TFY4205 Quantum Mechanics II
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NTNU
Institutt for fysikk

SUGGESTED SOLUTION

Problem 1

Proof by contradiction. Assume that $|W_n\rangle$ is not entangled. Then we can find coefficients $a_1,...,a_n,b_1,...,b_n$ such that:

$$(a_1|0\rangle+b_1|1\rangle)\otimes(a_2|0\rangle+b_2|1\rangle)\otimes...\otimes(a_n|0\rangle+b_n|1\rangle)=\frac{1}{\sqrt{n}}\big(|0...001\rangle+|0...010\rangle+|0...100\rangle+...+|1...000\rangle\big) \tag{1}$$

Expanding the left-hand side and comparing to the right-hand side, we can see that the coefficients must have the following relationship:

$$a_1...a_{n-1}b_n = a_1...b_{n-1}a_n = ... = b_1...a_{n-1}a_n = \frac{1}{\sqrt{n}} \tag{2}$$

Clearly, for this to be the case, all of the a_i and b_i must be non-zero for $i=1,...,n$. But if all of these coefficients were non-zero, then every possible n-qubit state would appear on the right-hand side of (1) with a non-zero coefficient.

However, we see that, for instance, the state $|1...111\rangle$ does not appear on the right-hand side of (1). This implies that one of the b_i's must be zero. This is a contradiction to what we stated earlier. Therefore the state $|W_n\rangle$ is entangled.

Problem 2

We are faced with the task of computing the correlation functions for the given density matrix describing the total system. For example, we have that $E(P,R)=\mathrm{Tr}(\rho P\otimes R)$. Computing the four correlations functions that comprise the total CHSH-like quantity, we get:

$$E(P,R) = p-1 \tag{3}$$

$$E(Q,R) = \frac{p-1}{\sqrt{2}} \tag{4}$$

$$E(P,S) = \frac{p-1}{\sqrt{2}} \tag{5}$$

$$E(Q,S) = 0 \tag{6}$$

Let us show the calculation for $E(P,R)$ in detail: the procedure for the remaining expectation values is similar.

We have

$$\mathrm{Tr}(\rho P\otimes R) = \sum_{n_A,n_B,n'_A,n'_B} \langle n_A n_B|\rho|n'_A n'_B\rangle\langle n'_A n'_B|\sigma_z^A\sigma_z^B|n_A n_B\rangle \tag{7}$$

by inserting a completeness relation

$$\sum_{n'_A,n'_B} |n'_A n'_B\rangle\langle n'_A n'_B| = 1 \tag{8}$$

and where $n_A, n_B = \{0,1\}$. Above, we have omitted the $\otimes$ symbol for brevity of notation. Inserting the given $\hat{\rho}$, we obtain:

$$\begin{aligned}
\mathrm{Tr}(\rho P\otimes R) = &\sum_{n_A,n_B,n'_A,n'_B} \left[\frac{p}{4}\delta_{n_A,n'_A}\delta_{n_B,n'_B} + \frac{1-p}{2}\langle n_A n_B|(|0^A\rangle|1^B\rangle - |1^A\rangle|0^B\rangle)(\langle 0^A|\langle 1^B| - \langle 1^A|\langle 0^B|)n'_A n'_B\rangle\right] \\
&\times[\delta_{n_A,n'_A}(\delta_{n_A,0}-\delta_{n_A,1})\delta_{n_B,n'_B}(\delta_{n_B,0}-\delta_{n_B,1})] \\
&= \sum_{n_A,n_B}[0 + \frac{1-p}{2}(\delta_{n_A,0}\delta_{n_B,1}+\delta_{n_A,1}\delta_{n_B,0})(\delta_{n_A,0}\delta_{n_B,0}-\delta_{n_A,0}\delta_{n_B,1}-\delta_{n_A,1}\delta_{n_B,0}+\delta_{n_A,1}\delta_{n_B,1})] \\
&= \frac{1-p}{2}(-1-1) = p-1.
\end{aligned} \tag{9}$$

Therefore, the CHSH-like quantity is equal to

$$\frac{\sqrt{2}+2}{\sqrt{2}}(1-p). \tag{10}$$

This violates the CHSH bound (is equal to >2) when $p < 0.1716$. Thus, we see that we do not have to deviate much from the maximally entangled state before failing to satisfy the CHSH inequality.