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Examination, course FY2450 Astrophysics
Friday May 21, 2010
Time: 9.00–13.00

Grades made public: Friday June 11, 2010

Allowed to use: Calculator, mathematical and physical tables.

A table of physical constants is found at the end of this problem set.
All subproblems are given the same weight in the grading.

Problem 1:

a) Mention two good reasons to build *big* telescopes.

And two good reasons to place telescopes outside the atmosphere of the Earth.

b) What is a sunspot?

At the transition between two 11 year sunspot cycles (as just now), how can one decide whether a sunspot belongs to the end of a cycle or the start of the next cycle?

c) Explain briefly what is meant by Doppler redshift, gravitational redshift and cosmological redshift.

d) The binary star system SS 433 is number 433 in a catalogue (compiled by Sanduleak and Stephenson) of stars with strong emission lines in their spectra. In addition to emission lines with essentially zero redshift, the spectrum of SS 433 contains at any time a set of lines with up to 15 % redshift, and another set of lines with almost as large blueshift. The shifts of these lines vary periodically with a period of 164 days. The average of the redshift and the blueshift is a nearly constant redshift corresponding to a velocity of 12 000 km/s.

The accepted explanation is that those spectral lines that are strongly redshifted and blueshifted come from two jets of matter which are ejected into opposite directions with velocities of 26 % of the speed of light. The direction of the jets varies periodically.

Explain how the large velocity gives rise to an extra redshift corresponding to a velocity of 12 000 km/s, in addition to the ordinary Doppler redshift.

e) Explain briefly what is meant by dark matter.

Which observations indicate that there exists dark matter?

What is the difference between dark matter and dark energy (vacuum energy)?

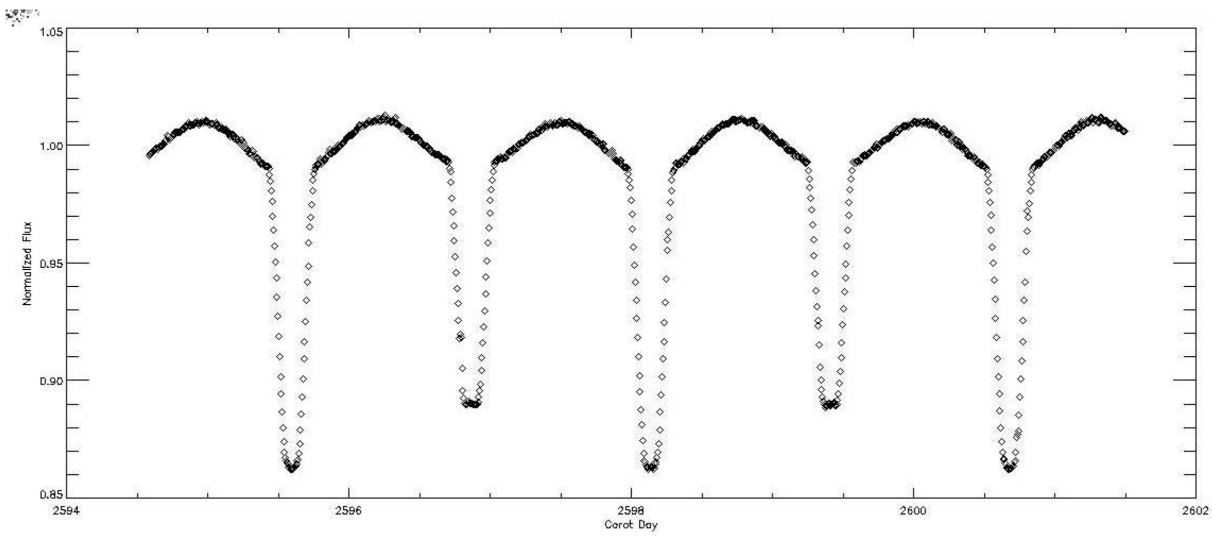

Figure 1: Light curve of an eclipsing binary star. The horizontal axis shows time in days, from 2594 to 2602. The vertical axis shows the brightness, in arbitrary units, from 0.85 to 1.05. Data from the satellite CoRoT.

Problem 2:

a) Figure 1 shows the light curve of an eclipsing binary star of magnitude 13. The curve was observed by the French satellite CoRoT, launched 27/12 2006, which is used e.g. for detecting planets orbiting a star and eclipsing the star.

What data about this binary star system can you read out of the light curve?

Can you, for example, say something about the eccentricity of the orbit? Explain your reasoning.

Try to explain as many details as possible of the light curve, such as the shape of the curve at maximum and at minimum.

b) The instruments aboard the CoRoT satellite are able to measure variations in the brightness of a star down to 5×10^{-5} of the brightness.

Would it be possible for an astronomer a few hundred light years away from us to detect the Earth with such an instrument?

In other words: if the Earth eclipses the Sun, how long is the duration of such an eclipse, seen from afar, and by how much is the light of the Sun reduced?

A rule of thumb, valid for both the Sun and the Moon, is that they are seen in the sky with an angular diameter of half a degree.

c) Figure 2 (next page) shows the light curve of the very special eclipsing binary star ϵ Aurigae, the fifth brightest star in the constellation Auriga, the Charioteer. It is eclipsed for nearly two years roughly every 27th year. The eclipse starting in August 2009 is expected to last until well into 2011. The eclipse is not total, but the brightness

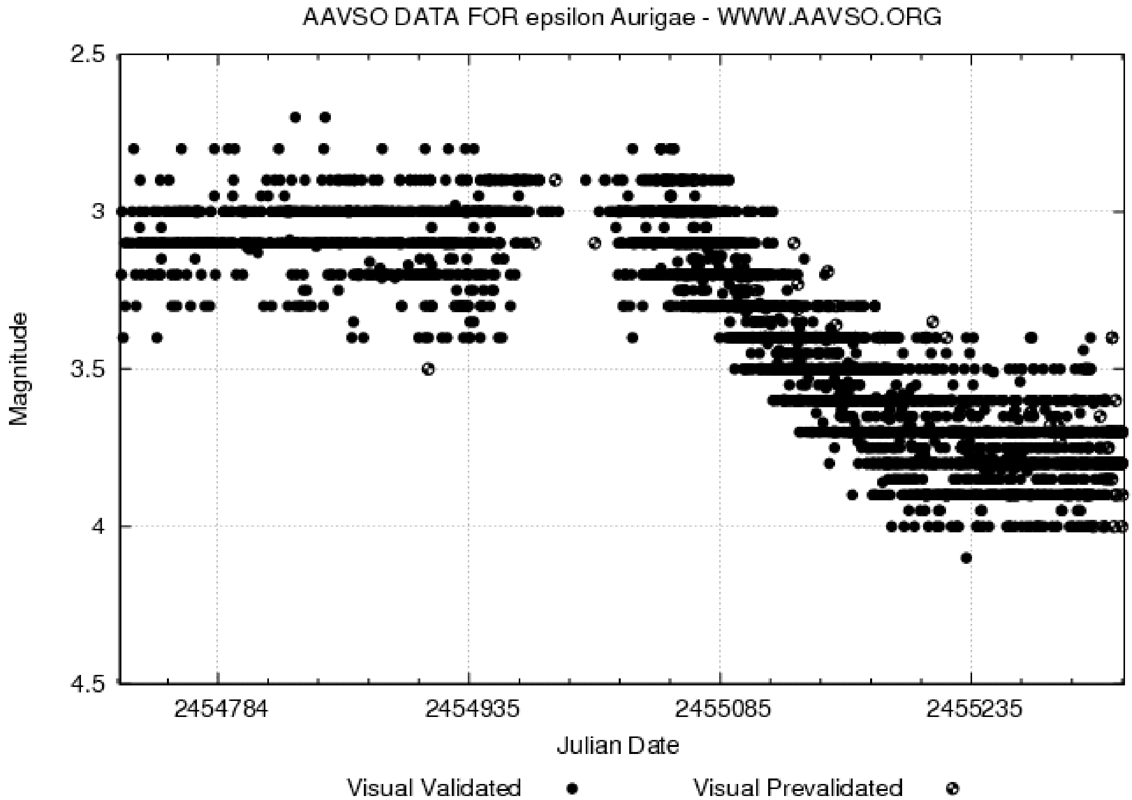

Figure 2: The light curve of the star ϵ Aurigae for 600 days. Data from AAVSO (American Association of Variable Star Observers). Julian Date 2455337 ends on May 21, 2010 at 2 PM, Norwegian daylight saving time.

is reduced by somewhat less than one magnitude, and the star brightens slightly in the middle of the two year period.

The distance is around 2000 light years. The visible star is a giant star of spectral class F0, having 2–3 solar masses, and a radius more than one hundred times the solar radius.

The secod component of the star system orbits with a period of 27 years, and is so large as to eclipse the main star for two years, but is too dim to be seen in visible light.

A model for the system (which appears to be confirmed by new observations) assumes that the dark component is a disk of dust orbiting anther massive star (maybe even two massive stars). The cloud of dust hides completely the star (or the two stars) that it orbits, at least in visible light.

If we, for example, assume that the main star has three solar masses, and that the second component of the system has five solar masses, what is then the distance between them?

What is the size of the component of the system (the dust cloud) which causes the eclipse of ϵ Aurigae?

Why could not the object eclipsing the star be just a large cloud of gas and dust?

Problem 3:

The mass of a neutral atom of ^{12}C is $m(6, 12) = 12$ u, this defines the atomic mass unit u. The mass of a neutral atom with atomic number Z and mass number A is, more generally,

$$m(Z, A) \approx A\,\mathrm{u}\ .$$

a) The pressure giving hydrostatic equilibrium inside a white dwarf star is due to the degenerate electron gas.

Assume that the atomic nuclei in the star have atomic number Z and mass number A.

What is then the relation between the mass density ρ in the star and the number density of electrons, n_e?

The equation of state for the degenerate electron gas gives the pressure P as a function of the mass density ρ,

$$P = K\rho^{\frac{5}{3}}\ ,$$

where the exponent 5/3 is the adiabatic index, and K is the following constant,

$$K = \frac{h^2}{20m_{\mathrm{e}}}\left(\frac{3}{\pi}\right)^{\frac{2}{3}}\left(\frac{Z}{A\,\mathrm{u}}\right)^{\frac{5}{3}}\ .$$

Here h is Planck's constant, and m_{e} is the electron mass.
The Lane–Emden equation describes the inner structure of a white dwarf, when the degenerate electron gas is in hydrostatic equilibrium. The equation may be integrated numerically, and if the star does not rotate, the radius R and mass M may be expressed in terms of the central density ρ_c as

$$R = ar_0\ , \qquad M = 4\pi b\, r_0^{\,3}\rho_c\ , \qquad r_0 = \rho_c^{\,-\frac{1}{6}}\sqrt{\frac{5K}{8\pi G}}\ ,$$

where $a = 3.653754$, $b = 2.714055$, and G is Newton's gravitational constant.
We may eliminate the central density ρ_c, then we get the following relation,

$$MR^3 = 4\pi a^3 b\, r_0^{\,6}\rho_c = \left(\frac{Z}{A}\right)^5 4{,}295\ 10^{52}\ \mathrm{kg\ m}^3\ . \qquad (1)$$

	Mass	Radius	
	$M_\odot$	km	$R_\odot$
Sirius B	1.00	5850	0.0084
Procyon B	0.602	8600	0.0123
40 Eridani B	0.50	9700	0.014

Table 1: Observed mass and radius of three white dwarf stars.

b) Table 1 gives the mass and radius of three of the closest white dwarf stars.

Use the formula in equation (1) to compute the ratio Z/A for the three stars.

Which elements might they consist of? Comment?

Some physical constants and formulae

Newton's gravitational constant:	$G = 6.673 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
The speed of light in vacuum:	$c = 299\,792\,458\,\mathrm{m/s}$
The permeability of vacuum:	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$
The permittivity of vacuum:	$\epsilon_0 = 1/(\mu_0 c^2) = 8.854 \times 10^{-12}\,\mathrm{F/m}$
The reduced Planck's constant:	$\hbar = h/(2\pi) = 1.055 \times 10^{-34}\,\mathrm{J\,s}$
The elementary charge:	$e = 1.602 \times 10^{-19}\,\mathrm{C}$
The fine structure constant:	$\alpha = e^2/(4\pi\epsilon_0\hbar c) = 1/137.036$
Boltzmann's constant:	$k = k_B = 1.3807 \times 10^{-23}\,\mathrm{J/K}$
The Stefan–Boltzmann constant:	$\sigma = 5.6704 \times 10^{-8}\,\mathrm{W/(m^2\,K^4)}$
The electron mass:	$m_\mathrm{e} = 9.109 \times 10^{-31}\,\mathrm{kg} = 0.511\,\mathrm{MeV}/c^2$
The proton mass:	$m_\mathrm{p} = 1.6726 \times 10^{-27}\,\mathrm{kg} = 938.28\,\mathrm{MeV}/c^2$
The neutron mass:	$m_\mathrm{n} = 1.6749 \times 10^{-27}\,\mathrm{kg} = 939.57\,\mathrm{MeV}/c^2$
The atomic mass unit:	$\mathrm{u} = 1.660\,54 \times 10^{-27}\,\mathrm{kg} = 931.46\,\mathrm{MeV}/c^2$
The mass of the Earth:	$M_\oplus = 5.974 \times 10^{24}\,\mathrm{kg}$
The radius of the Earth:	$R_\oplus = 6.378 \times 10^{3}\,\mathrm{km}$
The solar mass:	$M_\odot = 1.9891 \times 10^{30}\,\mathrm{kg}$
The solar radius:	$R_\odot = 6.960 \times 10^{5}\,\mathrm{km}$
The distance to the Sun (one astronomical unit):	$1\,\mathrm{AU} = 1.4960 \times 10^{8}\,\mathrm{km}$
The Hubble constant:	$H_0 = 70\,(\mathrm{km/s})/\mathrm{Mpc}$
	$1\,\mathrm{pc} = 1\,\mathrm{parsec} = 3.26\,\mathrm{lightyears}$
	$1\,\mathrm{lightyear} = 9.46 \times 10^{15}\,\mathrm{m}$

Kepler's third law, masses m_1 and m_2 in an elliptic orbit of semi-major axis a and period P:

$$P^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)} \; .$$

Stefan–Boltzmann's law (flux F of blackbody radiation of temperature T): $F = \sigma T^4$.

Relation between apparent magnitude m and absolute magnitude M of a star at distance d:

$$m - M = 5\;\log_{10}\left(\frac{d}{10\ \mathrm{parsec}}\right) .$$

For two stars 1 and 2 the following relations hold:

$$\begin{aligned} m_2 - m_1 &= 2.5\;\log_{10}\left(\frac{b_1}{b_2}\right) , \\ M_2 - M_1 &= 2.5\;\log_{10}\left(\frac{L_1}{L_2}\right) . \end{aligned}$$

Where b is (apparent) brightness and L is (absolute) luminosity.