DRIFT-FLUX MODELS

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Two-phase problem

A general transient two-phase flow problem is formulated using;

- Two-fluid model
- Drift-flux model

Two fluid model; Considers each phase separately. Two sets of conservation equations for mass, momentum and energy plus interactions (transfers of mass, momentum and energy) [Ishii and Hibiki, 2011].

Two-phase problem

Drift-flux model; considers two separate phases as a mixture phase. Mass conservation for each phase, mixture momentum equation and energy equation. In addition, slip law to cater for some relative motion of one phase with respect to the other [Ishii and Hibiki, 2011].

- General slip law

$$U_g = C_o U_s + V_{gu} \tag{1}$$

Here U_g is gas phase velocity ; C_o is the profile parameter; U_s is the total average superficial velocity and V_{gu} is the drift velocity of the gas [Shi et al, 2005].

Definitions

- Phase velocity; $U_g = \frac{Q_g}{A_g}$
- Superficial phase velocity; $U_{sg} = \frac{Q_g}{A} = \alpha_g U_g$
- Volume fraction; $\alpha_g = \frac{A_g}{A}$
- Total superficial phase velocity; $U_s = U_{sg} + U_{sl}$
- Drift velocity; $V_{gu} = U_g - U_s$.

Drift-flux model

We consider the following drift-flux model as given by [Evje and Fjelde, 2002]; assuming isothermal flow and no mass transfer between the phases.

$$\partial_t(\alpha_l \rho_l) + \partial_x(\alpha_l \rho_l U_l) = \Gamma_l \tag{2}$$

$$\partial_t(\alpha_g \rho_g) + \partial_x(\alpha_g \rho_g U_g) = \Gamma_g \tag{3}$$

$$\partial_t(\alpha_l \rho_l U_l + \alpha_g \rho_g U_g) + \partial_x(\alpha_l \rho_l U_l^2 + \alpha_g \rho_g U_g^2 + p) = -q \tag{4}$$

Source term q is defined as $q = F_w + F_g$.

Drift-flux model

Gravity term; $F_g = g(\alpha_l \rho_l + \alpha_g \rho_g) \sin\theta$.
Friction force term; $F_w = \frac{32 U_s \mu_{mix}}{d^2}$
Mixture viscosity; $\mu_{mix} = \alpha_l \mu_l + \alpha_g \mu_g$

We now have 7 unknowns, $\alpha_l, \alpha_g, \rho_l, \rho_g, U_l, U_g$ and p.
Since there are only 3 equations, we need 4 additional constraints to close the model.

Closure laws

Normally required in terms of density models for each phase, wall friction model and slip law.
We use closure laws as given by [Evje and Fjelde, 2002];

$$\alpha_l + \alpha_g = 1 \tag{5}$$

The liquid density model is assumed as;

$$\rho_l = \rho_{l,0} + \frac{p - p_{l,0}}{a_l^2} \tag{6}$$

The gas density model is as follows;

$$\rho_g = \frac{p}{a_g^2} \tag{7}$$

Discretization

Staggered grid for spatial discretization. Phase velocities are defined at the faces and other variables at nodes.

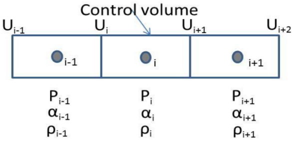


Figure: Staggered grid

Integrating over the control volume;

$$\int_{CV} \partial_t(\alpha_l \rho_l) dV + \int_{CV} \partial_x(\alpha_l \rho_l U_l) dV = 0$$

Discretization

$$\frac{d(\alpha_{li}\rho_{li})}{dt} = (\hat{\alpha}_{li}\hat{\rho}_{li}U_{li} - \hat{\alpha}_{li+1}\hat{\rho}_{li+1}U_{li+1})/\Delta x \tag{8}$$

$$\int_{CV} \partial_t(\alpha_g \rho_g) dV + \int_{CV} \partial_x(\alpha_g \rho_g U_g) dV = 0$$

which gives

$$\frac{d(\alpha_{gi}\rho_{gi})}{dt} = (\hat{\alpha}_{gi}\hat{\rho}_{gi}U_{gi} - \hat{\alpha}_{gi+1}\hat{\rho}_{gi+1}U_{gi+1})/\Delta x \tag{9}$$

$$\int_{CV} \partial_t(\alpha_l \rho_l U_l + \alpha_g \rho_g U_g) dV + \int_{CV} \partial_x(\alpha_l \rho_l U_l^2 + \alpha_g \rho_g U_g^2 + p) dV = \int_{CV} -\frac{32 U_s \mu_{mix}}{d^2} dV$$

which gives

Discretization

$$\frac{d(\alpha_{li}\rho_{li}U_{li} + \alpha_{gi}\rho_{gi}U_{gi})}{dt} = (\hat{\alpha}_{li}\hat{\rho}_{li}U_{li}^2 - \hat{\alpha}_{li+1}\hat{\rho}_{li+1}U_{li+1}^2)/\Delta x$$
$$+(\hat{\alpha}_{gi}\hat{\rho}_{gi}U_{gi}^2 - \hat{\alpha}_{gi+1}\hat{\rho}_{gi+1}U_{gi+1}^2)/\Delta x + (p_i - p_{i+1})/\Delta x - \frac{32U_{si}\mu_{mix,i}}{d^2} \tag{10}$$

Finally, using the 1^{st}-order upwind scheme as shown below;

$$\hat{x} = a_{i+1}x_i + (1 - a_{i+1})x_{i+1} \tag{11}$$

where a_{i+1} is given by;

$$a_{i+1} = \begin{cases} 1, & \text{if} \quad U_{i+1} \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Simulation example

Table: Parameters

Parameters	value	unit
a_g	316	m/s
a_l	1000	m/s
L	10	m
d	0.022	m
C_o	1.2	$-$
V_{gu}	0.25	m/s
μ_l	$5 * 10^{-2}$	Pa s
μ_g	$5 * 10^{-6}$	Pa s

$$\vec{x} = \overset{\text{State vector}}{\begin{pmatrix} \alpha_g \rho_g \\ \alpha_l \rho_l \\ \alpha_g \rho_g U_g + \alpha_l \rho_l U_l \\ U_l \\ U_g \\ \alpha_g \end{pmatrix}}$$

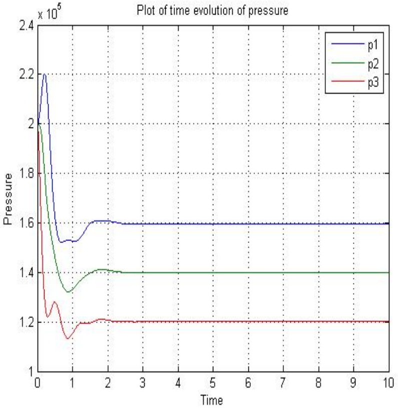
Simulation case

A stream of gas and liquid is injected at the inlet of the pipeline diameter 2.2 cm and length 10 m with superficial velocities of gas and liquid being 0.57 m/s and 1.6 m/s respectively. For simplicity, we assume a slip law with $C_o = 1.2$ and $V_{gu} = 0.54\sqrt{gd}$. We are interested in modelling the transient behaviour.

Table: Initial condition

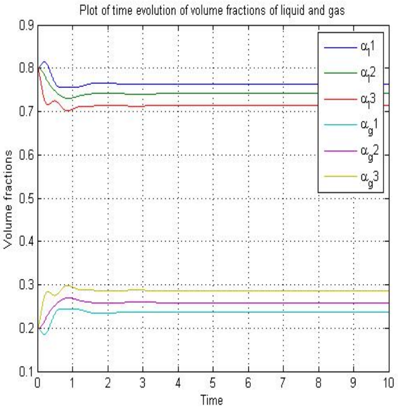
Variables	value	unit
α_g	0.2	$-$
α_l	0.8	$-$
ρ_g	2	kg/m^3
ρ_l	1000	kg/m^3
U_l	2	m/s
U_g	2.86	m/s
p	2	bar

Results and discussion



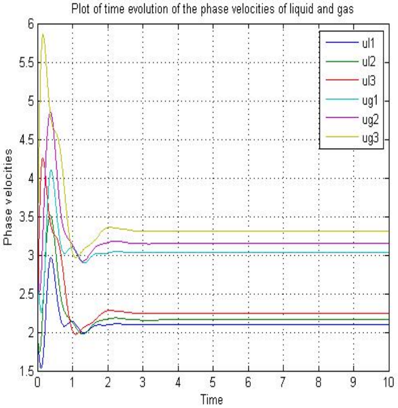
- Sharp peak at the inlet
- Pressure development afterwards is mainly due to friction forces.

Results and discussion



- Decreasing pressure causes gas expansion.
- Gas fractions increase while liquid fractions decrease.

Results and discussion



- Expansion results in increased gas mass flow rates.
- Liquid in front of gas is moved with larger velocities.
- Sharper peak at the outlet because of larger expansion.

After some time, flow is stabilized and steady state conditions achieved.

Conclusion

A drift-flux model has been used to simulate a two-phase flow problem. The results generally depict actual flow characteristics of two-phase flow in a pipeline.

References

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