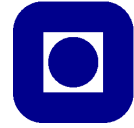


TFY4205 Quantum Mechanics II

NTNU

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Institutt for fysikk

SUGGESTED SOLUTION

Problem 1

When the spin part of the wavefunction is symmetric for identical fermions, the orbital part must be antisymmetric. Then, the differential scattering cross section is

$$\frac{d\sigma}{d\Omega} = |f(\vartheta) - f(\pi - \vartheta)|^2. \quad (1)$$

At low energies, only the partial waves with the lowest l -values contribute to the general expression

$$f(\vartheta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin \delta_l P_l(\cos \vartheta). \quad (2)$$

Since $P_0 = 1$, independent of the angle, the contribution from $l = 0$ to $f(\vartheta) - f(\pi - \vartheta)$ vanishes. The dominant contribution will therefore come from $l = 1$. We need that $P_1[\cos \vartheta] = \cos \vartheta$ and thus $P_1[\cos(\pi - \vartheta)] = -\cos \vartheta$. The angle dependence of the differential scattering cross section is therefore:

$$\frac{d\sigma}{d\Omega} \propto \cos^2 \vartheta. \quad (3)$$

Problem 2

We have $\mathbf{p} + e\mathbf{A} = -i\hbar\nabla + e\mathbf{A}$. Let us first differentiate a product of the form:

$$\nabla(e^{-ie^2t/(4\pi\epsilon_0\hbar r)} F) = e^{-ie^2t/(4\pi\epsilon_0\hbar r)} (\nabla F + \frac{ie^2t}{4\pi\epsilon_0\hbar} \frac{\mathbf{r}}{r^3} F), \quad (4)$$

where F is a function of space and time. Thus,

$$(\mathbf{p} + e\mathbf{A})e^{-ie^2t/(4\pi\epsilon_0\hbar r)} F = e^{-ie^2t/(4\pi\epsilon_0\hbar r)} \mathbf{p}F \quad (5)$$

and

$$(\mathbf{p} + e\mathbf{A})^2 e^{-ie^2t/(4\pi\epsilon_0\hbar r)} F = (\mathbf{p} + e\mathbf{A})e^{-ie^2t/(4\pi\epsilon_0\hbar r)} \mathbf{p}F = e^{-ie^2t/(4\pi\epsilon_0\hbar r)} \mathbf{p}^2 F. \quad (6)$$

Setting $\Psi = e^{-ie^2t/(4\pi\epsilon_0\hbar r)} \tilde{\Psi}$ in the Schrodinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{(\mathbf{p} + e\mathbf{A})^2}{2m} \Psi \quad (7)$$

we get, with the help of Eq. (??) and the elimination of the exponential functions on both sides:

$$i\hbar \frac{\partial \tilde{\Psi}}{\partial t} + \frac{e^2}{4\pi\epsilon_0 r} \tilde{\Psi} = \frac{\mathbf{p}^2}{2m} \tilde{\Psi}, \quad (8)$$

which is what we are used to. It is clear that Ψ and $\tilde{\Psi}$ provide the same probability density for position and they also provide the same probability current density (you can check this explicitly). Hence, the physics is unchanged.