THE NORWEGIAN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF PHYSICS

Contact person:
Name: Jan Myrheim
Telephone: 93653, mobile 90 07 51 72

Examination, course FY2450 Astrophysics
Wednesday May 20, 2009
Time: 9.00–13.00

Grades made public: Wednesday June 10, 2009

Allowed to use: Calculator, mathematical and physical tables.

A table of physical constants is at the end of this examination set.
All subproblems are weighted equally in the grading.

Problem 1:

a) Mention some characteristic differences between globular clusters, on the one hand, and open clusters (galactic clusters), on the other hand.

b) Which observable differences exist between the stars in globular clusters (population II) and stars in open clusters (population I)?

How are the observed differences explained theoretically?

c) Harlow Shapley studied how the globular clusters are distributed in three dimensions, and used his results to localize the centre of our galaxy, the Milky Way.

Explain briefly how.

Problem 2:

a) The Hubble *constant* H_0 is the present value of the Hubble *parameter* H, which is time dependent. The value $H_0 = 70\,(\mathrm{km/s})/\mathrm{Mpc}$ is believed to be correct within $\pm 5\,\%$.

What is Hubble's law, and how do we measure H_0?

The Hubble time $1/H_0$ is roughly equal to the age of the universe. Why?

How many years is the Hubble time $1/H_0$?

That the Hubble time is almost exactly equal to the age of the universe, according to the newest results from cosmological studies, is regarded as a pure coincidence.

b) The Friedmann model of the universe presupposes that the mass density ρ and the energy density ρc^2 are constant in space but vary in time.

Observation, e.g. of the fluctuations in the cosmic background radiation, indicates that the universe can be described by a Friedmann model in which the three dimensional space is flat. The condition for the three dimensional space to be flat is that the mass density ρ has a value which is exactly equal to the critical value

$$\rho_c = \frac{3H^2}{8\pi G} \; .$$

The present value of the critical mass density is

$$\rho_{c0} = \frac{3{H_0}^2}{8\pi G} = 9.2 \; 10^{-27} \; \text{kg/m}^3 \; .$$

What is the corresponding critical energy density $\rho_{c0}c^2$?

What is ρ_{c0} expressed in solar masses per $(\text{lightyear})^3$?

If a typical galaxy has 10^{11} solar masses, what would be the average distance between the galaxies if the galaxies alone provided all of the critical mass density ρ_{c0} ?

Comment?

c) An electromagnetic field in vacuum with an electric field $\vec{E}$ and magnetic flux density $\vec{B}$ has an energy density which is

$$\rho_{\text{em}}c^2 = \frac{1}{2}\left(\epsilon_0|\vec{E}|^2 + \frac{|\vec{B}|^2}{\mu_0}\right) .$$

If the energy density of a purely electric field is equal to the critical energy density of the universe, $\rho_{c0}c^2$, how large is then $|\vec{E}|$?

If the energy density of a purely magnetic field is $\rho_{c0}c^2$, how large is then $|\vec{B}|$?

Comment?

d) Thermal electromagnetic radiation at a temperature T has an energy density which may be written as an integral over the frequency ν,

$$\rho_{\text{em}}c^2 = \int_0^\infty \mathrm{d}\nu \; \frac{8\pi h\nu^3}{c^3} \, f \; ,$$

where

$$f = \frac{1}{\mathrm{e}^{\frac{h\nu}{kT}} - 1}$$

is the occupation number in a photon state of frequency ν. At zero temperature, in the limit $T \to 0$, we get $f = 0$. However, when we take $f = 0$ at $T = 0$, we have not included the vacuum energy of the quantized electromagnetic field.

A quantum mechanical harmonic oscillator of frequency ν has energy levels

$$E_n = h\nu \left(n + \frac{1}{2}\right) \qquad \text{with} \qquad n = 0, 1, 2, \ldots\ .$$

In the ground state, with $n = 0$, the oscillator has a positive energy $E_0 = h\nu/2$.

Hence, we find the density of vacuum energy, $\rho_{\rm v}c^2$, by taking $f = 1/2$ in the above integral. The problem is that the vacuum energy density becomes infinite,

$$\rho_{\rm v}c^2 = \int_0^\infty {\rm d}\nu\, \frac{4\pi h\nu^3}{c^3} = \infty\ .$$

From the physical point of view it gives little meaning to integrate all the way to infinitely high frequency.
In any case, it has no meaning to include wave lengths smaller than the Planck length

$$L_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6\ 10^{-35}\ \text{m}\ ,$$

since in that region we expect a (hitherto unknown) quantum theory of gravitation to play an essntial role.

We get a finite answer if we integrate only up to a finite maximal frequency ν_2. For example, if we cut off the frequency integral at the frequency

$$\nu_2 = \frac{c}{L_P}\ ,$$

what is then the estimate we get for the density of vacuum energy of the electromagnetic field?

Comment?

Problem 3:

The second most brilliant star in the constellation of Libra (The Scales) is called α Librae, even though it ought to be called β according to the naming convention. The arabic name Zubenelgenubi (the southern claw, that is, the southern claw of the Scorpion) makes it a public favourite, but it is interesting also for other reasons than its name.
Closer studies indicate that it is a star system of altogether 5 stars.
With a telescope it may be resolved into the following three components:

Name	Magnitude	Spectral class
A or α_2	2.8	A3
B or α_1	5.2	F4
C	13.2	

Since the three stars A, B, and C are at roughly the samme distance, 77 lightyears = 23.6 parsec, and have roughly the same velocity vector, we conclude that they orbit each other.

a) What is the absolute magnitude M of each of the three stars A, B, and C?

Can you guess what spectral class the faintest star, C, belongs to?
Justify your answer.

b) The angular distance is 231″ between A and B, 276″ between A and C, and 111″ between B and C.

Calculate an approximate period for two av these stars, assuming that they orbit each other.

Note that you are asked to give an order of magnitude estimate, not an exact value.

c) If the three stars are photographed through a telescope, how large should the telescope be in order that the photograph may show all three of them separately?

How large a telescope do you need, and how much magnification should you use if you want to see all the three stars separately with your eye through the telescope?

A rule of thumb is that a telescope with an objective diameter of 10 cm has an angular resolution of 1″, which is the limit set because turbulence in the atmosphere makes the image unsharp.

Assume that the light opening of your eye is 5 mm. Stars of magnitude $m = 6$ are barely visible with the naked eye.

d) The B star, α_1, is a spectroscopic binary star, in which the spectral lines of only one star are visible. By means of a telescope with adaptive optics correcting for turbulence in the atmospere. the Canada–France–Hawaii Telescope (CFHT), the two components of the binary star have been observed at an angular distance of 0.38″. See Figure 1.

The period is 5870 days. Use these data to calculate (roughly) the sum of the masses of the two stars.

Is the mass you calculate roughly consistent with the spectral class F4 of the most luminous star of the two, given that it is a main sequence star? Justify your answer briefly.

For comparison: an F4 main sequence star has a surface temperature roughly 1000 K higher than the Sun, which belongs to the spectral class G2.

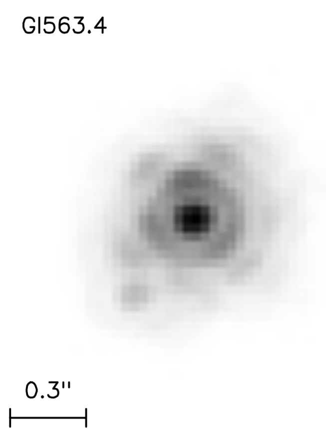

Figure 1: A negative image of α_1 Librae (= Gl563.4 = Zubenelgenubi B) photographed with the CFHT in February 1999. The secondary star is down left.

e) The story does not end there, because the A star, α_2, also seems to be a binary star, consisting of two almost identical stars of the same spectral class, A3. The angular distance between them is only $0.01''$.

How large is the distance between them, measured in astronomical units, AU?

If we assume that the two stars have the same luminosity, what is the magnitude of each of them separately, when the magnitude of the two together is 2.8 ?

f) An angular distance of $0.01''$ is so small that only a very special technique can be used for showing that α_2 consists of two stars. Zubenelgenubi is sometimes occulted (hidden) by the Moon, so that first one and then the other of the two stars disappear behind the Moon.

How large is the time interval between the occultation of the two stars of angular distance $0.01''$?

g) When we observe two stars orbiting each other, it may be that we observe only one point on the orbit, often because the period for a complete orbit is very long. Then it is usual to assume, like we have done above in this problem, that if we measure an angular distance γ, and the distance to the two stars is d, then the orbit is elliptic with a semi-major axis $a = \gamma d$.

The formula $a = \gamma d$ is actually a good approximation, it has a relatively large probability of giving a value no more than, say, 10 % from the correct value.

Explain briefly why. Only a qualitative argument is asked for.

Some physical constants and formulae

Newton's gravitational constant:	$G = 6.673 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
The speed of light in vacuum:	$c = 299\,792\,458\,\mathrm{m/s}$
The permeability of vacuum:	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$
The permittivity of vacuum:	$\epsilon_0 = 1/(\mu_0 c^2) = 8.854 \times 10^{-12}\,\mathrm{F/m}$
The reduced Planck's constant:	$\hbar = h/(2\pi) = 1.055 \times 10^{-34}\,\mathrm{J\,s}$
The elementary charge:	$e = 1.602 \times 10^{-19}\,\mathrm{C}$
The fine structure constant:	$\alpha = e^2/(4\pi\epsilon_0\hbar c) = 1/137.036$
Boltzmann's constant:	$k = k_B = 1.3807 \times 10^{-23}$ J/K
The Stefan–Boltzmann constant:	$\sigma = 5.6704 \times 10^{-8}$ W/(m^2 K^4)
The electron mass:	$m_\mathrm{e} = 9.109 \times 10^{-31}\,\mathrm{kg} = 0.511\,\mathrm{MeV}/c^2$
The proton mass:	$m_\mathrm{p} = 1.6726 \times 10^{-27}\,\mathrm{kg} = 938.28\,\mathrm{MeV}/c^2$
The neutron mass:	$m_\mathrm{n} = 1.6749 \times 10^{-27}\,\mathrm{kg} = 939.57\,\mathrm{MeV}/c^2$
The mass of the Earth:	$M_\oplus = 5.974 \times 10^{24}\,\mathrm{kg}$
The radius of the Earth:	$R_\oplus = 6.378 \times 10^{3}\,\mathrm{km}$
The solar mass:	$M_\odot = 1.9891 \times 10^{30}\,\mathrm{kg}$
The solar radius:	$R_\odot = 6.960 \times 10^{5}\,\mathrm{km}$
The distance to the Sun (one astronomical unit):	$1\,\mathrm{AU} = 1.4960 \times 10^{8}\,\mathrm{km}$
The Hubble constant:	$H_0 = 70\,(\mathrm{km/s})/\mathrm{Mpc}$
	1 pc = 1 parsec = 3.26 lightyears
	1 lightyear = 9.46×10^{15} m

Kepler's third law, masses m_1 and m_2 in an elliptic orbit of semi-major axis a and period P:

$$P^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)}\,.$$

Stefan–Boltzmann's law (flux F of blackbody radiation of temperature T): $F = \sigma T^4$.

Relation between apparent magnitude m and absolute magnitude M of a star at distance d:

$$m - M = 5\,\log_{10}\left(\frac{d}{10\ \text{parsec}}\right).$$

For two stars 1 and 2 the following relations hold:

$$\begin{aligned} m_2 - m_1 &= 2.5\,\log_{10}\left(\frac{b_1}{b_2}\right), \\ M_2 - M_1 &= 2.5\,\log_{10}\left(\frac{L_1}{L_2}\right). \end{aligned}$$

Where b is (apparent) brightness and L is (absolute) luminosity.