}}$ could melt ice in case this would be needed. Simulation parameters are presented in Table A.9 and the results in Figure A.7.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value [kW?]</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{\text{chiller}}$</td>
<td>1.0</td>
<td>15</td>
</tr>
<tr>
<td>$Q_{\text{chiller}}$</td>
<td>-1.0 (heating)</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure A.7 shows that the generation of ice increases as the capacity of $Q_{\text{chiller}}$ increases. The amount of mass increases from 2.8 kg to 56.7 kg in 5 hours in
Figure A.7: Changes in mass_{ice} and mass_{water} when changes in Q_{chiller}.

comparison with 29.7 kg when Q_{chiller} was 0.5 kW. The results also shows that Q_{chiller} is able to melt ice if this should be required (at time 22h).
B MATLAB Code and Block Diagram

The built-in function `sfuntmpl` was used in the simulations. Below is the MATLAB script `sfuntmpl`, Section B.1, with the state-equations. In the end the block diagram from Simulink is presented, Section B.2.

B.1 MATLAB script

```matlab
%% s-function sent into the simulink model.

% States:
% x1 = Temperature of the room
% x2 = Temperature of the radiator
% x3 = Mass of ice
% x4 = Mass water

% Inputs:
% u1 = Outdoor temperature
% u2 = splitfraction
% u3 = Q_chiller

function [sys,x0,str,ts,simStateCompliance] = ...
    sfuntmpl(t,x,u,flag,a)

switch flag,

%%%%%%%%%%%%%%%%
% Initialization%
%%%%%%%%%%%%%%%%

case 0,

[sys,x0,str,ts,simStateCompliance]=mdlInitializeSizes;
```
% Derivatives %
case 1,
sys=mdlDerivatives(t,x,u,a);

% Update & Terminate %
case {2,9},
sys=[]; %do nothing

% Outputs %
case 3,
sys=mdlOutputs(t,x,u);

otherwise
DAStudio.error('Simulink:blocks:unhandledFlag',num2str(flag));
end

% mdlInitializeSizes
% Return the sizes, initial conditions,
% and sample times for the S-function.
%=================================================================
function [sys,x0,str,ts,simStateCompliance]=mdlInitializeSizes

sizes = simsizes;
sizes.NumContStates = 4;
sizes.NumDiscStates = 0;
sizes.NumOutputs = 4;
sizes.NumInputs = 3;
sizes.DirFeedthrough = 0;
sizes.NumSampleTimes = 1; % at least one sample time is needed
sys = simsizes(sizes);

% initialize the initial conditions for the four states, x1, x2, x3 and 4x.

x0 = [296 288 50 80];

str = [];

% sample time
simStateCompliance = 'UnknownSimState';

% end mdlInitializeSizes

%================================================================

% mdlDerivatives
% Return the derivatives for the continuous states.
%================================================================

% The equality constraints of the system:

function sys=mdlDerivatives(t,x,u,a)

sys = [a.vent*(u(1)-x(1))/a.room ...
      a.U*a.A*(x(1)-x(2))/(a.room*a.cpair*a.densair);
      a.masssys*u(2)*(a.Tw-x(2))/(a.radiator*a.denswater) + ...
      a.U*a.A*(x(1)-x(2))/(a.radiator*a.cpwater*a.denswater);
      u(3)/a.fus - a.masssys*a.cpwater*u(2)*(x(2)-a.Tw)/a.fus];

sys(4) = -sys(3);

% end mdlDerivatives

%================================================================

% mdlOutputs
% Return the block outputs.
%================================================================

function sys=mdlOutputs(t,x,u)

sys = x;

% end mdlOutputs
%% Constants & Parameters

a.vent = 0.05; % [m^3/s]

a.room = 100; % [m^3]

a.U = 0.03; % [kW/m^2*K]

a.A = 1.8; % [m^2]

a.Tw = 273; % [K]

a.massys = 0.04; % [kg/s]

a.radiator = 0.060; % [m^3]

a.fus = 334; % [kJ/kg]

a.cpair = 1.005; % [kJ/kg K], 273.15 K

a.densair = 1.2929; % [kg/m^3], 0 [C]

a.cpwater = 4.220; % [kJ/kg K], 273.15 K

a.denswater = 999.87; % [kg/m^3]

%% Outside temperature

var.time = [0 7 12 15 17 20 24]*3600'; % [h]

var.signals.values = [20 23 30 30 25 23 20]'+273;

var.signals.dimensions = [1]';

%% Case switch

% normal values

alpha1 = 0;

tswitch1 = 36000; % 10h

alpha2 = 0;

tswitch2 = 54000; % 15h

Qchiller = 0;

tswitch3 = 72000; % 20h

Qchiller2 = 0;

tswitch4 = 0; % 0h

Qchiller3 = 0;

tswitch5 = 0;

switch 1

% case one

case 1

% case two

case 2

alpha1 = 1.0;

% case three

case 3
```matlab
alpha1 = .5;
alpha2 = -.5;
tswitch2 = 54000;
%case four
    case 4
        alpha1 = .5;
        alpha2 = -.5;
        Qchiller = 0.5;
        Qchiller2 = -.5;
        tswitch3 = 61200;  %17h
        tswitch4 = 79200;  %22h
    end

%case five
    case 5
        alpha1 = .5;
        alpha2 = -.5;
        Qchiller = .5;
        Qchiller2 = -.5;
        tswitch1 = 18000;  %5h
        tswitch2 = 36000;  %10h
        tswitch3 = 54000;  %15h
        tswitch4 = 72000;  %20h
    end

%varying ventilation
    case 6  % Manually varying the ventilation in the parameters

%varying mass_sys
    case 7  % Manually varying the mass_sys in the parameters

%varying splits
    case 8
        alpha1 = 0.2;  %varying the split manually and ...
        %storing the values
        %alpha1 = 0.5;
        %alpha1 = 1.0;

%capacity of Q_chiller
    case 9
        Qchiller = 2.0;
        Qchiller2 = -3;
end

% Simulink simulation

tsim = 3600*24;  %24 hour simulation over one day
```

XV
B.2 Simulink Model