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Specialization Project Thesis

Superstructure Optimization of a Subsea Separation System

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Summary

A superstructure was proposed for a subsea separation system, containing the possibilities for multiphase boosting, separate phase pumping and compression, multiphase transport/rising and separate phase transport/rising. From the superstructure, an optimization model was formulated. Binary variables were assigned to all potential pieces of equipment. The proposed constraints included mass balances for the system, and logical constraints related to the binary variables. Two approaches were evaluated for the objective function, resulting in two different cases. For the first case, the investment cost was minimized, using fixed costs for the equipment. For the second case, the net present value was maximized and a set of the equipment was cost estimated in the objective function, making it nonlinear. The models were implemented in GAMS, giving the optimal set of binary variables, from which the optimal configuration was constructed. Minimizing the investment cost gave multiphase boosting, transportation/rising as the optimal solution. The same solution was obtained by maximizing the NPV for early production. Maximizing the NPV for late production gave separate compression and pumping with multiphase transport/rising as the optimal solution.

1 Introduction

When proposing a flowsheet for a chemical process, there are several possible configurations to evaluate. A fundamental problem of chemical engineering is therefore to find the best possible process structure for a given process. This involves a series of decisions regarding units, equipment and interconnections of the flows. These decisions can be classified into two main categories: Structural decisions and operational decisions. The former deals with decisions such as interconnections of the flows and equipment as well the selection of the equipment for the process. Examples of the latter category include reactor temperatures, pressures and flow compositions. The complexity of these combinatorial problems can become quite high, and for a PSP (process synthesis problem) it is not uncommon to have 10^{15} different alternatives [1]. The need for generic methodologies to identify optimal configurations becomes evident.

There are mainly two different approaches when it comes to identifying the best possible process structure. The first approach involves solving the problem in sequential form by the use of decomposition [2]. Certain elements of the flowsheet are held constant while changes are introduced to study the effects and the possibility of an improved solution. The advantage of this method is that it can be simple to implement. However, due to its sequential nature, it may lead to sub-optimal solutions. The second approach involves the formulation of a superstructure, where all potentially useful units and interconnections are included in a process diagram. From the superstructure, an optimization model can be formulated, where the constraints include mathematical models for all the units and interconnections, as well as logical constraints. The objective function reflects the desired quantity to be maximized or minimized e.g the net present value or the investment costs, respectively. Due to the fact that integer(binary) variables often are used to represent the decisions, while continuous variables are used for the operating variables (flow, temperature, pressure etc.), the optimization model takes the form of a mixed integer programming (MIP) problem. By solving the mathematical programming problem, the optimal configuration of the flowsheet is identified, with respect to the specified objective function.

The objective of this report is to construct a simple superstructure for a subsea separation system, formulate a model, and solve it to obtain the optimal process design. The problem is relevant as subsea processing is a growing field which already has been used all over the world. As many of the oil and gas fields are approaching the final stages of their lifetime, there has been a need for more efficient oil recovery methods [3]. As a result, subsea processing was introduced, allowing for transport over greater distances, improved flow assurance and production. Statoil has developed subsea technology resulting in more than 500 subsea wells during the last 25 years as a part of their vision to develop and deploy all the necessary technology elements required for a subsea factory [4].

The next section will present the theoretical concepts that will be applied later on in the report. More specifically, section 2 gives insight in the concept of superstructure optimization, mathematical programming, generation and modeling of superstructures as well as some advantages and challenges related to the concept. The base case of the project will be described in section 3, before the superstructure generated for the process is presented in section 4. Section 5 gives the optimization model formulated from the superstructure. The results obtained from solving the model is presented in section 6. Some key aspects of the results and the model is given in section 7, before the results are concluded in section 8.

2 Background & theory

Superstructure optimization provides an approach for obtaining the optimal configuration of a flowsheet while at the same time optimizing the operational conditions of the process. The general procedure can be broken down into three general steps as illustrated by figure 1. The first step of the procedure is to generate a superstructure for the process, which is a diagram consisting of all feasible units and interconnections between the flows of the system [5]. Based on the superstructure, an optimization model is formulated and solved to produce the optimal configuration of the process flowsheet. The solution is a subsystem of the initially formed superstructure, where certain units and flows are chosen based on the objective function and the constraints specified.

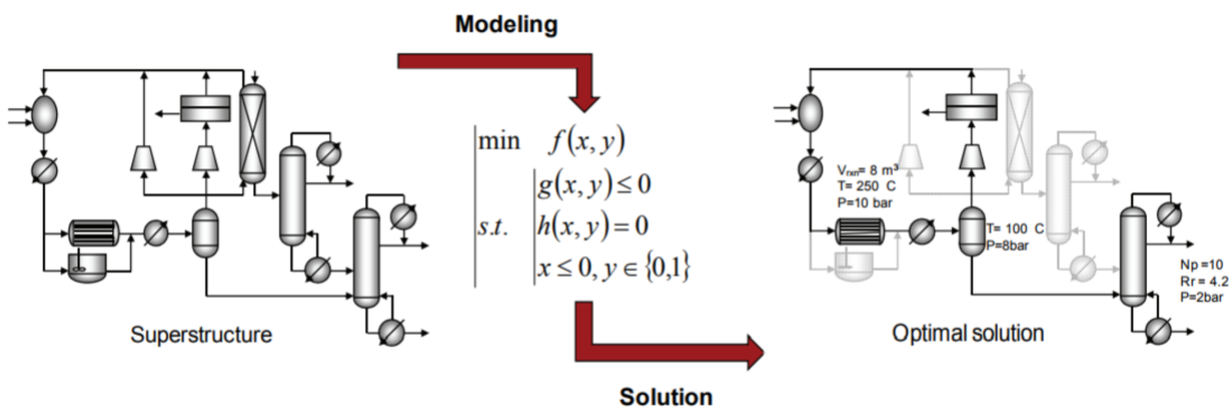

Figure 1: An optimization model is formulated from the superstructure, and solved to produce the optimal process flowsheet [1].

2.1 Mathematical programming and previous work

Contributions made by scientists and engineers combined with the development of mathematical programming and algorithms has made superstructure optimization a powerful tool for designing flowsheets. Important contributions include solution strategies for mixed integer linear programming (MILP) and mixed integer non linear programming (MINLP).

2.1.1 Linear programming (LP)

The modern era of optimization began with the development of the SIMPLEX algorithm by Dantzig in the late 1940s [6]. Despite the fact that the algorithm works exclusively for problems with both linear objectives and constraints, it still remains one of the most widely used of all optimization tools today. A linear problem can be formulated as follows:

$$\min Z = c^T x \tag{1a}$$

$$s.t\ Ax \leq b \tag{1b}$$

$$x \geq 0 \tag{1c}$$

where x represents the continuous variables restricted to non-negative values. Since the objective function is linear and equation 1b and 1c make up a convex feasible region, the programming problem in itself is convex and has a unique minimum. The SIMPLEX algorithm searches for this optimum value by exploiting the fact that it is located at one of the vertices of the feasible region. An illustration of a linear programming problem is given in figure 2.

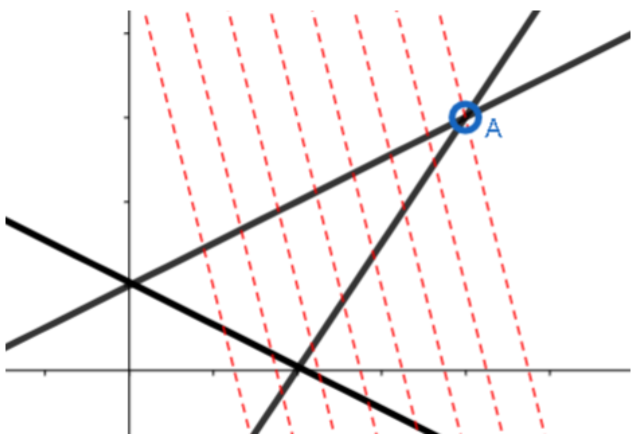

Figure 2: The minimum of a linear problem is located at one the vertices of the feasible region.

The black lines in figure 2 represent the constrainsts which define the feasible region. The red lines illustrate how the objective function can be increased until the end of the feasible region, where the optimum is located at a vertex of the feasible region, point A.

2.1.2 Mixed integer linear programming (MILP)

In many practical problems it makes more sense to assign integer or binary values to some of the variables. Examples include representations of non-continuous quantities and decisions (binary variables). When the objective function and the constraints are linear, the problem is referred to as a mixed integer linear programming (MILP) problem. The general formulation of the problem can be considered as an extension of the problem definition given in equation 1, with the addition of a constraint restricting a subset of the variables to integer values:

$$\min Z = a^T y + c^T x \tag{2a}$$
$$s.t\ By + Ax \leq b \tag{2b}$$
$$y \in \{0, 1\} \quad x \geq 0 \tag{2c}$$

Here, y represents the subset of variables that is restricted to integer (binary) variables, and x represents the continuous variables. This problem type is especially well suited for the modelling of choices as discrete values, which is essential for superstructure optimization. Some examples are given below [7].

$$\sum_{j=1}^{t} y_j = 1 \tag{3a}$$
$$\sum_{j=1}^{t} y_j \leq 1 \tag{3b}$$
$$y_j - y_k \leq 0 \tag{3c}$$
$$x_i - Uy_j \leq 0 \tag{3d}$$
$$g_1(x) - Uy \leq 0, \qquad g_2(x) - U(1-y) \leq 0 \tag{3e}$$

Equation 3a and 3b represent the selection of only one item, and the selection of at most one item respectively. Equation 3c and 3d are implication constraints. The former indicates that if item j is selected, item k must be selected as well, but not vice versa. The latter sets the continuous variable x_i to zero in the case that item j is not selected, where U is the upper limit for x. Equation 3e is a disjunction. If y is selected then constraint g_2 must hold. If y is not selected g_1 must hold.

A simple approach for solving MILP-problems would be to relax the binary constraint given in equation 2c and apply the SIMPLEX algorithm described earlier. However, this approach would not be recommended as it could lead to infeasible or non-optimal solutions. Another approach is the *branch and bound algorithm* which underlying concept is to *divide and conquer* [8]. The original problem is divided into smaller and smaller sub-problems by relaxing and adding constraints. These sub-problems are solved as linear programs to give the best possible solution of that particular subset (bound). Subsets are discarded if its bound indicates that it can not contain the optimal solution. The principle is illustrated in figure 3 [9].

2.1.3 Mixed integer non-linear programming (MINLP)

Mixed integer non-linear problems often arise in optimization of process synthesis problems and, are generally the hardest to solve. The branch and bound method described above can

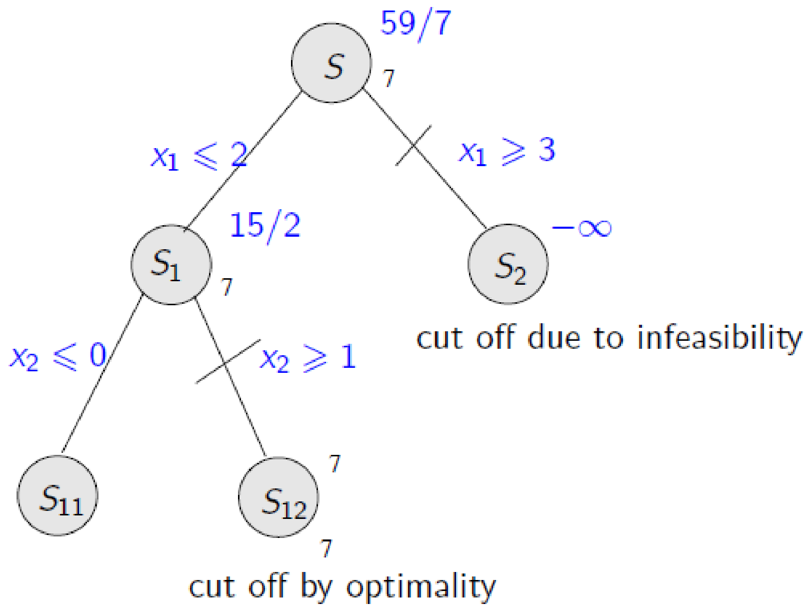

Figure 3: Illustration of a branch-and-bound tree [9]. The main problem is divided into sub-problems by relaxing and adding constraints.

in principle also be applied to these problems. However, instead of linear programs, each node will now correspond to solving a NLP, which can be expensive and not very effective. Other alternatives include Generalized Benders decomposition and Outer-Approximation [7]. The latter method involves solving a series of NLP-problems and master MILP problems. For a minimization problem, the NLP-subproblems provide upper bounds as well as values for the continuous variables while the MINLP master problems predicts lower bounds for the solution. The algorithm terminates when no lower bound can be found below the current upper bound. The formulation of a MINLP has a significant impact on the optimality of the obtained solution as well as the algorithmic performance [10]. For process synthesis problems it is not uncommon that MINLP-formulations can be trapped in local solutions. Therefore, great care must be taken when formulating the problems. It is generally favourable to place the nonlinearities in the objective function.

2.1.4 Previous work with superstructure optimization

The earliest work done in the field of superstructure optimization was focused on sub-systems and specific problems. Several models have been proposed for the selection of distillation column sequences during the last 50 years as well as reactor networks (Balakrishna and Biegler (1992)) and heat exhchanger networks (Yee and Grossmann (1990)) [2]. There has also been done work to incorporate different sub-systems into the formulation of a single process synthesis problem. Papoulias and Grossman proposed a MILP-formulation incorporating plant design, utility design and design of the recovery network [5]. One of the main challenges related to the creation of such superstructures is the fact that the mathematical programming problem can become highly complex. Especially if rigorous mathematical models are used for the units of the flow sheet. To circumvent this problem there has been developed so called short-cut methods, where rigorous unit-models are replaced by approximate models. An example is the linear and bi-linear approximations of component splitting fractions proposed by Aggarwall and Floudas.

2.2 Formulation and modelling

There are several ways of formulating a superstructure. Early approaches focused on combining different configurations that could perform the objective of the process into structures of greater complexity. Simple flowsheets were formulated in advance based on experience and engineering judgement and added together [1]. The idea is presented in figure 4.

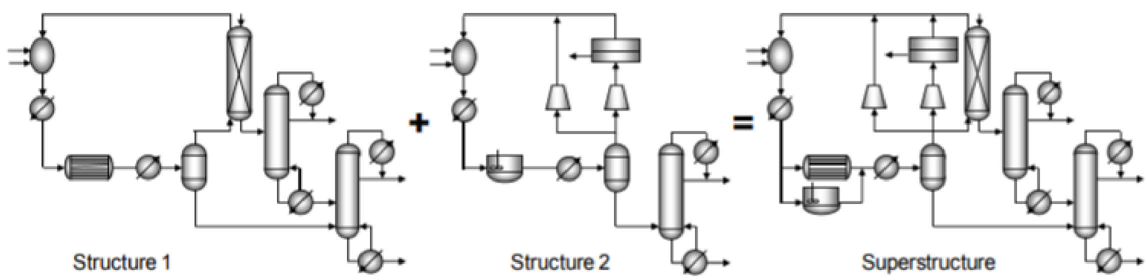

Figure 4: Combination of two simple flowsheet alternatives to create a superstructure [1].

Another option is to develop superstructures for the individual subsystems of the process e.g the reaction network and the heat recovery network, before combining the subsystems to create a superstructure for the overall process [7]. This method however, is limited by the generation of the individual superstructures, which again limits the total number of options.

2.2.1 Superstructure representations

The representation of a superstructure can play an important part in the implementation of the optimization. The examples given in figure 1 and 4 are so called state equipment network (SEN) representations, which were initially proposed by Smith in 1996 [2]. The main concept behind these representation is to initially select a set of equipment able to fulfill the objective of the process, and later on assign tasks for the equipment that is selected by the optimization. This way of generating superstructures gives the advantage of a more compact formulation of the optimization problem, due to the fact that the size of the combinatorial problem is reduced.

The main alternative to SEN-representations are state task network (STN) representations. Here, a task can be considered as a processing step transforming one particular state into another state. Examples include pressure changes, heat transfer, phase changes etc. The states are the chemical and physical properties of the process streams. Contrary to the SEN-representation, the concept behind STN is to initially identify a set of tasks necessary to fulfill the objective of the process and later assign appropriate equipment to the set of tasks. This method however, can lead to a complex superstructure if many states and tasks are needed to fully represent a unit.

2.3 Advantages and challenges

The obvious advantage of superstructure optimization is the ability to evaluate a large number of complex structures and interconnections in order to make good design decisions. By formulating the problem in an effective way, the method can give simultaneous optimization of both operating conditions and process structure. However, the truly optimal solution can not be found if it is not contained by the superstructure. This calls for rich superstructures of high complexity, which can be computationally demanding. Additionally, the accuracy of the results is highly dependent on the mathematical models of the units. Applying complex rigorous models will give better accuracy but at the same time lead to large scale non-convex MINLP-models, for which global optimality cannot be guaranteed. Therefore the greatest challenge regarding superstructure optimization is how to generate a structure that is rich and gives a wide search space, while at the same time not being too computationally demanding.

3 Problem Definition

The concepts described in section 2 can be applied to identify the optimal configuration of a subsea separation system with boosting. In order to do this, the process must first be broken down into its essential sub-processes and analyzed. This will be done in section 3.1. From the process analysis, the relevant discrete decisions, interconnections and potential pathways can be identified. This makes it possible to generate a superstructure for the process. From the superstructure, a mathematical optimazation model will be formulated and solved by the algebraic modeling system GAMS. The procedure is illustrated in figure 5

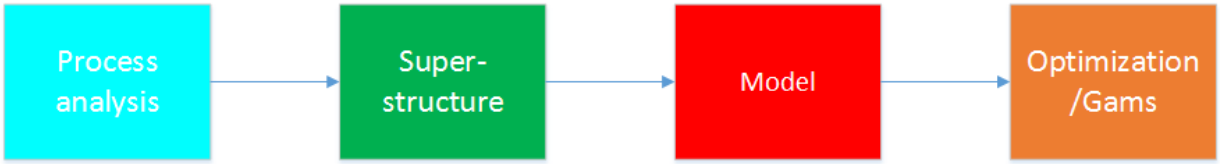

Figure 5: Procedure for identifying the optimal configuration of the subsea separation system

3.1 Process description

The case that will be studied is based on the problem presented in the report *Subsea separation* [11] as part of the process design project at NTNU. The case considers the design of a subsea separation system for a low energy oil field (low temperature and pressure). The main steps of the process is separation, treatment of the oil and the gas, boosting, transport to FPSO and handling of the produced water and sand. The general process flow is presented in figure 6.

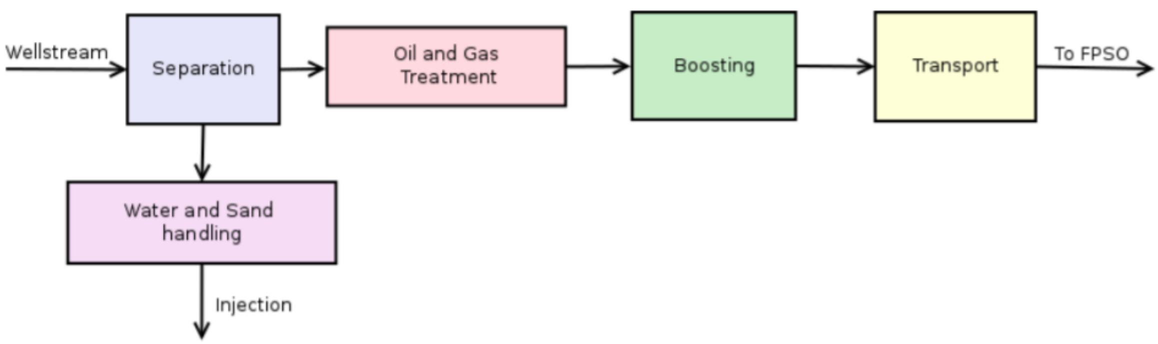

Figure 6: Main steps of the subsea separation process [11]

The well stream consists primarily of gas, oil, water and sand, and the first step of the process is to separate these main components in a 4-phase gravity separator. Since the water is separated off subsea, it is not transported to the surface, which solves the problem of limited water handling topsides as well as being energy efficient. The water and the sand are handled separately after the separation before being injected in a reservoir for disposal.

Due to the low energy of the reservoir, it is necessary to increase the pressure in order to ensure efficient transport of the oil and the gas topsides. This can be done separately for the two phases by the use of a pump for the liquid and a compressor for the gas. The alternative is to use multiphase boosting by the installment of a multiphase pump.

The final step is the transportation of the oil and the gas to the FPSO. The produced fluids must be transported 150 km before they are rised half a kilometer up to the FPSO. Based on the previous decions regarding boosting, the fluids can be transported separately or in a single pipe. Naturally, the fluids will not be transported separately in the case of multiphase boosting.

3.2 Possible scenarios and flowsheets

It becomes evident that there exists more than one feasible configuration for the process. The main decision lies in the transport of the fluids and how to handle the pressure increase.

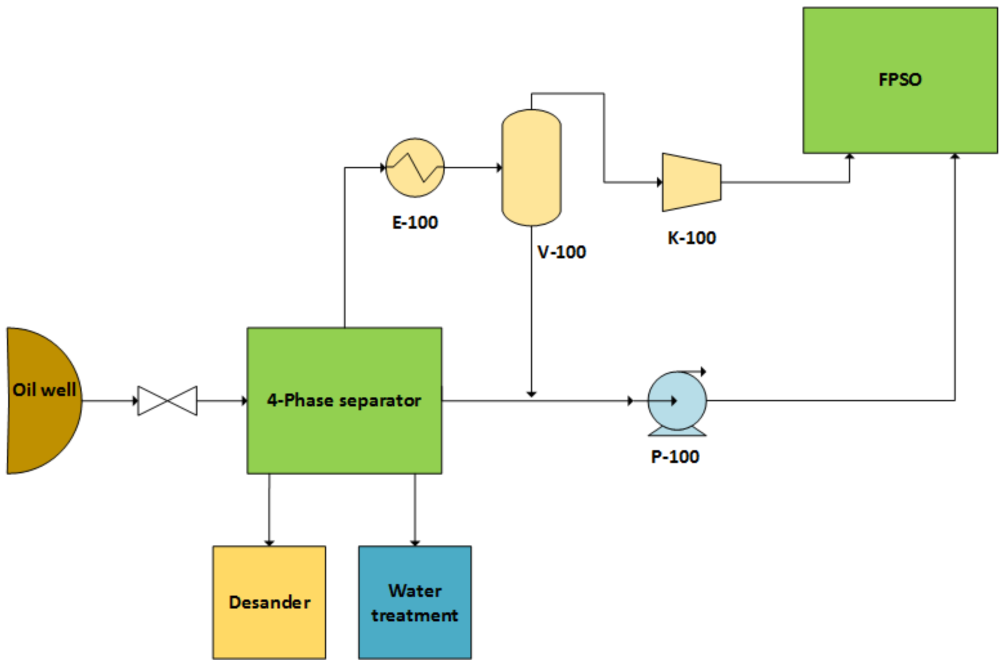

Figure 7: Separate boosting and compression and separate transport of the fluids to the FPSO.

Figure 7 shows a possible configuration for the process with separate boosting and compression for the two phases. The gas is cooled (E-100), to condense out the remaining liquid before the dry gas is compressed and sent to the FPSO. Since the phases are transported separately to FPSO, installment of two transport pipes and risers are necessary.

Another alternative is to use a multiphase pump for the boosting of both phases. This makes it possible to use a single transport line and riser to the FPSO. However, the phases must be separated topsides, so a gravity separator must be installed on the FPSO. This configuration is illustrated in figure 8.

A third option is to use separate boosting and compression like in figure 7, but mix the phases before transportation in order to use a single transport pipe and riser. This configuration also requires a topsides gravity separator. The configuration is illustrated in figure 9.

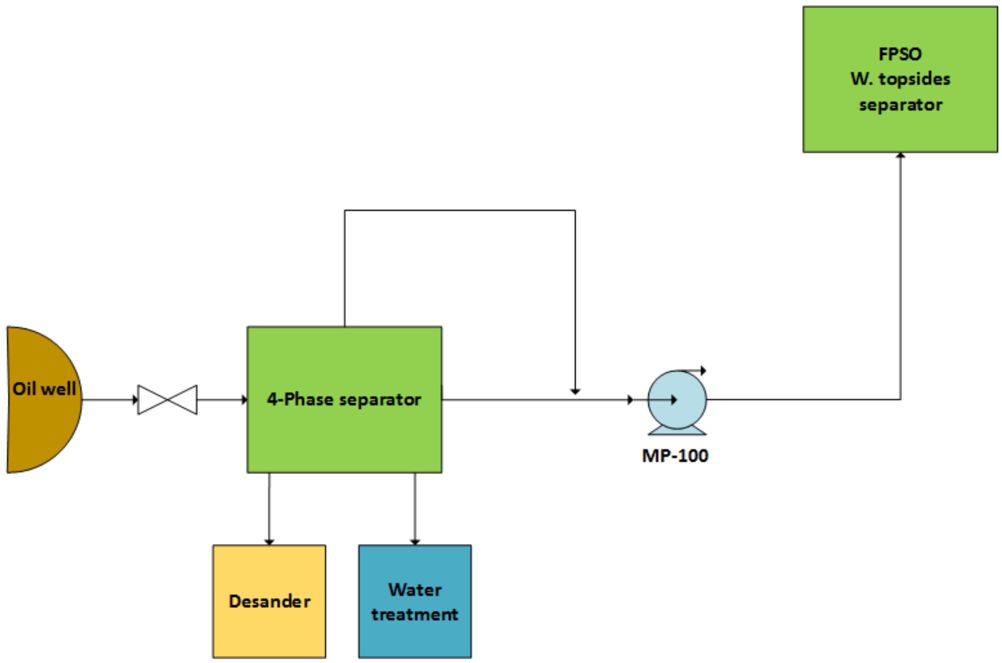

Figure 8: Multiphase boosting and transport with gravity separator topsides.

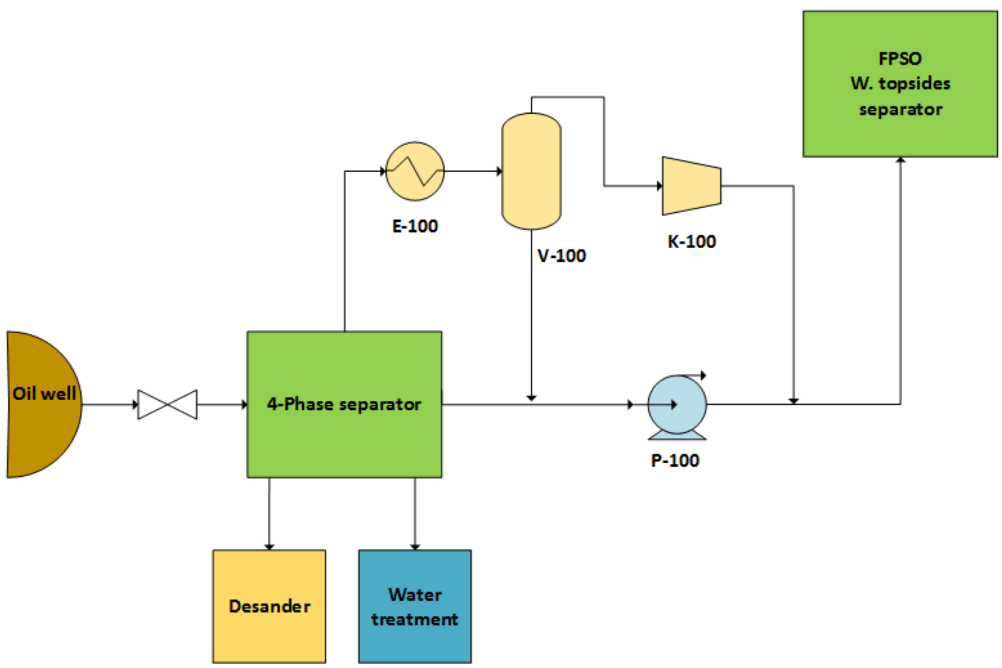

Figure 9: separate boosting and compression, transport in single pipe and riser with gravity separator topsides.

4 Superstructure

Based on the process description and possible configurations presented in section 3, a superstructure can be formulated for the process. As described in section 2.2, this can be done by combining simple flowsheets to include all the potential useful units and interconnections between them. Figure 10 is a superstructure based on the configurations given in figure 7, 8 and 9. The superstructure accounts for the possibilities of multiphase boosting and separate compression/pumping, as well as the transportation alternatives discussed in section 3. Possible splitters are represented by yellow nodes in the diagram, while the green nodes represent possible mixing points for the flows.

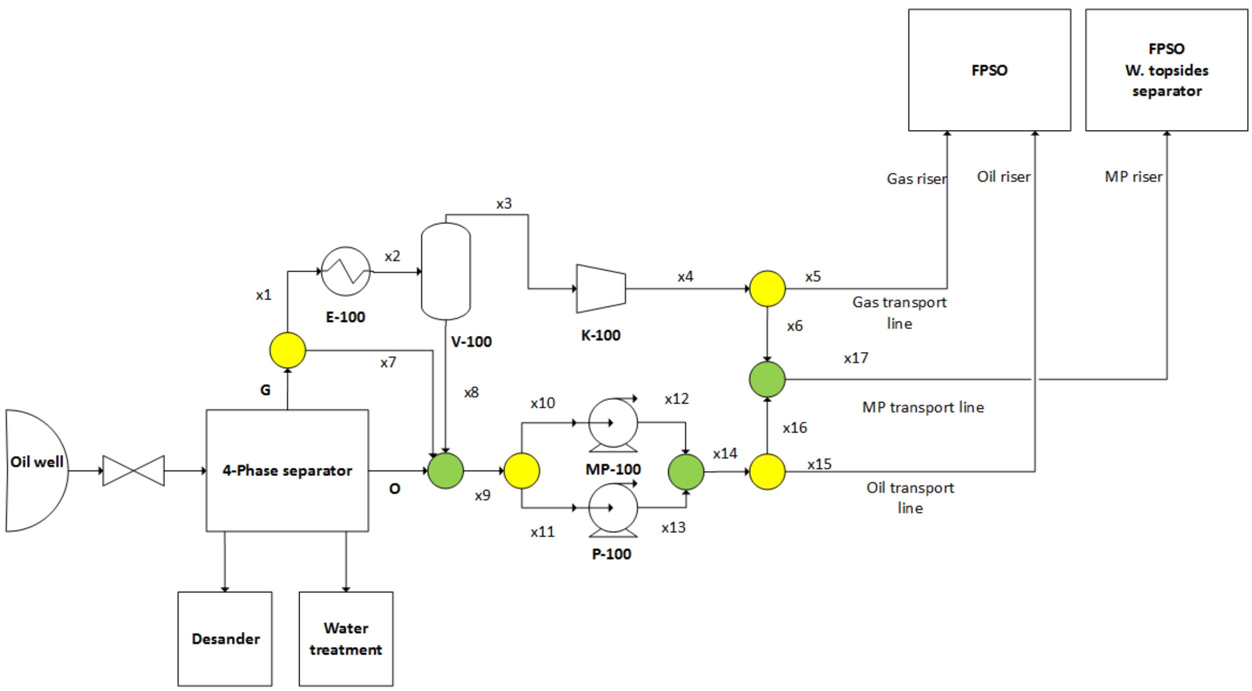

Figure 10: Proposed superstructure for the process.

The superstructure is mainly focused on the gas and oil flow from the 4-phase gravity separator. The water handling and sand handling are of lesser relevance for the optimization since no alternative options are proposed for these sub-processes. This leaves the gas and the oil flow as the initial starting point of the model. The gas flow is denoted as G in the diagram. After leaving the gravity separator, a splitter presents the option of either combining the two phases or sending the gas to the cooler (E-100) to condense out the remaining liquid. If the former alternative is chosen, multiphase boosting (MP-100) should be implemented, and the phases should be transported by the multiphase transport line and riser to the FPSO. The diagram indicates that the FPSO requires a topside separator in the case of multiphase transport. If the latter alternative is chosen, the the cooled fluid from the condenser is sent to a separator (V-100) to remove the condensed liquid from the dry gas. The condensed liquid is then joined with the oil flow and sent to the pump (P-100), while the gas is sent to the compressor (K-100). The boosted fluids can then either be transported separately to the FPSO, or in the multiphase transport line.

5 Optimization model

An optimization model can be formulated based on the superstructure presented in figure 10. The optimization model must account for every potential configuration present in the superstructure as well as the corresponding equipment. To achieve this, binary variables are assigned to each individual piece of equipment as to represent discrete decisions. The binary variables for the units that will be present in the optimal solution take the value 1, while the binary variables corresponding to the unused units will take the value 0. Table 1 shows the relationship between the binary variables for the optimization model and the corresponding equipment.

Table 1: Binary variables for the potential equipment in figure 10.

Binary variable	Unit
y_1	Cooler
y_2	Separator
y_3	Compressor
y_4	Oil pump
y_5	Multiphase pump
y_6	Gas transport line
y_7	Gas riser
y_8	Multiphase transport line
y_9	Multiphase riser
y_{10}	Topsides separator
y_{11}	Oil transport line
y_{12}	Oil riser

While the existence of the units are represented by binary variables, the mathematical models for the units are given by continuous variables. As a result, the problem contains both binary and continuous variables which makes the optimization problem take the form of a mixed integer problem (MIP) as described in sections 2.1.2 and 2.1.3. Equation 4 shows the general formulation for the optimization model.

$$\min f(x, y) \tag{4a}$$
$$s.t\ Ax = b \tag{4b}$$
$$x - Uy \leq 0 \tag{4c}$$
$$\Omega(y) = 0 \tag{4d}$$
$$y \in \{0, 1\} \quad x \geq 0 \tag{4e}$$

Here, x is a vector containing the flows of the system in total mass, and y is the binary variables for the equipment. Equation 4a is the objective function used to evaluate the economic performance of the potential configurations. The objective function is dependent on both the mass flows and the binary variables. Equation 4b-4e are the constraints for the problem. Each of the equations will be described in detail in the following sections.

5.1 Constraints

The constraints of the problem must be satisfied in order to get a feasible solution. The model constraints can be divided into 4 different sets: Mass balances, constraints for the flows of the unused units, logical conditions and variable restrictions. The variable restrictions are given by equation 4e and simply specifies that the mass flows must be nonzero, and that the y-variables must take binary variables.

5.1.1 Mass balances

Equation 4b specifies that the mass balances must be satisfied for any part of the system. This includes the units as well as the connections developed in the superstructure. As discussed in section 2, it is desirable to formulate the modeling equations in linear form if possible, which can be achieved by considering the balances in total mass. This is not always possible because many models call for component flows, especially in the case of reaction modeling. However, there is no reaction present in the system, and the mass balances can be specified in total form. The x-vector therefore represents the total mass of each flow as labeled in figure 10.

In order to model the separator (V-100) accurately, detailed data for the thermodynamic properties of the gas flow is needed. However, from HYSYS-simulations it can be seen that a very small percentage of fluid is condensed. To simplify the modeling, it is therefore assumed that 5 percent of the fluid is condensed and separated. By assuming steady state, the mass balances for the system can be formulated as follows:

$$
\underbrace{\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.05 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{bmatrix}}_{A}
\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \\ x_{16} \\ x_{17} \end{bmatrix}}_{x}
=
\underbrace{\begin{bmatrix} \text{G} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\text{L} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{b}
\tag{5}
$$

Here it is assumed that the gas and the oil flow from the four-phase-separator is known. Equation 5 can be modeled in GAMS by using sums. By constructing a set i = {1,2...n} for the mass flows and k = {1,2...m} for the mass balances, the equation takes the form:

$$
\sum_{i=1}^{n} A_{i,k}\, x_i = b_k \qquad \text{for } k = 1, 2...m \tag{6}
$$

where x_i is mass flow i, and b_k is the right hand side of equation k. By solving the mass balances for the system, the equipment can be sized according to relevant flows.

5.1.2 Unused units

Equation 4c in the model formulation specifies that the flows that correspond to unused pieces of equipment should be set to zero, in order to satisfy the logic of the system. This can be implemented by forcing the inflow of a unit with a zero-binary variable to be set to zero as well. From the mass balance in equation 5, the consecutive flows will also be set to zero. The general form of the constraint is similar to the one given in equation 3d in section 2.1.2. The set of constraints can be formulated as in equation 7.

$$x_1 - U\ y_1 \leq 0 \tag{7a}$$
$$x_7 - U\ y_5 \leq 0 \tag{7b}$$
$$x_{10} - U\ y_5 \leq 0 \tag{7c}$$
$$x_{11} - U\ y_4 \leq 0 \tag{7d}$$
$$x_5 - U\ y_6 \leq 0 \tag{7e}$$
$$x_6 - U\ y_8 \leq 0 \tag{7f}$$
$$x_{15} - U\ y_{11} \leq 0 \tag{7g}$$
$$x_{16} - U\ y_8 \leq 0 \tag{7h}$$

Here U acts as an upper limit for the mass flows of the system. Equation 7a ensures that if the cooler is not chosen (y_1=0), the corresponding inflow (x_1) is zero as well. However, if the cooler is chosen (y_1=1), x_1 can not exceed the specified upper limit limit U for the flows. All the other constraints in equation set 7 follow the same logic. Equation 7b specifies that the phases will not be mixed prior to the pressure increase if multiphase boosting is not chosen. In addition, the flow to the multiphase pump will be set to zero, as specified by equation 7c. If the pump is not chosen, equation 7d will set the inflow to zero. Equation 7f and 7h ensures that the phases are not mixed after the pressure increase if the multiphase transport line is not installed. The remaining constraints (7e and 7g) set the single phase flows to the FPSO to zero in the case where the transport lines for oil and gas are not installed.

By the logic of the system (described in detail in section 5.1.3), only four of the constraints of the set will be active at the same time. By the dimensions of the A-matrix given in equation 5, it follows that the mass balances will have an unique solution for the given set of optimal binary variables.

5.1.3 Logical conditions

Equation set 4d in the optimization model consists of logical constraints related to the binary variables. These constraints are included in order to ensure that the optimal solution will be logically feasible. The logical conditions are linear in the binary variables and are given in equation set 8 below.

$$y_4 + y_5 = 1 \tag{8a}$$
$$y_6 + y_8 = 1 \tag{8b}$$
$$y_5 - y_8 \leq 0 \tag{8c}$$
$$y_1 = y_2 = y_3 = y_4 \tag{8d}$$
$$y_6 = y_7 = y_{11} = y_{12} \tag{8e}$$
$$y_8 = y_9 = y_{10} \tag{8f}$$

Equation 8a and 8b both ensure the selection of only one item among two choices. The former specifies that multiphase pumping can not be chosen at the same time as the oil pump. The latter specifies that multiphase transport can not be chosen at the same time as the gas transport line. Equation 8c is an implication constraint. If the multiphase pump is installed, the multiphase transport line must also be installed. However, the multiphase transport line can be installed without installing the multiphase pump. This opens up the possibility to mix the phases after the pressure increase in order to transport them in a single pipe. Equation 8d-8f relates the equipment that can not be chosen independently from the other equipment in the same set. By equation 8d, the cooler, the separator, the compressor and the oil pump, can not be installed without the additional installment of the other units in the set. Similarly the gas transport line can not be installed without a riser for the gas, as well as a transport line and riser for the oil flow. This is ensured by equation 8e. Finally, equation 8f specifies that the multiphase transport and rising of the gas leads to the installment of a topsides separator. The units can not be chosen without the installment of the related units.

Equation 8d-8f indicates that several binary variables can be eliminated from the model before the implementation in GAMS. This would of course simplify the model by giving it a more compact form. However, by including all binary variables for the equipment, the output file will be simple in the sense that it gives the existence of all the proposed equipment directly. The model will therefore be implemented as presented in this section. Equation 8d-8f have to be decomposed into several equations in order to be implemented in GAMS. Equation 8d is implemented as follows:

$$\begin{aligned} y_1 - y_2 &= 0 \\ y_2 - y_3 &= 0 \\ y_3 - y_4 &= 0 \end{aligned} \tag{9}$$

Equation 8e-8f is implemented the same way as shown in equation 9.

5.2 Objective function

The objective function is given by equation 4a in the model formulation and is used to evaluate the economic performance of the alternatives present in the superstructure. The selection of the optimizing criteria can strongly influence the generation of the optimal solution. Common criteria include total investment costs, annual costs, internal rate of return and net present value. For this paper, two different approaches will be evaluated. For the first case, the investment costs of the plant will be minimized by using fixed costs for the equipment. For the second case the net present value (NPV) will be maximized. The second case will therefore include revenues and operating costs for the plant. In addition, a set of the equipment will be cost estimated based on the mass flows of the system, provided by equation 5.

5.2.1 Case I: Minimizing the investment costs

In order to evaluate the investment costs of the plant, the cost for all potential equipment must be estimated. For the case of fixed operating conditions, fixed costs can be used for the units. Table 2 shows the fixed costs for the equipment for the base case [11].

Table 2: Pre-estimated costs for the equipment in the base case [11].

Unit	Symbol	Cost [USD]
Cooler	C_c	423 885
Separator	C_s	1 003 096
Compressor	C_k	22 760 726
Oil pump	C_p	3 045 354
Multiphase pump	C_{MP}	16 000 000
Gas transport line	C_{fg}	93 675 000
Gas riser	C_{rg}	714 000
Multiphase transport line	C_{fm}	154 500 000
Multiphase riser	C_{mr}	2 177 700
Topsides separator	C_{ts}	462 183
Oil transport line	C_{fo}	129 165 000
Oil riser	C_{fr}	1 438 200

From the data in table 2 the objective function takes the form of equation 10.

$$C_0 = \sum_{j=1}^{13} C_j \, y_j \tag{10}$$

Here, the cost of each potential unit is multiplied by its corresponding binary variable from table 1. This makes the optimization model a mixed integer linear problem (MILP).

5.2.2 Case II: Maximizing the net present value (NPV)

In order to calculate the net present value of the plant, revenues and operating costs must be taken into account. The general formula for the net present vaule is given by equation 11.

$$NPV = \sum_{t=1}^{T} \frac{C_F}{(1+r)^t} - C_0 \tag{11}$$

Here, C_F is the cash flow, r is the interest rate, C_0 is the initial investment and T is the time horizon for the project. The cash flow can be calculated by adding the revenues from the gas and the oil flows and subtracting the cost of the power as illustrated by equation 12

$$C_F = (F_{oil}\ p_{oil} + F_{gas}\ p_{gas})\ h_y - (P_k + P_p + P_{MP}) h_y\ p_e \tag{12}$$

Here, F_{oil} and F_{gas} are the flows of oil and gas given in barrels and MMBTU respectively. The mass flows must therefore be converted into the respective units, as implemented in the script given appendix A.2. p_{oil}, p_{gas} and p_e denotes the prices for oil, gas and electricity. The operating hours per year is given by h_y, and P_k, P_p and P_{MP} are the power-consumption of the compressor, pump and multiphase pump respectively. The mass flows and power will be calculated by the model. The remaining parameters are given in table 3.

Table 3: Parameters for calculation of cash flows

Parameter	Value	Unit
p_{oil} [12]	57.30	[USD/bbl]
p_{gas} [12]	2.61	[USD/MMBTU]
p_e [13]	0.12	[USD/kWh]
h_y	8000	[hours/year]

To calculate the utility cost of the system, the power consumption of each unit in the flowsheet must be calculated as a function of the mass flow vector x. The power consumption of the compressor was calculated by equation 13.

$$P_k = \frac{p_1}{\eta_k} \left(\frac{x_3}{\rho_g\ 3600} \right) \frac{\gamma}{\gamma - 1} \left(\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right) \tag{13}$$

Here, p_1 and p_2 are the inlet- and outlet pressures of the compressor respectively.. Both pressures are given in kPa. ρ_g is the density of the gas in kg/m^3 and γ is the dimensionless heat capacity ratio of the gas. x_3 is the mass flow to the compressor calculated by the mass balances given in equation 5. η_k denotes the adiabatic efficiency of the compressor. The power consumption of the pump was calculated by equation 14 [13].

$$P_p = \frac{x_{11}\ \Delta P}{3.6 \times 10^6\ \rho_{oil}\ \eta_p} \tag{14}$$

x_{11} is the mass flow to the pump, and ΔP is the pressure difference. Equation 15 gives the power consumption of the multiphase pump [14].

$$P_{MP} = \frac{x_{10}\ g\ h}{3.6 \times 10^6\ \eta_{MP}} \tag{15}$$

Here, h is the head in meters converted from the differential pressure by:

$$p = 0.0981\ h\ SG,$$

where SG is the specific gravity of the fluid. From the constraints given in section 5.1.2, the inflows of the unused units are set to zero. From equation 13-15 it is then clear that the power consumption of the unused units will be zero as well, which is why the power of all three components are included in equation 12 for the cash flow. The parameters presented in the equations above were obtained from HYSYS and the *subsea separation report* [11]. The parameters are presented in table 4.

Table 4: Parameters power consumption calculations

Parameter	Value	Unit
p1	6500	kPa
p2	30500	kPa
ΔP	30e6	Pa
γ	1.56	-
ρ_g	53.45	kg/m^3
ρ_o	844	kg/m^3
η_k	0.75	-
η_p	0.75	-
η_{MP}	0.75	-
SG	0.7514	-
h	2713.25	m

C_0 in equation 11 is the investment costs. In order to evaluate cases with varying mass flows from the 4-phase gravity separator, the potential equipment was sized and cost estimated in the objective function. The cost of the equipment was calculated by the cost estimation equations proposed by *Sinnott and Towler* [13], for which the general form is presented in equation 16.

$$Ce = a + b\ S^n \tag{16}$$

The equation gives the purchased equipment cost as the sum of a constant cost coefficient a, and a term dependent on the size factor S. In addition the cost in equation 16 was multiplied by cost indices to account for installation, subsea environment operation and price changes in time. The cost indices are given in table 5.

Table 5: Cost indices

Cost index	Value	Description
f_{inst} [11]	4.208	Installation
f_{sub} [11]	3	Subsea operation
f_I [11]	1.1035	Cost changes in time

Equation 17 was used to estimate the cost of the compressor.

$$C_k = (490000\ y_3 + 16800\ P_k^{0.6})\ f_{inst}\ f_{sub}\ f_I \tag{17}$$

Here P_k is the power consumption of the compressor in kW. This quantity is provided by solving equation 13. The cost of the pump was estimated by equation 18.

$$C_p = C_{pm} + \left(6900\ y_4 + 206\ \left(\frac{x_{11}}{\rho_o\ 3.6}\right)^{0.9}\right)\ f_{inst}\ f_{sub}\ f_I \tag{18}$$

where x_{11} is the mass flow to the pump and C_{pm} is the cost of the motor given by:

$$C_{pm} = (-950\ y_4 + 1770\ P_p^{0.6})\ f_{inst}\ f_{sub}\ f_I \tag{19}$$

Here P_p is the power consumption of the pump given by equation 14. The cost of the cooler was estimated by:

$$C_c = (24000\ y_1 + 46\ A^{1.2})\ f_{inst}\ f_{sub}\ f_I \tag{20}$$

Where A is the heat exchange area, calculated by equation 21.

$$A = \frac{x_1\ Cp_g\ \Delta T}{3600\ U_h\ \Delta T_{LM}} \tag{21}$$

Cp_g is the heat capacity for the gas. ΔT is the temperature difference between the in- and the out-flow of the cooler. U_h is the overall heat transfer coefficient and ΔT_{lm} is the logarithmic mean temperature difference. Table 6 shows the parameters used for the estimation of the heat transfer area.

Table 6: Parameters for estimation of heat transfer

Parameter	Value	Unit
Cp_g	2681	J/kgK
U_h [13]	20	W/m^2
ΔT	2.5	K
ΔT_{LM}	19.6	K

The cost estimations given by equation 17-20 are composed of a fixed cost as well as a term that is dependent on the capacity. The fixed cost is multiplied with the corresponding binary variable for the unit in order to only include it when the unit is installed. The second term is automatically set to zero when the unit is not chosen because the mass flow to the unit is forced to zero by the constraints proposed in section 5.1.2. The rest of the units are given by their fixed costs in table 2. This is of course not accurate, but for the simplicity of the model it is assumed that the costs does not change much in the region of mass flows that are studied in this case.

6 Results

The optimization models proposed in section 5 were implemented and solved by the use of GAMS. The solutions of the problems were given as optimal sets of binary variables that satisfied the constraints defined in section 5.1. From these sets of binary variables, the optimal configurations of units were extracted for the two cases.

6.1 Case I: Minimizing the investment costs

The code for minimizing the investment cost is given in appendix A.1. The model consists of the constraints defined in section 5.1 and the objective function defined in 5.2.1. Since the model was a MILP-problem, CPLEX was used to obtain the solution. The optimal set of binary variables is given in table 7, with the minimum investment. The corresponding optimal flowsheet is given in figure 11. The total mass flows calculated by GAMS is given in appendix A.3, with the initial values for the oil and gas flow from the 4-phase separator.

Table 7: Optimal set of binary variables and investment cost for case I.

Variable	Description	Value in optimal solution
C_0	Investment cost	173.140 [Mill USD]
y_1	Cooler	0
y_2	Separator	0
y_3	Compressor	0
y_4	Oil pump	0
y_5	Multiphase pump	1
y_6	Gas transport line	0
y_7	Gas riser	0
y_8	Multiphase transport line	1
y_9	Multiphase riser	1
y_{10}	Topsides separator	1
y_{11}	Oil transport line	0
y_{12}	Oil riser	0

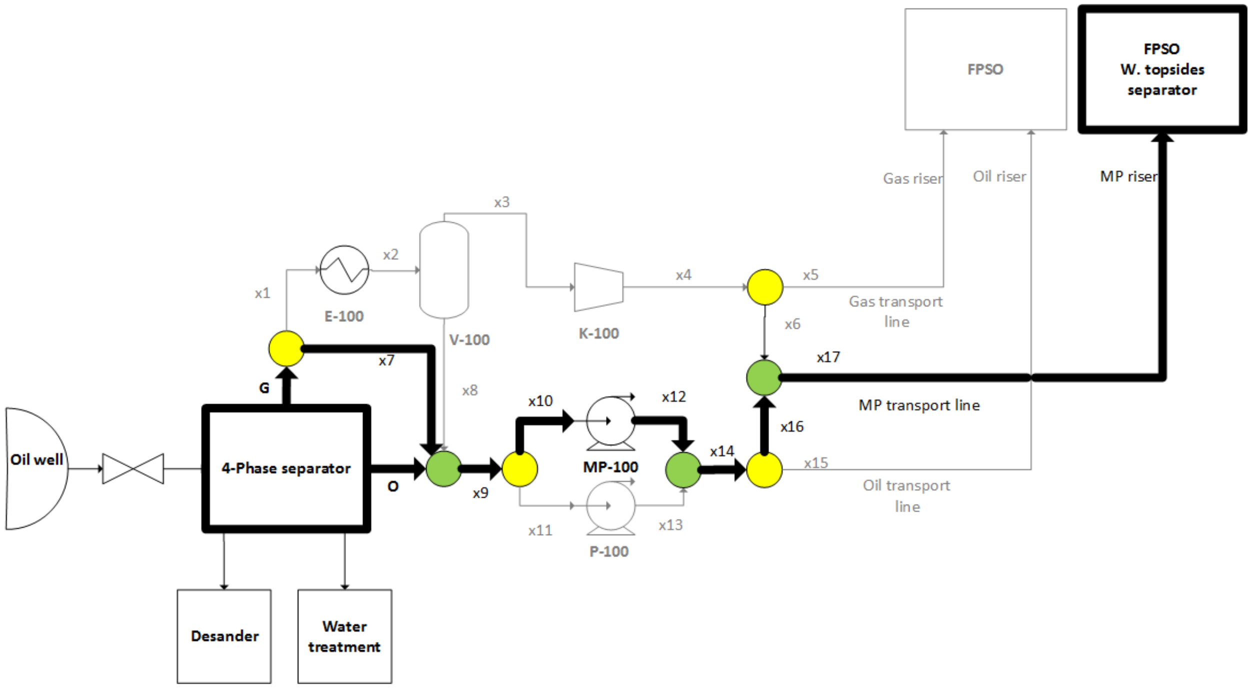

Figure 11: Optimal flowsheet for case I

6.2 Case II: Maximizing the net present value

The code for maximizing the net present value is given in appendix A.2. The model consists of the constraints defined in section 5.1 and the objective function and cost equations defined in 5.2.2. Since the model was a MINLP-problem, DICOPT was used to obtain the optimal solution. The problem was solved for two stages of the reservoirs life-time: early production and late production. The flow rates for these cases are given in appendix A.3, with the solution of the mass balances calculated by GAMS.

6.2.1 Early production

The optimal solution for early production was the same as for case I. The optimal set of binary variables is given in table 7. The corresponding optimal flowsheet is given in figure 11. Table 8 shows the costs, power consumption and heat transfer area calculated by GAMS.

6.2.2 Late production

For the case of late production, the optimal set of binary variables are given in table 9. The corresponding optimal flowsheet is given in figure 11. Table 10 shows the costs, power consumption and heat transfer area calculated by GAMS.

Table 8: Costs, power consumption and heat transfer area calculated by GAMS for maximizing the NPV for early production.

Variable	Value	Unit
NPV	4146.847	Mill USD
C_0	173.140	Mill USD
C_F	703.058	Mill USD
C_k	0	Mill USD
C_p	0	Mill USD
C_c	0	Mill USD
P_k	0	kW
P_p	0	kW
P_{MP}	2158.144	kW
A	0	m2

Table 9: Optimal set of binary variables for case II with late production rates.

Binary variable	Unit	Value in optimal solution
y_1	Cooler	1
y_2	Separator	1
y_3	Compressor	1
y_4	Oil pump	1
y_5	Multiphase pump	0
y_6	Gas transport line	0
y_7	Gas riser	0
y_8	Multiphase transport line	1
y_9	Multiphase riser	1
y_{10}	Topsides separator	1
y_{11}	Oil transport line	0
y_{12}	Oil riser	0

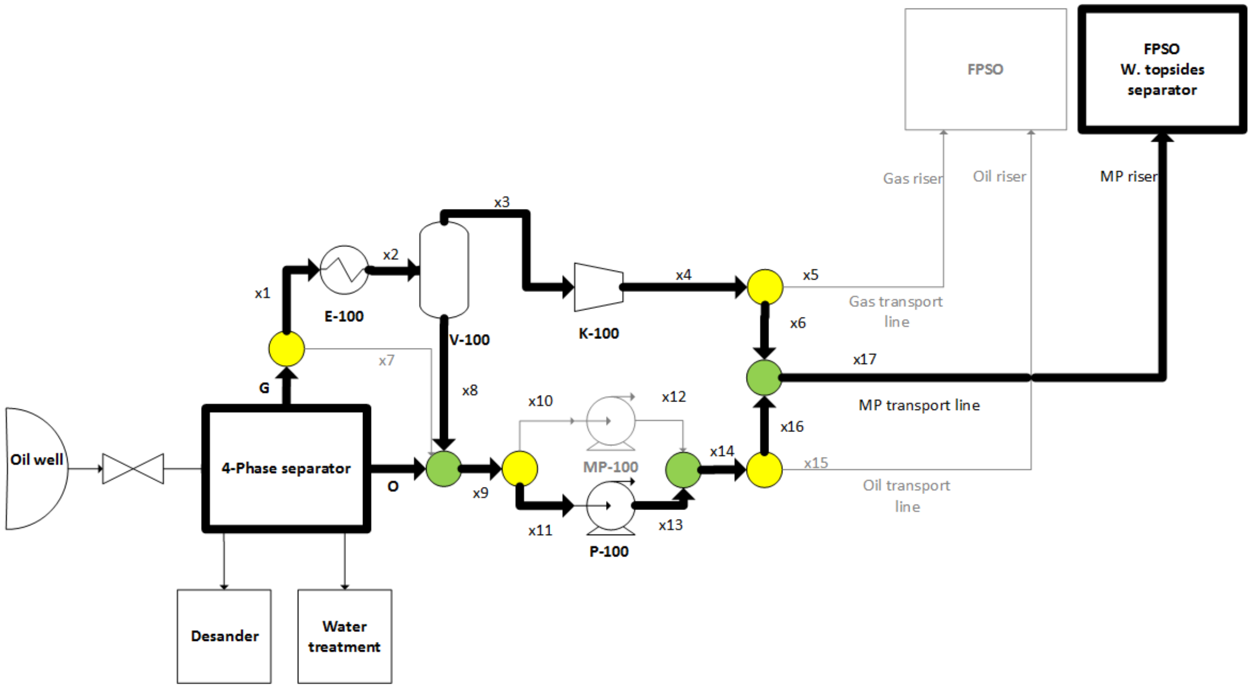

Figure 12: Optimal flowsheet for case II late production

Table 10: Costs, power consumption and heat transfer area calculated by GAMS for maximizing the NPV for late production.

Variable	Value	Unit
NPV	72.922	Mill USD
C_0	169.199	Mill USD
C_F	39.404	Mill USD
C_k	10.121	Mill USD
C_p	0.597	Mill USD
C_c	0.338	Mill USD
P_k	82.079	kW
P_p	153.060	kW
P_{MP}	0	kW
A	4.412	m2

7 Discussion

Figure 11 shows that the minimum investment cost of the subsea separation system is obtained by using multiphase boosting, transportation and rising with the additional installment of a topsides separator. Due to the limited size of the combinatorial problem, it is easy to verify that this is indeed the optimal solution. Furthermore, the optimal set of binary variables from table 7 satisfies the specified logical constraints in equation 8 as well as the mass balances in equation 5, which confirms that the solution is in fact feasible. This indicates that the optimization model works as intended.

The same optimal configuration is obtained by maximizing the net present value for early production. This indicates that the calculated investment cost should be equal to the one obtained in case I, which is verified by table 8. In addition, the data in table 8 shows that the calculated power consumption and costs for the unused units are set to zero, as specified in section 5.2.2. The results in table 9 show that the optimal set of binary variables changes for the late production. For these conditions, the maximum net present value is obtained by using separate compression and pumping for the phases before mixing them in order to use multiphase transport and rising. The reason behind this is that the reduced flows from the 4-phase separator lowers the costs of the oil pump and the compressor as can be seen from comparing the fixed costs in table 2 with the costs calculated by GAMS in table 10. However, these results should should be interpreted with skepticism since the cost of the multiphase pump has a high level of uncertainty. There are no current cost relations for multiphase pumps, and the fixed cost estimate used in this report does not account for the flow size, which makes the accuracy of the results questionable.

It is important to emphasize that the investment costs and net present values presented in table 7, 8 and 10 does not reflect the actual costs or profitability of the total plant. These values only account for the equipment and costs relevant for the optimization problem, and are therefore used exclusively for comparing the alternative configurations within the search space of the superstructure. This means that essential equipment and costs that are common for the configurations like the cost of the desander, water treatment system, 4-phase separator as well as drilling are excluded from the model in order to make it as simple as possible for the purpose of the optimization. The values for the investment costs and net present values should therefore not be used as a basis for evaluating profitability of the process. It is also important to emphasize that the net present values are calculated based on constant production rates of gas and oil during the time horizon for both the early and late production cases. This is of course not accurate as the production rate will change gradually in time. The truly optimal configuration should therefore account for estimated flow profiles as a function of time, such that accurate results for the operational costs are obtained.

It is clear from the process description in section 3.1 that the base case described in this report has a relatively limited number of combinatorial possibilities. As a result, superstructure optimization may not be the most efficient approach for obtaining the optimal configuration in this case. In fact, the optimal configuration can be identified relatively easily by evaluating each possible alternative and comparing them to each other. However, this approach would be very time consuming for systems of higher complexity. Superstructure optimization is therefore most commonly applied when there are numerous alternatives and units spanned by the superstructure, which is not the case for the problem studied in this report. Nevertheless, the model presents an approach for quickly identifying the optimal configuration for a variety of flow rates, which makes it applicable to other systems.

As discussed in section 2.3, one of the main advantages of superstructure optimization is the ability to simultaneously identify the best possible configuration of a flow sheet while at the same time optimizing the operating conditions of the process. The model presented in this report however, does not utilize optimization of the operating conditions. This means that the solutions presented for the system in theory only are valid for the specific set of operating conditions applied in the base case. It is likely that a different set of operating conditions could give a higher value for the NPV, but it would not necessarily change the optimal configurations of units in the plant. A more complete model would include the operating conditions as decision variables, as well as constraints for the operating conditions to ensure safe and efficient production.

8 Conclusion

Minimizing the investment costs of the subsea separation system gave multiphase boosting and transport/rising as the optimal solution. The same solution was obtained by maximizing the net present value. However the optimal solution is dependent on the production rate, and for late production separate pumping/compression with multiphase transport/rising was identified as the optimal solution (when maximizing the net present value). Uncertainty in the cost estimated for the multiphase pump may have affected the results. In addition, the operating conditions were not optimized for the process. The optimization model could be improved by including the operating conditions as decision variables with appropriate constraints, and accounting for variable production rates as a function of time.

9 List of Symbols

Symbol	Description
a	Constant cost coefficient
A	Heat transfer area
b	Cost coefficient for variable cost
C	Cost
C_0	Investment cost
C_F	Cash flow
C_p	Heat capacity
f_{inst}	Cost factor for installation
f_{sub}	Cost factor for subsea operation
f_I	Cost factor for price changes in time
h_y	Operating hours
n	Exponential cost factor
NPV	net present value
p	Pressure
P	Power
r	Interest rate
SG	Specific gravity
T	Temperature
ΔT_{lm}	Logarithmic mean temperature difference
U	Upper limit for continuous variable
U_h	Heat transfer coefficient
x	Total mass flow
y	Binary variable for equipment
γ	heat capacity ratio
η	Efficiency
ρ	Density

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Appendix

A GAMS

A.1 Case I: Minimizing the investment costs

```
$Title Minimizing the investment cost of a subsea separation system.

$Ontext
Optimization of the model presented in the report: "Superstructure
optimization of a subsea separation system".
Objective: Find the combination of process units that minimizes the investment
costs and satisfies the the specifications of the process.
$Offtext

*Defining a symbol for an end of line comment.
$eolcom ->
*.

*Defining the sets
Set
         i mass flow index /i1*i17/
         j bin var index /j1*j12/
         k eq number /k1*k13/;

*Parameters for the model
Parameters
         G gas stream /12720/
         O oil stream /206200/
         U upper limit for mass flows /250000/
         b(k) RHS coeff for mass balances /set.k 0/

*Fixed costs for the units                                  Binary variables
         C_c Cost for cooler /423885/                       -> y1.
         C_s Cost for separator /1003096/                   -> y2.
         C_k Cost for compressor /22760726/                 -> y3.
         C_p Cost for pump / 3045354/                       -> y4.
         C_MP Cost for multiphase pump /16000000/           -> y5.
         C_fg Cost for flowline gas /93675000/              -> y6.
         C_rg Cost for riser gas /714000/                   -> y7.
         C_fm Cost for flowline multiphase /154500000/      -> y8.
         C_rm Cost for riser multiphase /2177700/           -> y9.
         C_ts Cost for toptside separator /462183/          -> y10.
```

```
        C_fo Cost for flowline oil /129165000/        -> y11.
        C_ro Cost for riser oil /1438200/;            -> y12.


*Assigning numbers to the vector.
b('k1') = G;
b('k7') = -O;

Table A(k,i)
        i1  i2   i3  i4  i5  i6  i7  i8  i9  i10 i11 i12 i13 i14 i15 i16 i17
   k1   1   0    0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
   k2   1  -1    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   k3   0  0.05  0   0   0   0   0  -1   0   0   0   0   0   0   0   0   0
   k4   0  -1    1   0   0   0   0   1   0   0   0   0   0   0   0   0   0
   k5   0   0   -1   1   0   0   0   0   0   0   0   0   0   0   0   0   0
   k6   0   0    0  -1   1   1   0   0   0   0   0   0   0   0   0   0   0
   k7   0   0    0   0   0   0   1   1  -1   0   0   0   0   0   0   0   0
   k8   0   0    0   0   0   0   0   0  -1   1   1   0   0   0   0   0   0
   k9   0   0    0   0   0   0   0   0   0  -1   0   1   0   0   0   0   0
   k10  0   0    0   0   0   0   0   0   0   0  -1   0   1   0   0   0   0
   k11  0   0    0   0   0   0   0   0   0   0   0   1   1  -1   0   0   0
   k12  0   0    0   0   0   0   0   0   0   0   0   0   0  -1   1   1   0
   k13  0   0    0   0   0   1   0   0   0   0   0   0   0   0   0   1  -1;

*Variables
Positive variables x(i) mass flows;

Binary variables y(j);

Variable c0 investment
        inv_cost investment cost;

*Declaring equations
Equations
        obj objective function
        MB(k) mass balances
        q1 flow to zero
        q2 flow to zero
        q3 flow to zero
        q4 flow to zero
        q5 flow to zero
        q6 flow to zero
        q7 flow to zero
        q8 flow to zero
        l1 logic eq 1
        l2 logic eq 2
        l3 logic eq 3
        l4 logic eq 4
        l5 logic eq 5
        l6 logic eq 6
        l7 logic eq 7
```

```
          l8 logic eq 8
          l9 logic eq 9
          l10 logic eq 10
          l11 logic eq 11;

*Defining the equations
obj..     inv_cost =e= 1E-6*(C_c*y('j1') + C_s*y('j2') + C_k*y('j3') +
          C_p*y('j4')+ C_MP* y('j5') + C_fg*y('j6') + C_rg*y('j7') +
          C_fm*y('j8') + C_rm*y('j9') + C_ts*y('j10') +
          C_fo*y('j11') + C_ro*y('j12'));
MB(k)..   sum(i,A(k,i)*x(i)) =e= b(k);                       ->Mass balances.
q1..      x('i1') - U*y('j1') =l= 0;
q2..      x('i7') - U*y('j5') =l= 0;
q3..      x('i10') - U*y('j5') =l= 0;
q4..      x('i11') - U*y('j4') =l= 0;
q5..      x('i5') - U*y('j6') =l= 0;
q6..      x('i6') - U*y('j8') =l= 0;
q7..      x('i15') - U*y('j11') =l= 0;
q8..      x('i16') - U*y('j8') =l= 0;
l1..      y('j4') + y('j5') =e= 1;
l2..      y('j6') + y('j8') =e= 1;
l3..      y('j5') - y('j8') =l= 0;
l4..      y('j1') - y('j2') =e= 0;
l5..      y('j2') - y('j3') =e= 0;
l6..      y('j3') - y('j4') =e= 0;
l7..      y('j6') - y('j7') =e= 0;
l8..      y('j7') - y('j11') =e= 0;
l9..      y('j11') - y('j12') =e= 0;
l10..     y('j8') - y('j9') =e= 0;
l11..     y('j9') - y('j10') =e= 0;

Model Subpro /all/;

Option optcr=0.0;         ->Zero optimality gap.
Option reslim=9E9;        ->Increased time limit.
option mip = cplex;

*Solving the model
Solve Subpro using MIP minimizing inv_cost;

display y.l;
```

A.2 Case II: Maximizing the net present vaue

```
$Title Maxiizing the NPV of a subsea separation system.

$Ontext
```

```
Optimization of the model presented in the report: "Superstructure optimization
of a subsea separation system".
Objective: Find the combination of process units that maximizes the NPV and
satisfies the the specifications of the process.
$Offtext

*Defining a symbol for an end of line comment.
$eolcom ->
*.

*Defining the sets
Set
         i mass flow index /i1*i17/
         j bin var index /j1*j12/
         k eq number /k1*k13/
         t years /1*10/;

*Parameters for the model
Parameters
         U upper limit for mass flows /300000/
         b(k) RHS coeff for mass balances /set.k 0/

*Early production
         G gas stream /12720/
         O oil stream /206200/

*Late production
*        G gas stream /929/
*        O oil stream /11580/


*Fixed costs for the units                                 Binary variables
*        C_c Cost for cooler /0.750855/                     -> y1.
         C_s Cost for separator /1.003096/                  -> y2.
*        C_k Cost for compressor /22.760726/                -> y3.
*        C_p Cost for pump /0.222857/                       -> y4.
         C_MP Cost for multiphase pump /16.000000/          -> y5.
         C_fg Cost for flowline gas /93.675000/             -> y6.
         C_rg Cost for riser gas /0.714000/                 -> y7.
         C_fm Cost for flowline multiphase /154.500000/     -> y8.
         C_rm Cost for riser multiphase /2.177700/          -> y9.
         C_ts Cost for toptside separator /0.462183/        -> y10.
         C_fo Cost for flowline oil /129.165000/            -> y11.
         C_ro Cost for riser oil /1.438200/                 -> y12.

*Economic factors
         f_inst installation factor /4.208/
         f_sub factor for installation subsea /3/
         f_I scaling economics for inflation /1.1035/
```

```
*compressor
        p1 pressure in  /6500/
        p2 pressure out /30500/
        gamma Cp over Cv /1.557488545/
        d_gas density /53.44791051/
        eff_k effectivity /0.75/

*oil pump
        d_oil density of oil /844/
        dp pressure difference /30000000/
        eff_p adiabatic efficiency pump /0.75/

*Multiphase pump
        H head converted from differential pressure /2713.25/
        eff_MP multiphase pump efficiency /0.75/

*Heat exchanger
        LMTD logarithmic mean temp difference /19.6/
        dT temperature difference (out-in) /2.5/
        Cp_g Heat capacity of gas /2681/
        U_h heat transfer coefficient /20/

*cost and plant data
        r discount rate /0.1/
        p_bbl price per barrel oil /57.30/
        p_g price natural gas /2.61/
        bbl_m3 barrels per cubic meter /6.29/
        MMBTU_m3 MMBTU per cubic meter /0.0354/
        h_y operating hours per year /8000/
        p_e price for electricity /0.09/;


*Assigning numbers to the vector
b('k1') = G;
b('k7') = -O;

Table A(k,i)
        i1  i2   i3  i4  i5  i6  i7  i8  i9  i10 i11 i12 i13 i14 i15 i16 i17
   k1   1   0    0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
   k2   1  -1    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   k3   0  0.05  0   0   0   0   0  -1   0   0   0   0   0   0   0   0   0
   k4   0  -1    1   0   0   0   0   1   0   0   0   0   0   0   0   0   0
   k5   0   0   -1   1   0   0   0   0   0   0   0   0   0   0   0   0   0
   k6   0   0    0  -1   1   1   0   0   0   0   0   0   0   0   0   0   0
   k7   0   0    0   0   0   0   1   1  -1   0   0   0   0   0   0   0   0
   k8   0   0    0   0   0   0   0   0  -1   1   1   0   0   0   0   0   0
   k9   0   0    0   0   0   0   0   0   0  -1   0   1   0   0   0   0   0
   k10  0   0    0   0   0   0   0   0   0   0  -1   0   1   0   0   0   0
   k11  0   0    0   0   0   0   0   0   0   0   0   1   1  -1   0   0   0
   k12  0   0    0   0   0   0   0   0   0   0   0   0   0  -1   1   1   0
   k13  0   0    0   0   0   1   0   0   0   0   0   0   0   0   0   1  -1;
```

```
*Variables
Positive variables x(i) mass flows;

Binary variables y(j);

Variable NPV net present value
         c0 investment cost
         CF cash flow
         P_k power consumption of compressor
         C_k cost of compressor
         P_p power consumption of pump
         C_p cost of pump
         C_pmo cost of pump motor
         P_MP power consumption multiphase pump
         Area heat exchange area
         C_c cost of cooler;


*Declaring equations
Equations
         obj objective function
         cinv investment costs
         rev revenue
         MB(k) mass balances
         pwr_k power consuption of compressor
         Ck cost of compressor
         pwr_p power consumption of pump
         Cp cost of pump
         Cpmo cost of pump motor
         pwr_MP power consumption multiphase pump
         Aeq Heat transfer area
         Cc cost of cooler
         q1 flow to zero
         q2 flow to zero
         q3 flow to zero
         q4 flow to zero
         q5 flow to zero
         q6 flow to zero
         q7 flow to zero
         q8 flow to zero
         l1 logic eq 1
         l2 logic eq 2
         l3 logic eq 3
         l4 logic eq 4
         l5 logic eq 5
         l6 logic eq 6
         l7 logic eq 7
         l8 logic eq 8
         l9 logic eq 9
         l10 logic eq 10
```

```
          l11 logic eq 11;

*Defining the equations
obj..     NPV =e= -C0 + sum(t,CF/((1+r)**ord(t)));
cinv..    C0 =e= C_c + C_s*y('j2') + C_k + C_p + C_MP* y('j5') + C_fg*y('j6')
           + C_rg*y('j7') + C_fm*y('j8') + C_rm*y('j9') + C_ts*y('j10') +
           C_fo*y('j11') + C_ro*y('j12');
rev..     CF =e= (1/1000000)*((O*bbl_m3*p_bbl/d_oil + G*MMBTU_m3*p_g/d_gas)*h_y
           - (P_k+P_p+P_MP)*h_y*p_e);
pwr_k..   P_k =e= p1*(x('i3')/(d_gas*3600))*(gamma/(gamma-1))*
          ((p2/p1)**((gamma-1)/gamma)-1)/eff_k;
Ck..      C_k =e= (1/1000000)*(490000*y('j3') + 16800*P_k**0.6)*
          f_inst*f_sub*f_I;
pwr_p..   P_p =e= dp*x('i11')/(d_oil*3600*eff_p*1000);
Cp..      C_p =e= C_pmo + (1/1000000)*(6900*y('j4') +
          206*(x('i11')/(d_oil*3.6))**0.9)*f_inst*f_sub*f_I;
Cpmo..    C_pmo =e= (1/1000000)*(-950*y('j4') + 1770*P_p**0.6)*
          f_inst*f_sub*f_I;
pwr_MP..  P_MP =e= x('i10')*9.81*h/(3600000*eff_MP);
Aeq..     Area =e= x('i1')*dT*Cp_g/(3600*U_h*LMTD);
Cc..      C_c =e= (1/1000000)*(24000*y('j1')+46*Area**1.2)*f_inst*f_sub*f_I;
MB(k)..   sum(i,A(k,i)*x(i)) =e= b(k);                         ->Mass balances.
q1..      x('i1') - U*y('j1') =l= 0;
q2..      x('i7') - U*y('j5') =l= 0;
q3..      x('i10') - U*y('j5') =l= 0;
q4..      x('i11') - U*y('j4') =l= 0;
q5..      x('i5') - U*y('j6') =l= 0;
q6..      x('i6') - U*y('j8') =l= 0;
q7..      x('i15') - U*y('j11') =l= 0;
q8..      x('i16') - U*y('j8') =l= 0;
l1..      y('j4') + y('j5') =e= 1;
l2..      y('j6') + y('j8') =e= 1;
l3..      y('j5') - y('j8') =l= 0;
l4..      y('j1') - y('j2') =e= 0;
l5..      y('j2') - y('j3') =e= 0;
l6..      y('j3') - y('j4') =e= 0;
l7..      y('j6') - y('j7') =e= 0;
l8..      y('j7') - y('j11') =e= 0;
l9..      y('j11') - y('j12') =e= 0;
l10..     y('j8') - y('j9') =e= 0;
l11..     y('j9') - y('j10') =e= 0;

C_pmo.up = 1000;
Model Subpro /all/;

Option optcr=0.0;         ->Zero optimality gap.
Option reslim=9E9;        ->Increased time limit.

*option sysout = on ;
option minlp = dicopt;
*option nlp = conopt;
```

```
*option mip = gurobi;

*solving the model
Solve Subpro using MINLP maximizing NPV;

display y.l;
```

A.3 Mass balances

Table 12: Mass flows in t/h calculated by the model implemented in GAMS

	Case I	Case II	
Flow		Early production	Late production
G [11]	12.720	12.720	0.929
O [11]	206.200	206.200	11.580
x_1	0	0	0.929
x_2	0	0	0.929
x_3	0	0	0.882
x_4	0	0	0.882
x_5	0	0	0
x_6	0	0	0.882
x_7	12.720	12.720	0
x_8	0	0	0.046
x_9	218.920	218.920	11.626
x_{10}	218.920	218.920	0
x_{11}	0	0	11.626
x_{12}	218.920	218.920	0
x_{13}	0	0	11.626
x_{14}	218.920	218.920	11.626
x_{15}	0	0	0
x_{16}	218.920	218.920	11.626
x_{17}	218.920	218.920	12.509