TFY4205 Quantum Mechanics II
Problemset 7 fall 2022

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SUGGESTED SOLUTION

Problem 1

When the spin part of the wavefunction is symmetric for identical fermions, the orbital part must be antisymmetric. Then, the differential scattering cross section is

$$\frac{d\sigma}{d\Omega} = |f(\vartheta) - f(\pi - \vartheta)|^2. \tag{1}$$

At low energies, only the partial waves with the lowest l-values contribute to the general expression

$$f(\vartheta) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l P_l(\cos\vartheta). \tag{2}$$

Since $P_0 = 1$, independent of the angle, the contribution from $l = 0$ to $f(\vartheta) - f(\pi - \vartheta)$ vanishes. The dominant contribution will therefore come from $l = 1$. We need that $P_1[\cos\vartheta] = \cos\vartheta$ and thus $P_1[\cos(\pi - \vartheta)] = -\cos\vartheta$. The angle dependence of the differential scattering cross section is therefore:

$$\frac{d\sigma}{d\Omega} \propto \cos^2\vartheta. \tag{3}$$

Problem 2

We have $\boldsymbol{p} + e\boldsymbol{A} = -i\hbar\nabla + e\boldsymbol{A}$. Let us first differentiate a product of the form:

$$\nabla(e^{-ie^2t/(4\pi\varepsilon_0\hbar r)}F) = e^{-ie^2t/(4\pi\varepsilon_0\hbar r)}(\nabla F + \frac{ie^2t}{4\pi\varepsilon_0\hbar}\frac{\boldsymbol{r}}{r^3}F), \tag{4}$$

where F is a function of space and time. Thus,

$$(\boldsymbol{p} + e\boldsymbol{A})e^{-ie^2t/(4\pi\varepsilon_0\hbar r)}F = e^{-ie^2t/(4\pi\varepsilon_0\hbar r)}\boldsymbol{p}F \tag{5}$$

and

$$(\boldsymbol{p} + e\boldsymbol{A})^2 e^{-ie^2t/(4\pi\varepsilon_0\hbar r)}F = (\boldsymbol{p} + e\boldsymbol{A})e^{-ie^2t/(4\pi\varepsilon_0\hbar r)}\boldsymbol{p}F = e^{-ie^2t/(4\pi\varepsilon_0\hbar r)}\boldsymbol{p}^2F. \tag{6}$$

Setting $\Psi = e^{-ie^2t/(4\pi\varepsilon_0\hbar r)}\tilde{\Psi}$ in the Schrodinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{(\boldsymbol{p} + e\boldsymbol{A})^2}{2m}\Psi \tag{7}$$

we get, with the help of Eq. (**??**) and the elimination of the exponential functions on both sides:

$$i\hbar\frac{\partial\tilde{\Psi}}{\partial t} + \frac{e^2}{4\pi\varepsilon_0 r}\tilde{\Psi} = \frac{\boldsymbol{p}^2}{2m}\tilde{\Psi}, \tag{8}$$

which is what we are used to. It is clear that Ψ and $\tilde{\Psi}$ provide the same probability density for position and they also provide the same probability current density (you can check this explicitly). Hence, the physics is unchanged.