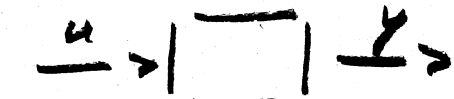
Review: Canonical Representations

5150

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}\,u$$
$$y = \underline{c}^T \underline{x}$$



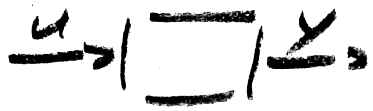
w/ unique:

$$\underline{\underline{T}}\,\dot{\underline{x}} = \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{x} + \underline{\underline{T}}\,\underline{b} + u$$

$$\tilde{x} := Tx$$

$$\Rightarrow \dot{\tilde{\underline{x}}} = \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1}\tilde{\underline{x}} + \tilde{\underline{b}}\,u$$
$$y = \underline{c}^T \underline{\underline{T}}^{-1}\tilde{\underline{x}}$$

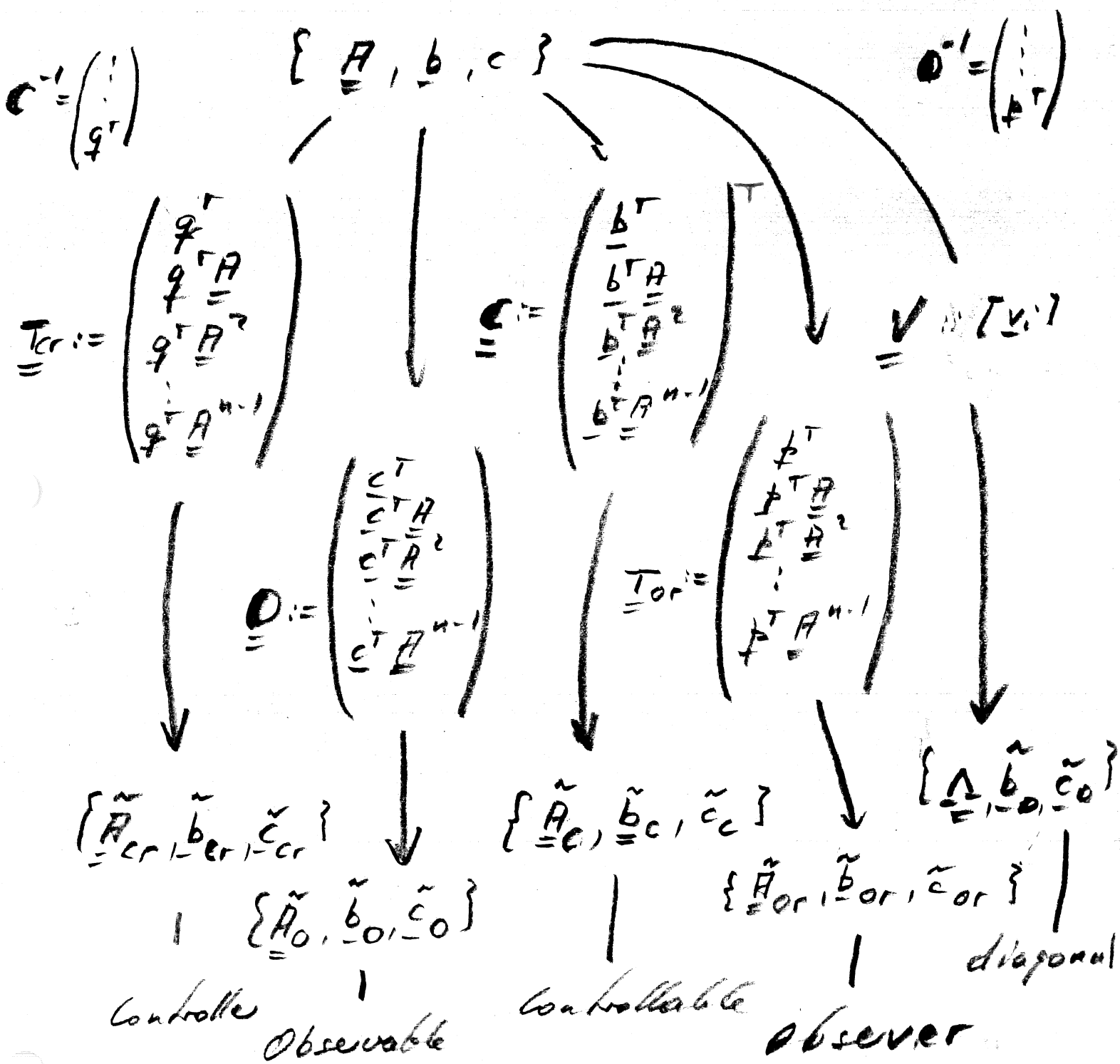
$$\dot{\tilde{\underline{x}}} = \tilde{\underline{\underline{A}}}\,\tilde{\underline{x}} + \tilde{\underline{b}}\,u$$
$$y = \tilde{\underline{c}}^T \tilde{\underline{x}}$$



n^{th} order ODE:

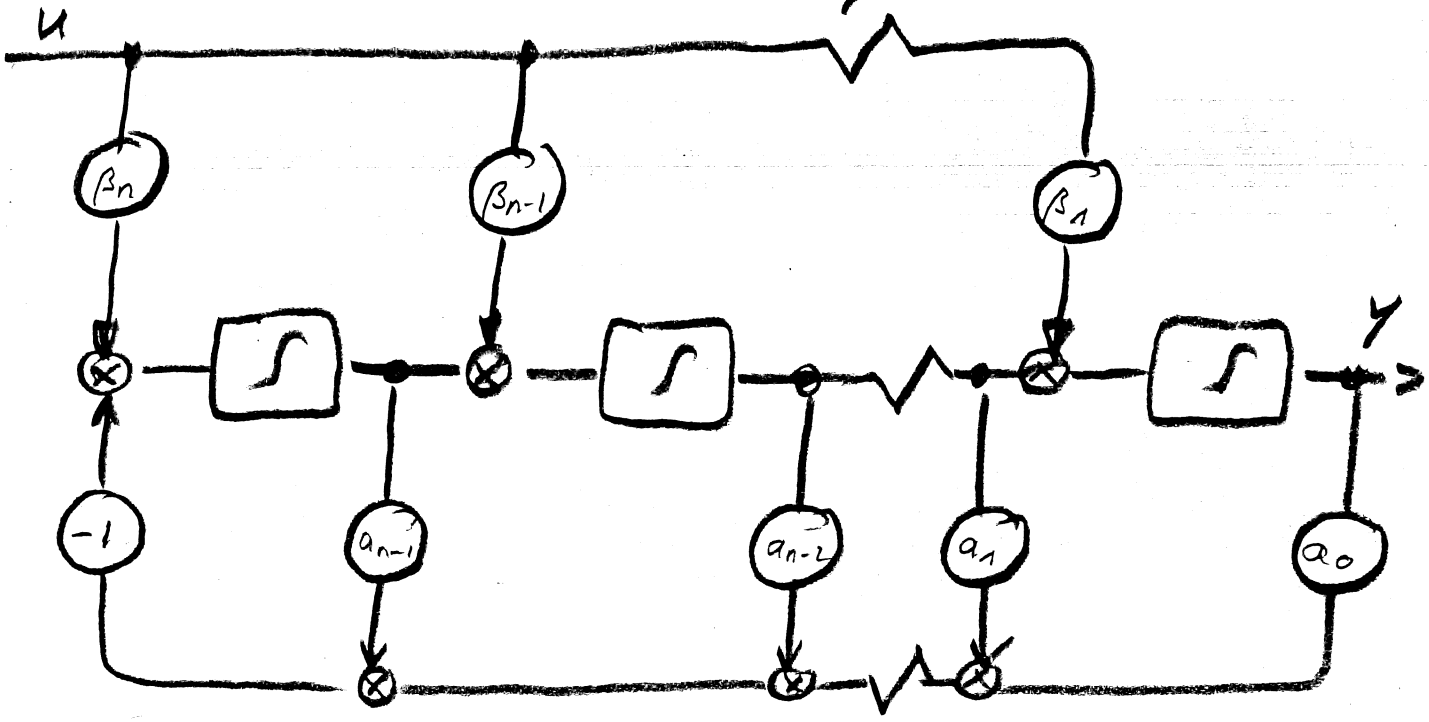
$$\sum_{i=1}^{n} a_i y^{(i)} = \sum_{i=1}^{m} b_i u^{(i)}$$

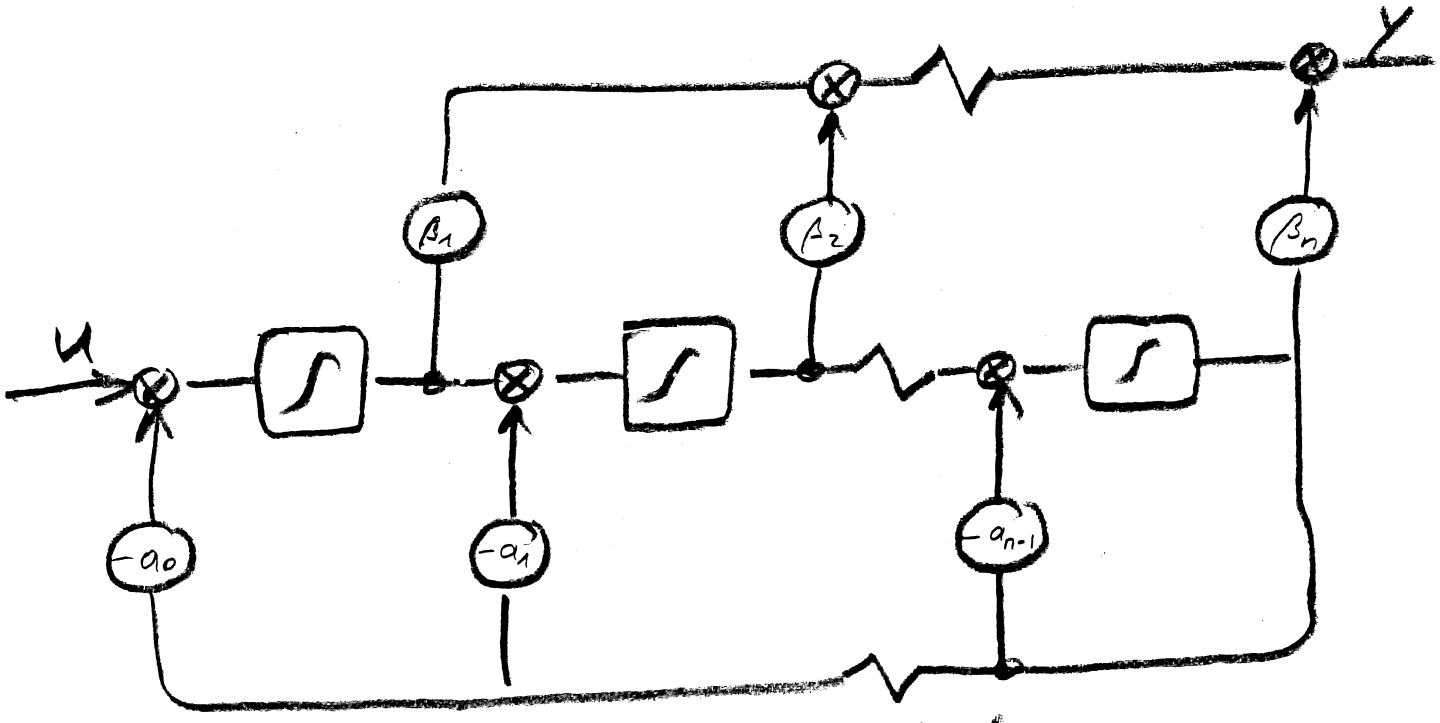
Transformations



$$\sum_{i=0}^{n} a_i y^{(i)} = \sum_{i=0}^{m} b_i u^{(i)}$$

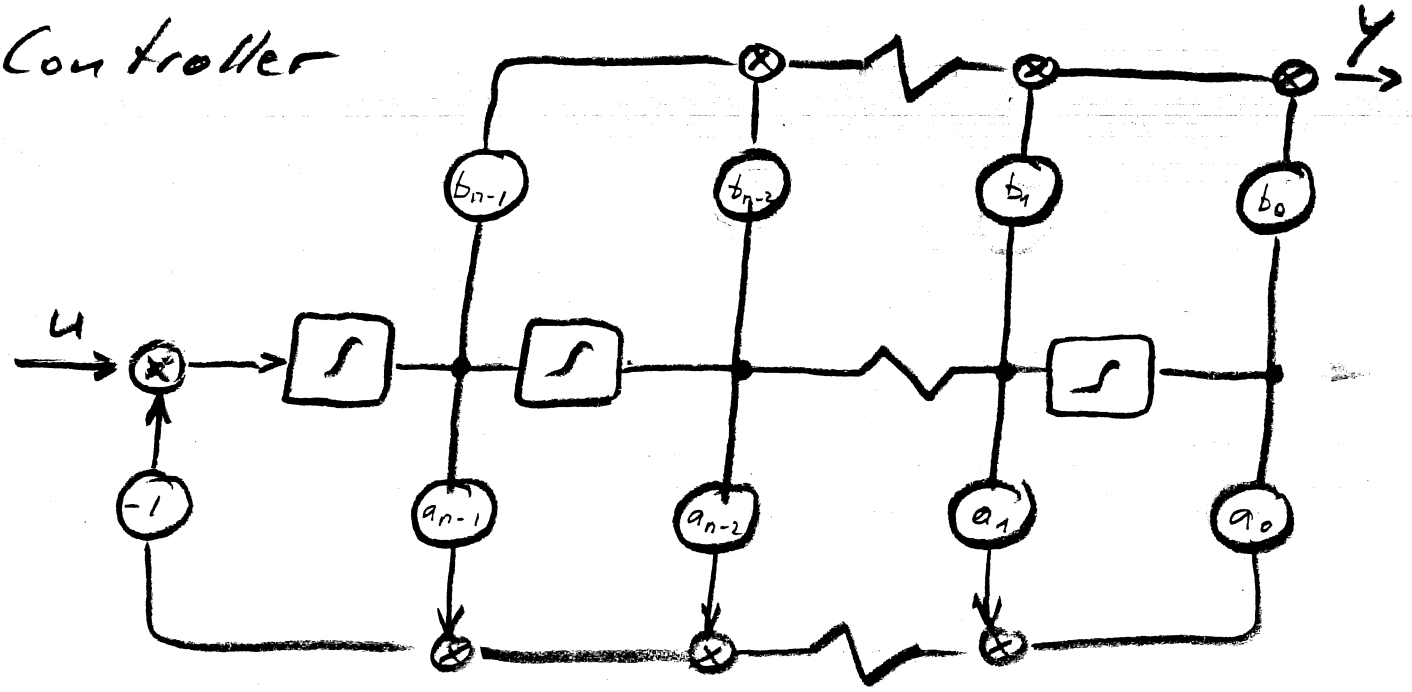
Observability

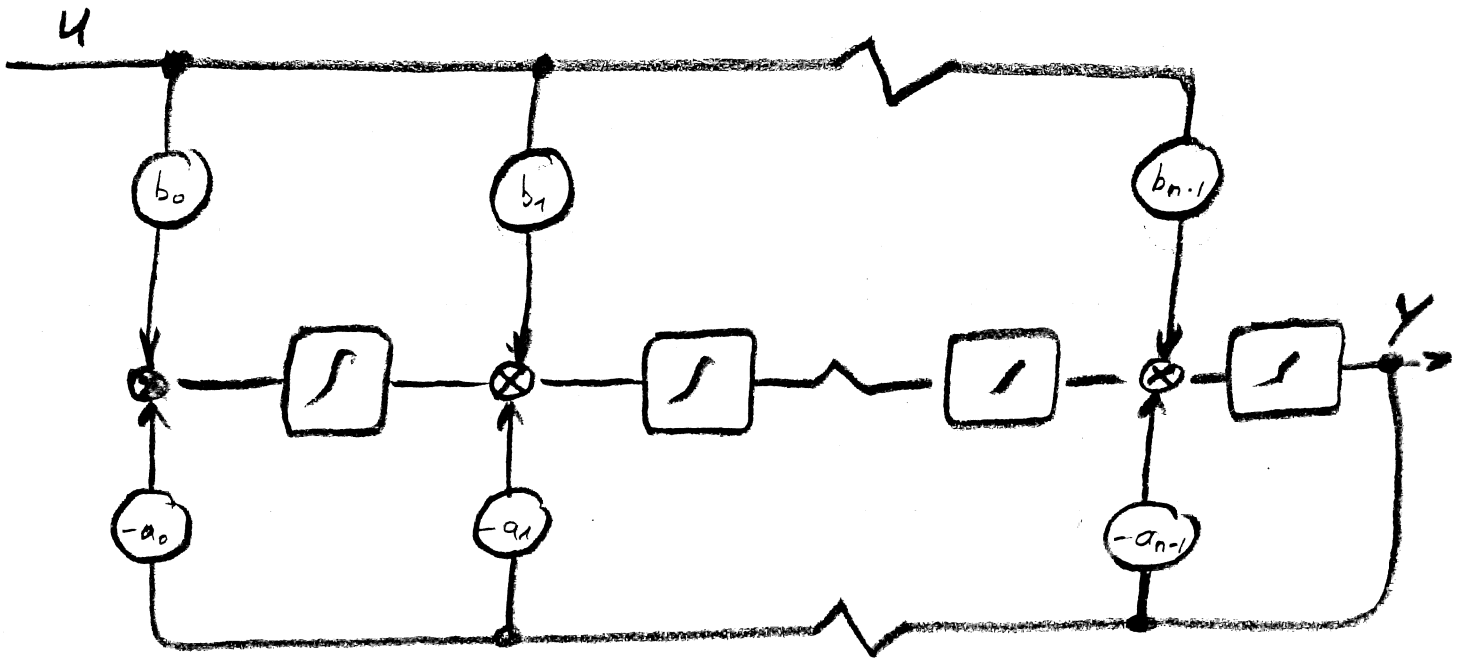




Controllability

Controller





Observer

Controller

$$\underline{\underline{\tilde{A}}}_{cr} := \begin{bmatrix} 0 & 1 & & \\ & 0 & 1 & \underline{\underline{0}} \\ & \underline{\underline{0}} & & \ddots \\ & & & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{bmatrix}$$

$$\underline{\tilde{b}}_{cr}^T := [0, 0, 0, \ldots, 1]$$

$$\underline{\tilde{c}}_{cr}^T := [b_0, b_1, b_2, \ldots, b_{n-1}]$$

Observer

$$\underline{\underline{\tilde{A}}}_{or} := \begin{bmatrix} 0 & & & -a_0 \\ 1 & 0 & \underline{\underline{0}} & -a_1 \\ & 1 & 0 & \\ & & \ddots & \\ & & 1 & -a_{n-1} \end{bmatrix} = \underline{\underline{\tilde{A}}}_{cr}^T$$

$$\underline{\tilde{b}}_{or} := [b_0, b_1, b_2, \ldots, b_{n-1}]$$

$$\underline{\tilde{c}}^T := [0, 0, 0, \ldots, 1]$$

Controllability

$$\underline{\underline{\tilde{A}}}_c = \underline{\underline{\tilde{A}}}_{cr}$$

$$\underline{\tilde{b}}_c^T = [1, 0, \ldots, 0]$$

$$\underline{\tilde{c}}_c^T = [\beta_0, \beta_1, \ldots, \beta_n]$$

Observability

$$\underline{\underline{\tilde{A}}}_o = \underline{\underline{\tilde{A}}}_{or}$$

$$\underline{\tilde{b}}_o^T = [\beta_n, \beta_{n-1}, \ldots, \beta_0]$$

$$\underline{\tilde{c}}_o^T = [1, 0, 0, \ldots, 0]$$

$$\beta_i = b_{n-i} - \sum_{j=1}^{i-1} a_{n-j} \beta_{i-j}$$