A Distributed Coding Cooperative Scheme for Wireless Communications

Pierluigi Salvo Rossi, Sarod Yatawatta and Francesco Palmieri,
Università di Napoli “Federico II”,
Dipartimento di Ingegneria Sistemi,
Via Claudio 21, 80125 Napoli, Italy,
salvsal@unina.it

Athina P. Petropulu, Giulio Iannello,
Drexel University, Seconda Università di Napoli,
Electrical and Computer Engineering Department,
3141 Chestnut Street, Dipartimento di Ingegneria dell’Informazione,
Philadelphia, PA 19104, US,
Via Roma 29, 81031 Aversa (CE), Italy,
sarof@cbs.ece.drexel.edu
athi@cbio.ece.drexel.edu
francesco.palmieri@unina2.it
g.iannello@unicampus.it

Abstract — Cooperative transmission schemes have
recently been proposed to improve the uplink performance
of wireless communications systems by increasing
diversity without requiring use of multiple transmit
antennas. We here propose a novel cooperative scheme
that performs distributed coding of user data, uses
only one channel per user and allows a flexible trade-off
between rate and diversity, thus achieving higher
data rates than previously proposed coded cooperative
schemes. The point of operation of the system on the
diversity-rate curve can be arbitrarily chosen according
to the number of cooperating users. A trade-off is
increased complexity as more users are required to
be simultaneously decoded by the BS. We describe
the construction of the distributed code for a n-user sys-
tem, that results in reduced complexity decoding op-
erations at the BS. The performance of the proposed
method is studied via analytical bounds and numerical
simulations.

I. INTRODUCTION

Wireless communications play a central role in modern information society. The increasing demand of high-quality services and the intrinsic problems affecting the radio channel make this research area always requiring different and various solutions. One of the main problems affecting the performance of wireless communications is fading, caused by multipath propagation, introducing severe variations in signal strength as function of the user position. Spatial, temporal and frequency domain diversity has been used to effectively obtain independent fading paths for the transmitted signal [9][6]. Space-time coding combined with multiple antennas [2] is the most typical means for creating spatial diversity, but user constraints often do not allow the use of antenna arrays at a mobile handset, thus confining the deployment of multiple antennas to the Base Station (BS).

Cooperation has recently emerged as a means to obtain spa-
tial diversity while each user still employs a single antenna [11][12]. In a cooperative scenario, the users share their single antennas, with each user decoding and transmitting information on behalf of other users as well as transmitting user’s own information. This makes the problem different from the relay prob-

lem [1]. Cooperation may be thought as a virtual array MIMO system obtained via antenna sharing. Increasing the number of cooperating users allows one to exploit the trade-off between diversity and rate, corresponding to the trade-off between multiplexing and diversity in MIMO channels [9]. Maximum order of diversity and maximum data rate are not both achievable at the same time [14].

In the coded cooperation scheme [13], 2 users propagate portions of their codeword through different paths. They both can listen and transmit on two orthogonal channels. Each user computes a codeword to be sent to the BS, and transmits only one half. The partner node decodes the codeword based on the portion it received and transmits a complementary portion of it. Both portions received at the BS are used to recover the original information. This scheme implies that there should be enough redundancy in the transmitted codeword portions for both the partner and BS to be able to recover the entire codeword and thus achieve full diversity. The bits transmitted in the first frame play a crucial role in decoding operations, and the need for such redundancy limits the rate of actual information, indeed rates 1/3 or less were considered [13].

In this paper we propose a new scheme, that uses block-coded cooperation-diversity. For simplicity we will refer to the cooperative users as the Cooperation Group (CG). We consider a CG of U-users which are assigned orthogonal channels. Each user transmits in one channel and listens in the remaining U - 1 channels. The information transmitted over each channel is a combination of source information of all users in the CG. This strategy can be globally viewed as a linear block coding built on the joint source information of all the users. This approach allows us greater flexibility in selecting data rate as users do not need to recover each other’s codewords but only decode each other’s current symbols. The task of recovering source information from the transmitted symbols is left to the BS.

Our scheme allows to select arbitrarily the point of operation of the system on the diversity-rate curve [14]. The price is paid in complexity as more users are required to be simultaneously decoded by the BS. We propose a construction of the distributed code for a U-user CG that results in reduced complexity decoding operations at the BS.

II. THE PROPOSED SCHEME

We consider the case in which each user transmits one symbol per Time Slot (TS). The bits that each user wants to communicate to the BS will be referred to as source-bits, while the bits transmitted by each user on his own channel will be referred to as code-bits. All users and the BS are assumed perfectly synchro-
Table 1: XOR-sum for BPSK symbols.

<table>
<thead>
<tr>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_1 \oplus b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>+1</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>+1</td>
<td>+1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Table 2: XOR-sum for QPSK symbols.

<table>
<thead>
<tr>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$q_1 \oplus q_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1, -1)</td>
<td>(-1, -1)</td>
<td>(-1, -1)</td>
</tr>
<tr>
<td>(-1, -1)</td>
<td>(-1, +1)</td>
<td>(-1, +1)</td>
</tr>
<tr>
<td>(-1, +1)</td>
<td>(+1, +1)</td>
<td>(+1, +1)</td>
</tr>
<tr>
<td>(+1, +1)</td>
<td>(+1, -1)</td>
<td>(+1, -1)</td>
</tr>
</tbody>
</table>

Table 3: Transmission bits of 3 users in a single frame.

<table>
<thead>
<tr>
<th>User</th>
<th>$c_i(1)$</th>
<th>$c_i(2)$</th>
<th>$c_i(3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$b_1(1)$</td>
<td>$b_2(1) \oplus b_3^{(2)}(1)$</td>
<td>$b_3^{(2)}(2) \oplus b_1^{(3)}(1) \oplus b_2^{(3)}(1)$</td>
</tr>
<tr>
<td>2</td>
<td>$b_1(2)$</td>
<td>$b_2(2) \oplus b_3^{(2)}(1)$</td>
<td>$b_3^{(2)}(2) \oplus b_1^{(3)}(1) \oplus b_2^{(3)}(1)$</td>
</tr>
<tr>
<td>3</td>
<td>$b_1(3)$</td>
<td>$b_2(3) \oplus b_3^{(2)}(1)$</td>
<td>$b_3^{(2)}(2) \oplus b_1^{(3)}(1) \oplus b_2^{(3)}(1)$</td>
</tr>
</tbody>
</table>

IIA. Construction of the distributed code

To introduce how the proposed algorithm works, let us consider the case of $k = n - 1$, i.e., rate $R = (n - 1)/n$ and order of diversity $L = 2$. Let us denote $\hat{c}_i(l)$ the estimate of the code-bit transmitted at the $l$-th TS by the $i$-th user performed by the $i$-th user, and let $\mu(i, n) = \begin{cases} \text{mod} (i, n) & \text{mod} (i, n) \neq 0 \\ n & \text{mod} (i, n) = 0 \end{cases}$.

Let us consider a circular ordering of the users in the CG, then the distributed code is implemented as follows:

- $c_i(1) = b_i(1)$,
- $c_i(l) = b_i(l) \oplus c_{\mu(i-1,n)}^{(l)}(l-1)$, $l \in \{2, \ldots, n-1\}$,
- $c_i(n) = c_{\mu(i-1,n)}^{(n)}(n-1) \oplus c_{\mu(i+1,n)}^{(n)}(1)$,

where $i \in \{1, \ldots, n\}$. In other words: in the first TS of a frame each user transmits his own first source-bit; in the internal TSs of a frame each user transmits the combination of his own current source-bit and the decoded bit of what his previous user has transmitted in the previous TS; in the last TS of a frame each user transmits the combination of the decoded bit of what his previous user has transmitted in the previous TS and the decoded bit of what his following user has transmitted in the first TS.

Fig. 1 shows a block diagram representing how the $i$-th user combines the symbols to construct the distributed code.

Tab. 3 shows code-bits of a frame for the case $n = 3$. Note that $\hat{b}_i^{(1)}(l)$ denotes the estimate of $b_i(l)$ performed by the $i$-th user, while $\hat{b}_i^{(m,0)}(l)$ denotes the estimate of $b_i(l)$ performed by the $i$-th user, which in turn was based on the estimate provided.

nized. All the channels (user-user and user-BS) are considered slow-Rayleigh-fading additive-white-Gaussian-noise channels.

Let us consider the single CG composed of $U$ cooperative users. Within the CG, each user transmits over $i$ out of $U$ orthogonal channels (e.g., TDMA, FDMA, CDMA), while it listens and decodes information from the remaining $U - 1$ cooperative users. For simplicity, we will consider BPSK modulation at the user level. Without loss of generality in the following we will use the term "bit" to denote a BPSK symbol. We will also assume that coherent detection is performed at every receiver (BS and cooperative users). Users transmit their information in frames of $n$ TSs, and each user transmits $k$ source-bits per frame. In this paper we only consider the case when the number of users in the CG equals the number of TSs composing a frame, i.e., $U = n$, though this is not a constraint. Each user transmits at rate $R = k/n$ and obtains, as shown in the sequel, order of diversity $L \leq n - k + 1$. The order of diversity is defined as the opposite of the slope of the Bit Error Rate (BER) vs Signal-to-Noise Ratio (SNR) curve in the log-log domain.

Let us denote $\{b_1(1), b_2(1), \ldots, b_k(l)\}$ the $k$ source-bits that the $i$-th user wants to transmit in a single frame. The baseband discrete-time signal (after matched filtering and sampling at the symbol rate) that the $j$-th user receives ($j = 0$ denotes the BS) from the $i$-th user during the $l$-th TS of a frame is:

$$y_j(l) = \alpha_{il} \sqrt{E_s} c_i(l) + w_j(l),$$

where $\alpha_{il}$ is the energy-per-source-bit of the single user, $R$ is the code rate, $c_i(l) \in \{-1, +1\}$ is the code-bit transmitted by the $i$-th user in the $l$-th time slot, and where $\alpha_{il} \sim \text{Rayleigh}(\sqrt{2})$ and $w_j(l) \sim N(0, \sigma_j^2)$ are the fading envelope and the additive noise over the channel between users $i$ and $j$, respectively. In this paper we assume that the fading is constant within a frame, i.e., we consider slow fading. Moreover we consider a symmetric configuration where

$$\sigma^2_j = \begin{cases} \sigma_j^2/2 & j = 0 \\ \sigma_j^2/2 & j \neq 0,i \end{cases}, \quad \sigma^2_i = \begin{cases} \sigma_i^2/2 & j = 0 \\ \sigma_i^2/2 & j \neq 0,i \end{cases}.$$

The code, $c_i(l)$, of the $i$-th user combines the different pieces of information coming from various channels to be transmitted over his own channel. The natural choice is to use the XOR-sum (denoted with $\oplus$). Tab. 1 shows the case for BPSK symbols. The XOR-sum is a fair way to introduce combing as it does not change the symbol alphabet nor the symbol energy. Thus we can focus on advantage given by cooperation rather than worrying about the effect of the signal constellation. Use of M-ary modulation requires the extension of the XOR-sum to an alphabet of $M$-symbols, e.g., considering a QPSK modulation we may refer to Tab. 2. The extension has to guarantee that the alphabet size is not altered.
by the $m$-th user, i.e. $\tilde{b}_j^{(m)}(t)$. For simplicity we do not consider how $\tilde{b}_j^{(m)}(t)$ depends on the particular $m$-th user. The source bits of every user are distributed over all the available channels, allowing to obtain diversity.

To show what happens at the BS, let us consider the case $R=2/3$. From Eq. (1) and Tab. 3 we obtain

$$
\begin{align*}
y_{10}(1) &= \alpha_{10}\sqrt{RE_0}b_{1}(1) + w_{10}(1) \\
y_{20}(1) &= \alpha_{20}\sqrt{RE_0}b_{2}(1) + w_{20}(1) \\
y_{30}(1) &= \alpha_{30}\sqrt{RE_0}b_{3}(1) + w_{30}(1) \\
y_{10}(2) &= \alpha_{10}\sqrt{RE_0}b_{2}(2) \oplus b_{3}(1) + w_{10}(2) \\
y_{20}(2) &= \alpha_{20}\sqrt{RE_0}b_{1}(2) \oplus b_{3}(1) + w_{20}(2) \\
y_{30}(2) &= \alpha_{30}\sqrt{RE_0}b_{1}(2) \oplus b_{2}(1) + w_{30}(2) \\
y_{10}(3) &= \alpha_{10}\sqrt{RE_0}b_{2}(2) \oplus b_{3}(1) + w_{10}(3) \\
y_{20}(3) &= \alpha_{20}\sqrt{RE_0}b_{1}(2) \oplus b_{3}(1) + w_{20}(3) \\
y_{30}(3) &= \alpha_{30}\sqrt{RE_0}b_{2}(2) \oplus b_{1}(1) + w_{30}(3)
\end{align*}
$$

Using Tab. 3, it is evident that every bit is related always to the same set of source bits (by different users). This allows to reduce computational complexity, since the obtained system of 9 equations can be decoupled into 3 subsystems. Let us define:

$$
\begin{align*}
u_i &= [b_1(1), b_{\mu(i+1,n)}(2)]^T, \\
v_i &= [y_{10}(1), y_{\mu(i+1,n)}(2), y_{\mu(i+2,n)}(3)]^T, \\
z_i &= [w_{10}(1), w_{\mu(i+1,n)}(2), w_{\mu(i+2,n)}(3)]^T, \\
f_i &= [\sigma_{10}, \sigma_{\mu(i+1,n)}, \sigma_{\mu(i+2,n)}]^T, \\
C &= \begin{bmatrix} 1 & 1 & 0 \\
0 & 0 & 1 \end{bmatrix}^T.
\end{align*}
$$

Assuming that all estimates performed by the users are error-free, Eq. (2) can be re-written as

$$
v_i = \sqrt{RE_0} \text{Diag}(f_i) \cdot (C \circ u_i) + z_i, \quad i \in \{1,2,3\},
$$

where $\circ$ denotes row-column product with sum replaced by XOR-sum; $\text{Diag}(x)$ denotes the ordinary row-column product; $\text{Diag}(x)$ is a diagonal matrix whose main diagonal is the vector $x$. Let us ignore fading and noise. The 3 equations of (3) represent a modulation scheme described by the signal-space constellation

$$
\begin{align*}
s^{(0)} &= \sqrt{RE_0}[-1,-1,-1]^T, \\
s^{(1)} &= \sqrt{RE_0}[-1,+1,+1]^T, \\
s^{(2)} &= \sqrt{RE_0}[+1,+1,-1]^T, \\
s^{(3)} &= \sqrt{RE_0}[+1,-1,+1]^T
\end{align*}
$$

illustrated in Fig. 2, which can be shown to be uniquely associated to

$$
\begin{align*}
u_i^{(0)} &= [-1,-1]^T, \\
u_i^{(1)} &= [-1,+1]^T, \\
u_i^{(2)} &= [+1,+1]^T, \\
u_i^{(3)} &= [+1,-1]^T
\end{align*}
$$

respectively. Decoupling Eqs. (2) into (3) allows the BS to decode by computation of 12 distances in an 3-dimensional space instead of 64 distances in an 9-dimensional space. To perform ML decoding by means of the minimum distance, the BS needs to know the correspondence between Eqs. (5) and (4). The latter can be represented by the Constellation Matrix $C$. More specifically, the BS can easily generate a table of correspondence by use of

$$
s^{(m)} = \text{mod}(C \cdot u^{(m)}, 2), \quad m \in \{0,1,2,3\},
$$

where $u^{(m)}$ is the binary representation of $m$ considered as a column vector.

The proposed algorithm can be easily extended for arbitrary $n$ in the case $R = (n-1)/n$ and $L=2$, and the computation saving at the BS for that case is replacing $2^n$ distances in an $n^2$-dimensional space with $n^{2n}$ distances in an $n$-dimensional space. It is easy to implement a similar scheme for a generic rate $R = k/n$, where each user decodes information from $L$ different users, though a general formula including all the cases it is not easy to find.

III. PERFORMANCE

In the absence of fading it can be shown, by use of the union bound [3], that the BER can be expressed approximately as

$$
P_c \approx \frac{3}{2} Q(2\sqrt{RE_0}/\sigma_0),
$$

with $Q(t) = \frac{1}{\sqrt{2\pi}} \int_{t}^{\infty} e^{-x^2/2}dx$. In the presence of fading the set of distances in the signal-space is altered by the fading coefficients, and Eq. (7) can be re-written as

$$
P_{c_f} \approx \frac{3}{2} Q(2\gamma),
$$

where $\gamma = (\sigma_{\mu}^2 + \sigma_{\rho}^2)/\sigma_0$ with $i \neq j$. Eq. (8) provides the conditional BER that has to be averaged according to fading statistics to obtain the average performance\footnote{Note that the algorithm relates each dimension of the signal space to a different channel, thus the fading scales each coordinate independently from the others.}. For the Rayleigh fading coefficients, the probability density function of $\gamma$ is $p(\gamma) = \frac{2\gamma}{\sigma_0^2} e^{-\gamma}$, thus obtaining, according to the procedure showed in [4],

$$
P_c \approx \frac{3}{4} (1 + \Gamma)^{-2}.
$$

where $\Gamma = \frac{2\rho^2}{\sigma_0^2}$, is the SNR at the BS.

Similarly, for the general case $R = k/n$, we have $\gamma \sim \text{Erlang}(\Gamma, d_{\min})$, where $d_{\min}$ is the minimum distance of the block code corresponding to Eq. (3), and

$$
P_c \approx \frac{2^n-1}{4} (1 + \Gamma)^{-d_{\min}}.
$$
Figure 3: BER for $R = 2/3$ and $L = 2$. Analytical results.

Figure 4: BER for $R = 2/3$ and $L = 2$. Simulation results.

i.e. order of diversity $L = d_{\text{min}} \leq n - k + 1$ is obtained.

In a more realistic scenario the presence of errors on the user-user channels affects the performance introducing an error floor. A bound on the error probability ($P_e$) is the following (for more details see [15]):

$$P_e \leq \frac{(2\Gamma_u + 1)}{(\Gamma_u + 1)^2} + \frac{3/4}{(\Gamma_u + 1)(1 + \Gamma)} - 1,$$

where $\Gamma_u$ is the SNR of the worst user-user channel. The derivation is based on the fact that Eq. (10) holds only in the error-free case (for user-user channels). Applying the Total Probability Theorem we obtain

$$P_e \leq \left(1 - P_{\text{ec}}\right) - \frac{1}{n}(1 - (n + 1)P_{\text{ec}})P_e,$$

where $P_{\text{ec}}$ is the probability that the error-free case happens, thus resulting in Eq. (11).

IV. SIMULATIONS

Our simulations were performed using Matlab software running 10000 trials per case. Each trial was made by generating BPSK symbols (with uniform probability distribution) for the users and iid Rayleigh fading coefficient (assumed constant in each frame). ML decoding was performed by selection of the minimum distance with reference to the scheme of Fig. 2.

Figs. 3 and 4 show the BER-vs-SNR curves for the case $R = 2/3$, $L = 2$, obtained via analytical bounds, see Eq. (11), and numerical simulations, respectively. By comparing the slope of the performance curve in case of cooperation with perfect decoding on the user-user channels and in case of absence of cooperation (see curves in Fig. 3 indicated by $\Gamma_u = \infty$ and "no-cooperation", respectively), we note how the proposed scheme improves the performance via spatial diversity. Our simulation results indicate that the performance follows the perfect decoding trend for the case $\Gamma < \Gamma_u$. In the range $\Gamma \in [0, 20] \text{ dB}$ the degradation vanishes for $\Gamma_u > 20 \text{ dB}$.

Fig. 5 shows the BER-vs-SNR curves for the case $R = 2/5$, $L = 3$, obtained via numerical simulations. The modulation scheme corresponding to this case has not been presented, due to lack of space, and it can be found in [15]. The same considerations of the previous case about obtained diversity hold for this case. The proposed scheme obtains flexible transmissions with desired order of diversity and rates (above and below 1/2). The worst user-user channel dominates, and degrades, the overall performance introducing an error floor.

Summarizing our proposal we note:

- it is easily extended to a generic number of users $U > 2$;
- it does not present a bottleneck due to user-decoding;
- it requires only 1 transmission channel per user;
- it can not switch from cooperation to non-cooperation resulting in an error floor.

Fig. 6 compares the analytical bounds for the proposed scheme with $R = 2/3$ and $L = 2$, for the proposed scheme with $R = 2/5$ and $L = 3$, and the scheme proposed by Janani et al. [13] with $R = 1/4$ and $L = 2$, all in the error-free case. Both the proposed schemes waste less resource (greater transmission rates) for creating cooperative diversity, and it can be evinced how for BER $= 10^{-2}$ the 5-user proposed scheme provides a reduction of 4 dB for the requested SNR, i.e., cooperation of 5 users allows 4 dB saving for each user (provided that their reciprocal SNR is greater than 20 dB). Fig. 7 compares the point of operations of the proposed schemes on the rate-diversity plane with the one of the Janani scheme.

The presence of the error-floor determined by the worst user-user channel reduces the utility of the proposed method to a scenario where the topology is known and the user-user channels...
are good. However in a scenario where user-user channels are bad, cooperation is not the best way to attack the problem.

In this paper, due to lack of space, we omitted results we obtained for different values of $R$ and $L$ and different modulation schemes (e.g. QPSK), these results can be found in [15].

V. CONCLUSION

A novel scheme for coded cooperative wireless communications has been proposed. It is based on linear combinations of symbols to be transmitted and estimates of the symbols previously transmitted by different users. It can be viewed as a distributed linear block code allowing to obtain a flexible trade-off between spatial diversity and transmission rates just by distinguishing information of all the users over all the available channels. Transmitting combined information of many users over each channel allows to obtain diversity. Appropriate cooperative algorithms can keep low the computational complexity at the Base Station. Analytical and simulation results for the performance have been derived in terms of Bit Error Rate vs Signal-to-Noise Ratio. The main problem is the presence of an error floor on the performance that makes necessary to have good user-user channels.

Future work concerns investigation of the possibility to eliminate the error floor by allowing each user to transmit on more channels. Enabling the user to switch from cooperation mode to non-cooperation and notify the BS, one can trade diversity order for mitigating the effect of a bad user-user channel. Also we are interested in determining, given a set of users, the optimum partition in CGs, and given a CG, who listen to whom in terms of diversity, complexity, transmission rates, user-user distances, other parameters evaluating the quality of the communications.

REFERENCES


