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TKP11 ADVANCED PROCESS SIMULATION

DRIFT-FLUX MODELS

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Abstract

The report gives an insight into some concepts and characteristics of multiphase flow so that the applicability of a drift-flux model to a two-phase flow problem becomes easier to understand. Then focuses on a given drift-flux model and how this model was used to simulate a two-phase flow problem in matlab.

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1 INTRODUCTION

Multiphase flow is a phenomena that is widely encountered in many engineering operations and therefore knowledge of its characteristics, modelling and simulation is very important. In petroleum engineering for example, multiphase flow effects in well bores and pipes can strongly influence the performance of reservoirs and surface facilities. For example, pressure losses in the well can lead to losses in production of oil and gas. Therefore accurate multiphase flow models are essential in reservoir simulators in order to model and thereby optimize the performance of wells and reservoirs coupled to surface facilities(Shi et al., 2005).

Three types of pipe flow models are used in the context of petroleum engineering namely; Empirical correlations,Homogeneous models and mechanistic models (Shi et al., 2005). Empirical correlations are based on curve fitting of experimental data and are generally applied to a limited range of variables considered in experiments. Homogeneous models represent the fluid properties by mixture properties and apply techniques for single phase flow to the mixture. Drift flux models are homogeneous models that allow the use of slip relation between the phases. Mechanistic models are in general the most accurate and modelling is based on detailed physics of each of the different flow patterns (Shi et al., 2005).

In two phase flow, there is always some relative motion of one phase with respect to the other and therefore a two phase flow problem is formulated in terms of two velocity fields. A general transient two phase problem is formulated using a two-fluid model or a drift-flux model and this depends on the degree of coupling between the phases (Goda et al., 2003).

The two-fluid model is formulated by considering separately each phase in terms of conservation equations for mass, momentum and energy, and coupling the phases through interphasial transport (Ishii and Mishima, 1984). The advantage of a two-fluid model over a drift-flux model disappears if phasic momentum interactions are not accurately modelled as is indicated by previous studies and in certain cases, numerical instabilities result(Ishii and Mishima, 1984). The drift-flux model on the other hand is formulated by considering the mixture as a whole rather than two phases separately (Ishii and Hibiki, 2011). Conservation of mass is considered for each phase and one momentum equation for the mixture. But since two phase flow involves some relative motion of one phase with respect to the other (Goda et al., 2003), a slip relation is used for a drift-flux model. Therefore, the drift-flux model is an approximate formulation compared to the more rigorous two-fluid model. However, it's of considerable importance because of it's simplicity and easy applicability to a wide range of two phase flow problems of practical interest (Goda et al., 2003). The drift-flux model is studied extensively because of it's practical importance and it's not surprising that the major focus of this report is on drift flux models.

2 MULTIPHASE FLOW CONCEPTS

It's quite important to understand some multiphase flow concepts before a detailed discussion of drift-flux models and their applicability to multiphase flow problems.

2.1 General slip law

The general slip law that is used in this report was introduced by (Zuber and Findlay, 1965) and is of the following form;

$$U_g = C_o U_s + V_{gu} \tag{1}$$

Here U_g is the gas phase velocity averaged across the pipe area; C_o is the profile parameter(distribution coefficient) describing the effect of velocity and concentration profiles within the mixture; U_s is the total average superficial velocity and V_{gu} is the drift velocity of the gas, describing the buoyancy effect (Shi et al., 2005).

The phase velocity of a phase k (gas or liquid) is defined as $U_k = \frac{Q_k}{A_k}$ where Q_k is the volumetric flow rate of k and A_k is the pipe cross sectional area of k.

The total average superficial velocity, $U_s = U_{sg} + U_{sl}$ where U_{sg} and U_{sl} are superficial velocities of gas and liquid respectively. These are the velocities each corresponding fluid would have if all the cross sectional area of the pipe were available for each of the fluids to flow alone in the pipe (Molvik, 2011). Thus $U_{sg} = \frac{Q_g}{A}$ and $U_{sl} = \frac{Q_l}{A}$ where A is the pipe cross sectional area.

Therefore, the relation between superficial velocities and phase velocities is such that $U_{sg} = \alpha_g U_g$ and $U_{sl} = \alpha_l U_l$ where α_g and α_l are volume fractions of gas and liquid respectively (Molvik, 2011). The volume fractions are defined as $\alpha_g = \frac{A_g}{A}$ and $\alpha_l = \frac{A_l}{A}$ and therefore $\alpha_g + \alpha_l = 1$.

The drift velocity of phase k (gas or liquid) is defined as $V_{ku} = U_k - U_s$.

Note that the above defined velocities and phase fractions are cross sectional averages and therefore ideally suitable for use in one dimensional fluid models (Molvik, 2011).

2.2 Flow patterns

The existence of an interface between the phases complicates the description of two phase flow in tubes. In gas-liquid flow, this interface can take on a variety of forms depending on flow rates and physical properties of the phases, and geometry and inclination of the tube (McQuillan and Whalley, 1985). These different interfacial structures are called flow regimes or patterns.

Knowing the flow pattern is very important because it allows one apply the appropriate fluid-dynamic or heat transfer theories and as suggested in (Wambsganss et al., 1991), knowing the flow pattern in two phase flow is analogous to knowing the whether the flow is laminar or turbulent in single-phase flow.

Numerous studies have been carried out on two-phase flow in circular tubes to identify these patterns for both vertical, horizontal and inclined orientations and transitions primarily defined from visual observations. These transitions are not “sharp line” but rather fairly wide bands (Wambsganss et al., 1991).

However the focus in this section is briefly on describing the horizontal and vertical flow patterns so that it’s possible to identify the pattern encountered in two-phase flow in horizontal or vertical pipe orientations.

2.2.1 Flow patterns in horizontal flow

These are controlled by gravity and vapor shear forces where gravity forces dominate at low flow velocities resulting in stratified and wave flow while the at high flow velocities, the shear forces dominate resulting in annular flow.

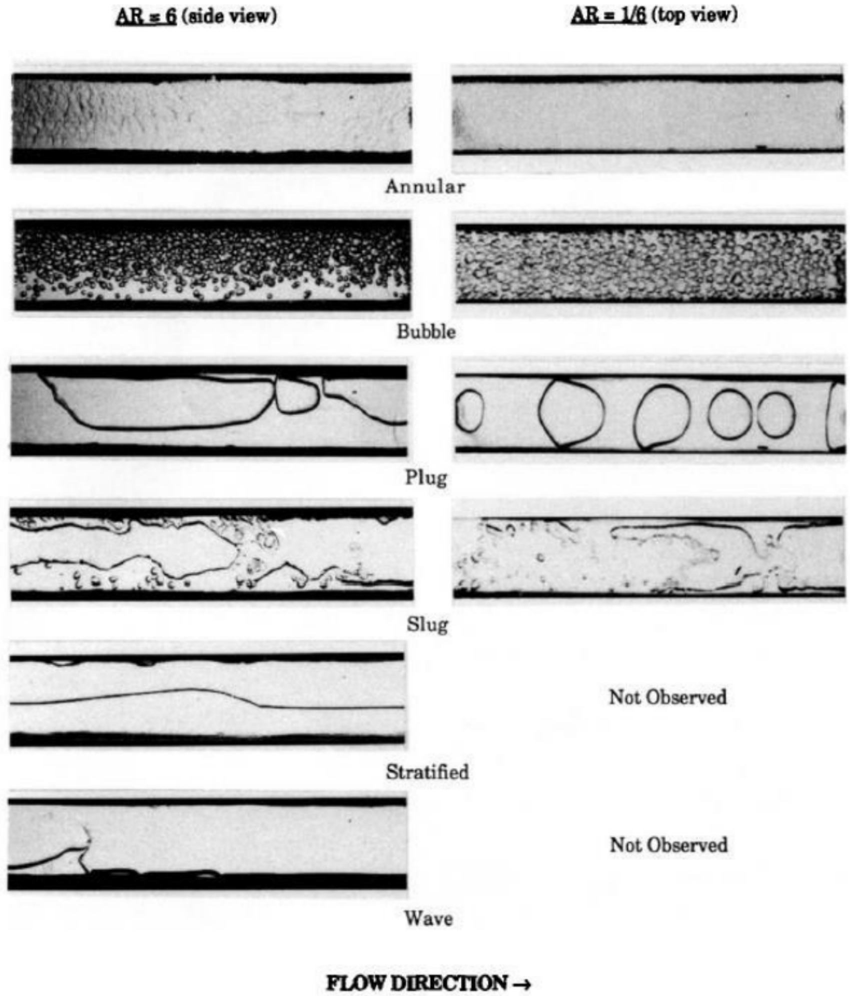

Figure 1: Horizontal flow patterns-aspect ratios of 6 and 1/6
Adapted from (Wambsganss et al., 1991)

Figure 1 shows a photograph of the patterns as identified in the study by (Wambsganss et al., 1991) and the description for each of the patterns is given below as given by (Wambsganss et al., 1991).

- Stratified flow; liquid flows along the bottom and gas on top with a smooth interface between the phases.

- Wave flow; similar to stratified flow but with waves at the interface travelling in the direction of flow.
- Plug flow; intermittent plugs of gas (elongated bubbles) in various sizes flowing in a continuous liquid phase, in the upper half of the flow channel.
- Slug flow; intermittent slugs of liquid and bubbles with waves growing to touch the upper part of the channel, propagating at high velocities.
- Bubble flow; vapor dispersed in a continuous liquid phase as small bubbles that tend to travel in the upper section of the flow channel.
- Annular flow; liquid forms a film around the channel wall with a vapour in the centre core that may contain entrained droplets.

2.2.2 Flow patterns in vertical flow

For two-phase upward flow in a vertical tube, a number of patterns may be identified depending on the way the two phases distribute axially or radially. The flow is normally chaotic and difficult to describe but we consider the flow patterns as described by (Taitel et al., 1980). These are as shown in figure 2 and a brief description is given for each as given by (Taitel et al., 1980).

- Bubble flow; gas phase is distributed in a continuous liquid phase in form of discrete small bubbles.
- Slug flow; most of the gas is in form of large bubbles with diameter almost equal to pipe diameter, often called “Taylor bubbles” separated by continuous liquid slugs that contain small gas bubbles.
- Churn flow; much more chaotic form similar to slug flow but with the “Taylor bubbles” becoming narrow and distorted in shape.

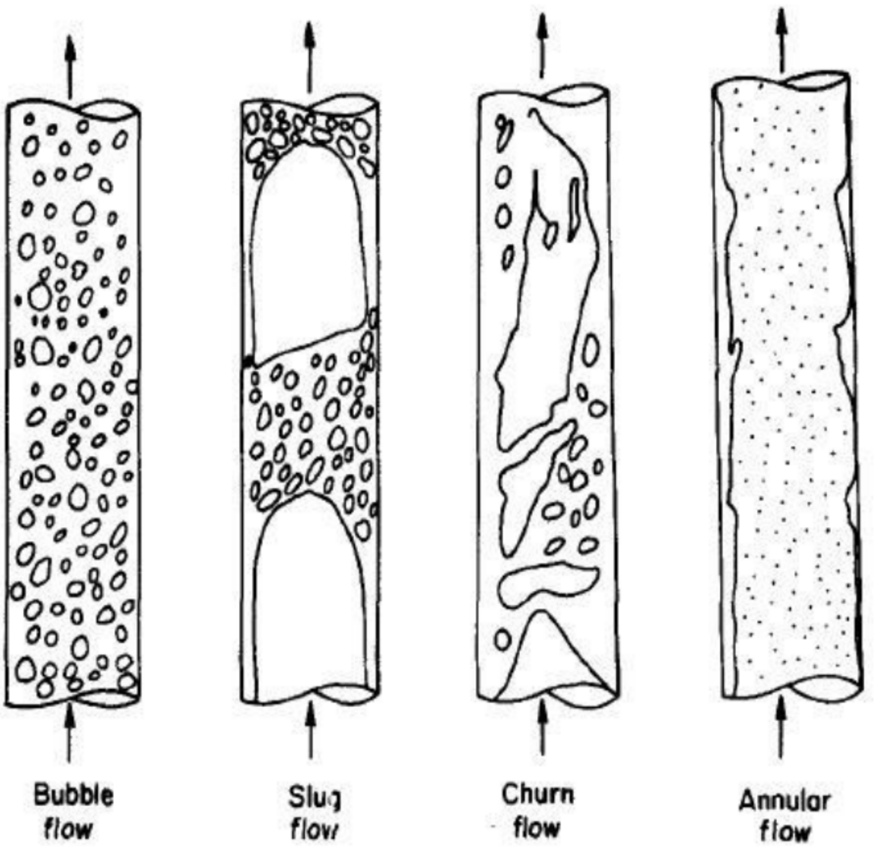

Figure 2: Flow patterns in vertical flow

Adapted from (Taitel et al., 1980)

- Annular flow; liquid moves upward partly in form of a liquid film on the wall and as droplets entrained in the gas core in the centre of the pipe.

Note that the stratifying gravity component is absent and therefore stratified flow disappears in vertical flow.

3 DRIFT FLUX MODEL

3.1 Drift-flux model

The basic concept of the drift-flux model is the consideration of two separate phases as a mixture phase. Therefore the fluid properties are represented by mixture properties making the drift-flux formulation simpler than the two-fluid formulation. This requires some drastic constitutive assumptions causing some two-phase flow characteristics to be lost. It's however this simplicity that makes the drift-flux model very useful in many engineering applications (Ishii and Hibiki, 2011).

For purposes of demonstration, this report considers a simple one dimensional model for two phases, with conservation of mass for each of the phases and a mixture momentum equation as follows;

$$\partial_t(\alpha_l \rho_l) + \partial_x(\alpha_l \rho_l U_l) = \Gamma_l \tag{2}$$

$$\partial_t(\alpha_g \rho_g) + \partial_x(\alpha_g \rho_g U_g) = \Gamma_g \tag{3}$$

$$\partial_t(\alpha_l \rho_l U_l + \alpha_g \rho_g U_g) + \partial_x(\alpha_l \rho_l U_l^2 + \alpha_g \rho_g U_g^2 + p) = -q \tag{4}$$

This model is a so-called drift-flux model (Evje and Fjelde, 2002). In this model, flow is assumed to be isothermal (no energy balance needed). We shall also assume that there is no mass transfer between the phases, thus $\Gamma_l = \Gamma_g = 0$

The α's are volume fractions, ρ's are densities and U's are phase velocities as defined in subsection 2.1. The term p in equation 4 is the common pressure for liquid and gas and q is the source term (Evje and Fjelde, 2002).

The source term q is defined as $q = F_w + F_g$ where

$$F_g = g(\alpha_l \rho_l + \alpha_g \rho_g) \sin\theta \tag{5}$$

is the gravitational contribution where g is the gravitational constant and θ is the inclination.

The friction force term F_w takes into account the viscous forces and forces between the wall and fluids and is given by

$$F_w = \frac{32 U_s \mu_{mix}}{d^2} \tag{6}$$

where d is the inner diameter, U_s is the total average superficial velocity as defined in subsection 2.1 and μ_{mix} is the mixture viscosity given by $\mu_{mix} = \alpha_l \mu_l + \alpha_g \mu_g$ and the viscosities for liquid and gas are assumed to be $\mu_l = 5 * 10^{-2} [Pas]$ and $\mu_g = 5 * 10^{-6} [Pas]$ respectively (Evje and Fjelde, 2002).

The model here considered has 7 unknowns and these are
$\alpha_l, \alpha_g, \rho_l, \rho_g, U_l, U_g$ and p. Since there are only 3 equations, we need 4 additional constraints to close the model so as to have a smooth solution. These are called closure laws and are discussed in the next section.

3.2 Closure laws

Several closure laws are normally required in terms of density models for each phase, a model for wall friction and a slip relation since we have some relative motion of one phase with respect to the other. These models are usually quite complex and often given in tabular form based on experimental data (Fjelde and Karlsen, 2002).

However, our objective is to specify some simple models that can be used for numerical demonstration purposes and the following closure laws as obtained from (Evje and Fjelde, 2002) are used.

The volume fractions are related as follows;

$$\alpha_l + \alpha_g = 1 \tag{7}$$

The slip law given in subsection 2.1 is used.

$$U_g = C_o U_s + V_{gu} \tag{8}$$

The following form of the liquid density model is assumed.

$$\rho_l = \rho_{l,0} + \frac{p - p_{l,0}}{a_l^2} \tag{9}$$

where $a_l = 1000$ m/s is the velocity of sound in the liquid phase and $\rho_{l,0}$ and $p_{l,0}$ are given constants. We shall assume $\rho_{l,0} = 1000$ kg/m^3 and $p_{l,0} = 1$ bar.

For the gas density, we assume the following form;

$$\rho_g = \frac{p}{a_g^2} \tag{10}$$

where $a_g = 316$ m/s is the velocity of sound in the gas phase.

The four closure laws are now given and therefore we can now solve the drift-flux model equations.

4 NUMERICAL SIMULATION

There could be various strategies that can be laid down to solve such a system of equations as the drift-flux model discussed in this report, from various schemes to commercial codes but because of the time constraints and lack of experience in this area, no information is given here.

Therefore, the focus here is on the strategy used in this report. The strategy is to carry out spatial discretization of equations 2, 3 and 4 on a staggered mesh grid and use a differential algebraic equation solver in matlab to integrate over time and predict the variation in the 7 variables here (variables), in space and time.

4.1 Staggered grid

The first obvious choice for the grid for use is a non-staggered grid. However the choice of storing variables such as pressure and velocities at the geometrical centre of the control volume usually leads to non-physical oscillations and associated difficulties in obtaining a converged solution(Rodi et al., 1989). The solution is therefore to use a staggered variable arrangement where the pressure is located at the control volume centres and the velocities at the faces. This arrangement removes the need for interpolation of pressure in the momentum equation and of velocity in the continuity equation (Rodi et al., 1989).

The grid to be used is shown in the figure 3 below;

As is seen in figure 3, the volume fractions α's, the densities ρ's and pressure p are defined at the nodes of the control volumes and phase velocities U's at the faces.

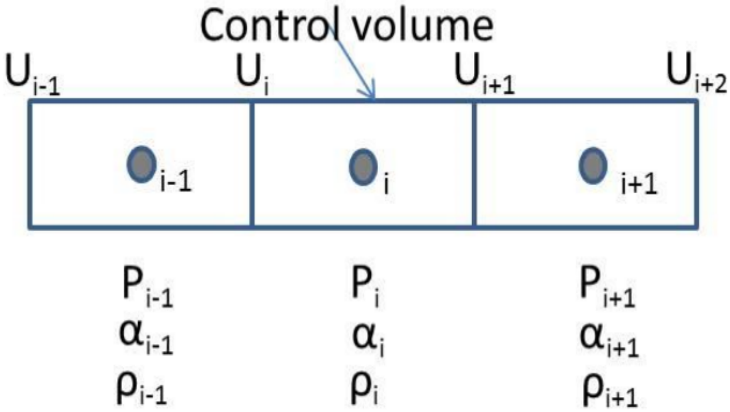

Figure 3: Staggered grid

4.2 Discretization

The finite volume method of discretization has been used. In this method, the flow domain is discretized by volumes of finite size and difference equations representing the balances of fluxes across the finite control-volume faces are obtained (Rodi et al., 1989).

These equations are obtained by integrating equations 2, 3 and 4 over the control volume as follows bearing in mind that the mass transfer rate terms are 0; We assume that volumes and faces across which fluxes propagate of the control volumes are equal.

$$\int_{CV} \partial_t(\alpha_l \rho_l) dV + \int_{CV} \partial_x(\alpha_l \rho_l U_l) dV = 0$$

This gives for the liquid phase;

$$\frac{d(\alpha_{li}\rho_{li})}{dt} = (\hat{\alpha}_{li}\hat{\rho}_{li}U_{li} - \hat{\alpha}_{li+1}\hat{\rho}_{li+1}U_{li+1})/\Delta x \tag{11}$$

where Δx is the length of each control volume. The variables with the hat represent variables at the faces of the control volume which are originally assigned to be node variables in the staggered grid.

Similarly for the gas phase;

$$\int_{CV} \partial_t(\alpha_g \rho_g) dV + \int_{CV} \partial_x(\alpha_g \rho_g U_g) dV = 0$$

which gives

$$\frac{d(\alpha_{gi}\rho_{gi})}{dt} = (\hat{\alpha}_{gi}\hat{\rho}_{gi}U_{gi} - \hat{\alpha}_{gi+1}\hat{\rho}_{gi+1}U_{gi+1})/\Delta x \tag{12}$$

In this case, horizontal two-phase flow is considered and therefore the gravitational term in the mixture momentum equation is ignored. Integration of equation 4 over a control volume now gives;

$$\int_{CV} \partial_t(\alpha_l \rho_l U_l + \alpha_g \rho_g U_g) dV + \int_{CV} \partial_x(\alpha_l \rho_l U_l^2 + \alpha_g \rho_g U_g^2 + p) dV = \int_{CV} -\frac{32 U_s \mu_{mix}}{d^2} dV$$

which gives

$$\begin{aligned} \frac{d(\alpha_{li}\rho_{li}U_{li} + \alpha_{gi}\rho_{gi}U_{gi})}{dt} &= (\hat{\alpha}_{li}\hat{\rho}_{li}U_{li}^2 - \hat{\alpha}_{li+1}\hat{\rho}_{li+1}U_{li+1}^2)/\Delta x \\ +(\hat{\alpha}_{gi}\hat{\rho}_{gi}U_{gi}^2 - \hat{\alpha}_{gi+1}\hat{\rho}_{gi+1}U_{gi+1}^2)/\Delta x &+ (p_i - p_{i+1})/\Delta x - \frac{32 U_{si}\mu_{mix,i}}{d^2} \end{aligned} \tag{13}$$

We now have the desired equations to solve using the ode15s solver in matlab. These are equations 11, 12, 13 as well as the algebraic equations 7, 8, 9 and 10.

Finally, the variables with a hat as seen in the differential equations are variables at control volume faces and are approximated from the variables in the neighbouring nodes using the 1^{st}-order upwind scheme as shown below;

$$\hat{x} = a_{i+1}x_i + (1 - a_{i+1})x_{i+1} \tag{14}$$

where x is either α or ρ and a_{i+1} is given by;

$$a_{i+1} = \begin{cases} 1, & \text{if} \quad U_{i+1} \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

4.3 Simulation case

For demonstrational purposes, the following problem was created from a similar example in (Evje and Fjelde, 2002).

We consider a horizontal pipeline with diameter 2.2 cm and length 10 m. A stream of gas and liquid is injected at the inlet of the pipeline with the superficial velocities of gas and liquid being 0.57 m/s and 1.6 m/s respectively. At the outlet, the pressure is kept constant at 1 bar. For simplicity, we assume a slip relation given by equation 8 with $C_o = 1.2$ and $V_{gu} = 0.54\sqrt{gd}$. We are interested in modelling the transient behaviour.

The simulation of this two-phase flow problem was done using the differential algebraic system of equations developed in previous sections and with the *ode*15*s* solver in Matlab. The values of the parameters used are shown in table 1 below.

Table 1: Parameters

Parameters	value	unit
a_g	316	m/s
a_l	1000	m/s
L	10	m
d	0.022	m
C_o	1.2	$-$
V_{gu}	0.25	m/s
μ_l	$5 * 10^{-2}$	Pa s
μ_g	$5 * 10^{-6}$	Pa s

The initial condition was set as shown in table 2.

Table 2: Initial condition

Variables	value	unit
α_g	0.2	$-$
α_l	0.8	$-$
ρ_g	2	kg/m^3
ρ_l	1000	kg/m^3
U_l	2	m/s
U_g	2.86	m/s
p	2	bar

The states chosen for the simulation are as shown in the state vector below;

$$\vec{x} = \begin{pmatrix} \alpha_g\rho_g \\ \alpha_l\rho_l \\ \alpha_g\rho_g U_g + \alpha_l\rho_l U_l \\ U_l \\ U_g \\ \alpha_g \end{pmatrix}$$

The state vector used for the Matlab solver is a stack of respective vectors for each of the control volumes.

The last step was implementation and was done very much with the help of John Morud and the Matlab script used is found in the Appendix.

5 RESULTS AND DISCUSSION

The results below show the time evolution of the variables at 3 different positions from inlet to outlet.

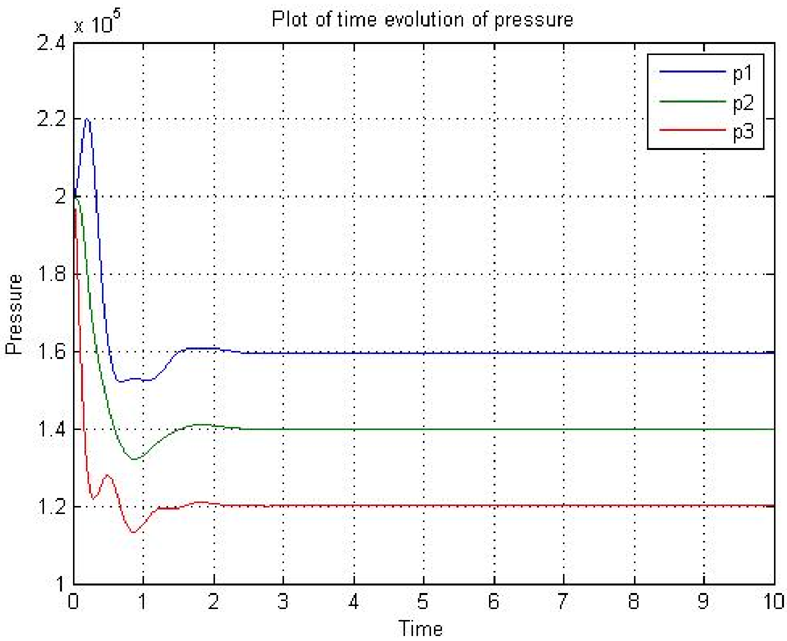

Figure 4: time evolution of presure

As seen in figure 4, a sharp peak is observed when liquid and gas are injected at the inlet. The cause of this behaviour are the acceleration terms which are responsible for the pressure pulses. The peak is also sharper at the inlet of the pipe since a large part of the stagnant fluid present must be set into motion. The peak vanishes after a short while and the pressure development afterwards is mainly due to friction forces.

The decreasing pressure experienced by the liquid and gas as they propagate through the pipe causes the gas to expand. Therefore the gas volume fractions also increase while the liquid volume fractions decrease as is seen from figure 5. This expansion results in increased gas mass flow rates thus increased gas phase velocities and this scenario becomes more extreme when

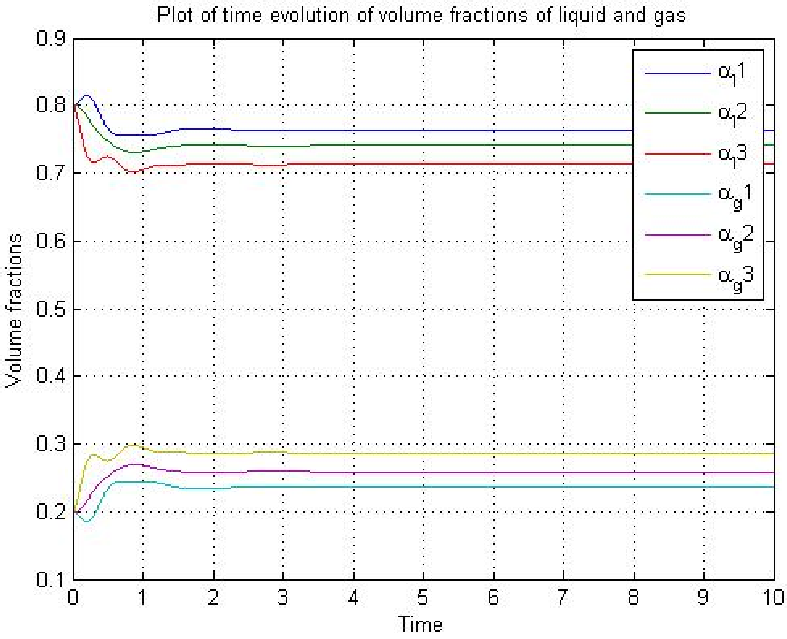

Figure 5: time evolution of volume fractions of liquid and gas

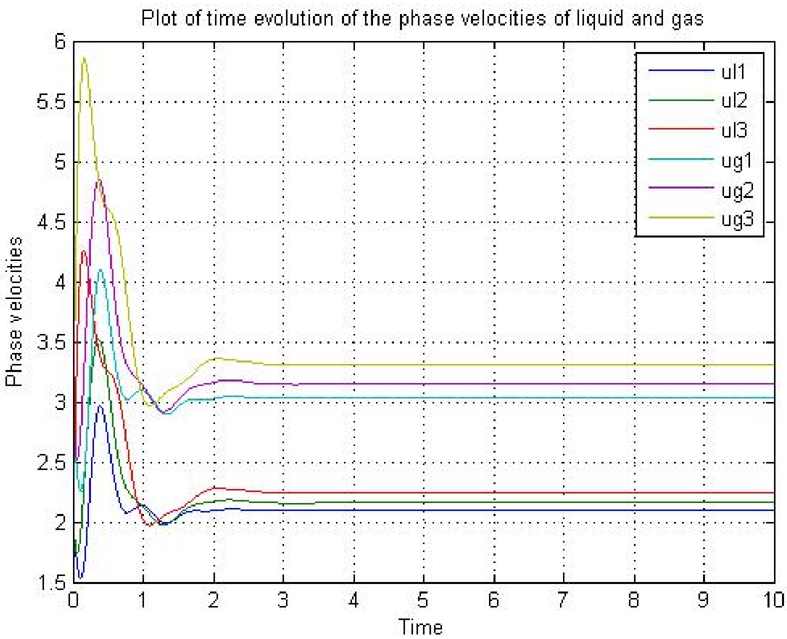

Figure 6: time evolution of phase velocities of liquid and gas

the gas reaches the outlet where a sharper peak is observed but drops quickly towards the steady state solution (see figure 6).

The increased gas mass flow rates result in the liquid in front of the gas to be moved with larger velocities at a given position until the gas passes and the velocity drops quickly afterwards(see figure 6).

After sometime, the flow is stabilized and steady state conditions are obtained. Results that give an insight into a similar discussion as one above can be obtained from (Evje and Fjelde, 2002).

6 CONCLUSION

In this report, we have been able to demonstrate that a drift-flux model can be used to simulate two-phase flow. The two-phase flow phenomenon was approximated by two mass conservation equations and one mixture momentum equation. But because of the relative motion between the phases, a slip law was used, together with other algebraic closure relations so as to have a solution.

This differential algebraic equation system was simulated in matlab using ode15s solver. The results obtained are satisfactory as they exhibit characteristic behaviour of two-phase flow.

Bibliography

Evje, S. and Fjelde, K. (2002). Relaxation schemes for the calculation of two-phase flow in pipes. *Mathematical and computer modelling*, 36(4):535–567.

Fjelde, K. K. and Karlsen, K. H. (2002). High-resolution hybrid primitive–conservative upwind schemes for the drift flux model. *Computers & Fluids*, 31(3):335–367.

Goda, H., Hibiki, T., Kim, S., Ishii, M., and Uhle, J. (2003). Drift-flux model for downward two-phase flow. *International journal of heat and mass transfer*, 46(25):4835–4844.

Ishii, M. and Hibiki, T. (2011). Drift-flux model. In *Thermo-Fluid Dynamics of Two-Phase Flow*, pages 361–395. Springer.

Ishii, M. and Mishima, K. (1984). Two-fluid model and hydrodynamic constitutive relations. *Nuclear Engineering and design*, 82(2):107–126.

McQuillan, K. and Whalley, P. (1985). Flow patterns in vertical two-phase flow. *International Journal of Multiphase Flow*, 11(2):161–175.

Molvik, S. (2011). A study of two-phase drift-flux modeling in wells and corresponding slip relations.

Rodi, W., Majumdar, S., and Schönung, B. (1989). Finite volume methods for two-dimensional incompressible flows with complex boundaries. *Computer methods in applied mechanics and engineering*, 75(1):369–392.

Shi, H., Holmes, J. A., Durlofsky, L. J., Aziz, K., Diaz, L., Alkaya, B., Oddie, G., et al. (2005). Drift-flux modeling of two-phase flow in wellbores. *Spe Journal*, 10(01):24–33.

Taitel, Y., Bornea, D., and Dukler, A. (1980). Modelling flow pattern transitions for steady upward gas-liquid flow in vertical tubes. *AIChE Journal*, 26(3):345–354.

Wambsganss, M., Jendrzejczyk, J., and France, D. (1991). Two-phase flow patterns and transitions in a small, horizontal, rectangular channel. *International journal of multiphase flow*, 17(3):327–342.

Zuber, N. and Findlay, J. (1965). Average volumetric concentration in two-phase flow systems. *Journal of Heat Transfer*, 87(4):453–468.

Appendix

```
function driftflux

%Parameters
cg= 316;
cl=1000;
rhol0=1000;
p0=2e5;
L=10;
d=0.022;
C0=1.2;
Vgu=0.25;
Ncell=3;
Nstates=6;
Dx=L/Ncell;
mul=0.05;
mug=0.000005;
Tend=50;

%Initial condition
agval=0.2;
ag=agval;
al=1-ag;
p=p0;
rhog=p/cg^2;
rhol=rhol0;
agin=agval; agout=agval; pout=1e5;
rhogout=345;rholout=1000; rhogin=rhog;
alout=0.8;alin=1-agin; rholin=rhol;

Ul=2;
Ug=(C0*al*Ul+Vgu)/(1-C0*ag);
Ulin=Ul; Ugin=Ug;
```

```
%Set up state
z0=[
    ag*rhog
    al*rhol
    ag*rhog*Ug+al*rhol*Ul
    Ul
    Ug
    ag
    ];

X0=z0*ones(1,Ncell);
x0=X0(:);

II=eye(Ncell);
m=diag([1 1 1 0 0 0]);
M=kron(II,m);
M=sparse(M);

Tri=diag(ones(Ncell,1)) + diag(ones(Ncell-1,1),1)
++ diag(ones(Ncell-1,1),-1);
Jpatt=kron(Tri,ones(Nstates));
Jpatt=sparse(Jpatt);
opt=odeset('mass',M,'jpattern',Jpatt,'reltol',1e-12);

[T,xx]=ode15s(@f,[0 Tend],x0,opt);

%Post processing:
for i=1:length(T)[~,UUL(i,:),UUG(i,:),AAL(i,:),AAG(i,:),PP(i,:)]
=f(T(i),xx(i,:)');
end

%Plotting

figure(1)
plot(T,UUL,T,UUG)
```

```
title('Plot of time evolution of phase velocities
 of liquid and gas')
xlabel('Time')
ylabel('Phase velocities')
legend ul1 ul2 ul3 ug1 ug2 ug3
grid

figure(gcf+1)
plot(T,AAL,T,AAG)
title('Plot of time evolution of volume fractions
 of liquid and gas')
xlabel('Time')
ylabel('Volume fractions')
legend \alpha_l1 \alpha_l2 \alpha_l3 \alpha_g1 \alpha_g2 \alpha_g3
grid

figure(gcf+1)
plot(T,PP)
title('Plot of time evolution of pressure')
xlabel('Time')
ylabel('Pressure')
legend p1 p2 p3
grid

%=================================================================
    function [xdot,UUl,UUg,aal,aag,pp]=f(t,x)
        X=zeros(Nstates,Ncell);
        X(:)=x(:);
        aag=X(6,:);
        aal=1-aag;
        rrhog=X(1,:)./aag;
        pp=cg^2*rrhog;
        rrhol=rhol0+(pp-p0)/cl^2;
        UUl=X(4,:);
        UUg=X(5,:);
```

```
      a=(UUg>0);
      ag_aug = [aag agout];
      I=1:Ncell;
      aghat=a.*ag_aug(I) + (1-a).*ag_aug(I+1);
      rhog_aug= [rrhog rhogout];
      rhoghat=a.*rhog_aug(I) + (1-a).*rhog_aug(I+1);

      a=(UUl>0);
      al_aug = [aal alout];
      I=1:Ncell;
      alhat=a.*al_aug(I) + (1-a).*al_aug(I+1);
      rhol_aug= [rrhol rholout];
      rholhat=a.*rhol_aug(I) + (1-a).*rhol_aug(I+1);

      ppaug = [pp pout];

      %Gas
      flux=aghat.*rhoghat.*UUg; flux=[agin*rhogin*Ugin flux];
      rhs1=-diff(flux)/Dx; % right hand side 1

      %Liquid
      flux=alhat.*rholhat.*UUl; flux=[alin*rholin*Ulin flux];
      rhs2=-diff(flux)/Dx;

      %Momentum
     flux_g=aghat.*rhoghat.*UUg.*UUg;
     flux_g=[agin*rhogin*Ugin*Ugin flux_g];

     flux_l=alhat.*rholhat.*UUl.*UUl;
     flux_l=[alin*rholin*Ulin*Ulin flux_l];

     rhs3=(-diff(flux_g)/Dx) + (-diff(flux_l)/Dx)
 + (-diff(ppaug)/Dx)
 -(32*(aal.*UUl+aag.*UUg).*(mul*aal+mug*aag)/d^2);

      Xdot=[
```

```
            rhs1
            rhs2
            rhs3
            UUg.*(1-C0*aag)-(C0*aal.*UUl+Vgu); %res1
            X(2,:)-aal.*rrhol;    %res2
            X(3,:)-(aghat.*rhoghat.*UUg+alhat.*rholhat.*UUl) %res3
            ];

        xdot=Xdot(:);

    end %f

end %driftflux
```