FY3464 Quantum Field Theory
Problemset 12

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SUGGESTED SOLUTION

Problem 1

The starting assumption is that $k_1 \neq -k_2$. In the case $n = m = 1$, we start with

$$\int \prod_{j=1}^{2} \frac{k_j^2 - m^2 + i\varepsilon}{i} e^{-ik_j y_j} \langle T\{\phi(y_1)\phi(y_2)\phi(x_1)\phi(x_2)\}\rangle_{\text{free}}. \tag{1}$$

Using Wick's theorem, this equals

$$\int \prod_{j=1}^{2} \frac{k_j^2 - m^2 + i\varepsilon}{i} e^{-ik_j y_j} = [\overbrace{\langle : \phi(y_1)\phi(y_2)\phi(x_1)\phi(x_2) : \rangle_{\text{free}}}^{=0 \text{ since } :ABC\ldots:|0\rangle_{\text{free}}=0} + \overbrace{\text{terms with two fields contracted}}^{=0 \text{ for the same reason}}$$
$$+ G_F(y_1 - y_2)G_F(x_1 - x_2) + G_F(y_1 - x_1)G_F(y_2 - x_2) + G_F(y_1 - x_2)G_F(y_2 - x_1). \tag{2}$$

Looking back at our definition of $G_F(x-y)$, we see that it satisfies $G_F(x-y) = G_F(y-x)$. Therefore, we have that Eq. (2) may be written

$$[\int \prod_{j=1}^{2} d^4y_j \frac{k_j^2 - m^2 + i\varepsilon}{i} e^{-ik_j y_j} G_F(y_1 - y_2)G_F(x_1 - x_2)] + e^{-i(k_1x_1 + k_2x_2)} + e^{-i(k_1x_2 + k_2x_1)}. \tag{3}$$

by using that

$$\tilde{G}_F(k) = \int d^4y e^{iky} G_F(y) = \frac{i}{k^2 - m^2 + i\varepsilon}. \tag{4}$$

Thus, Eq. (3) gives the desired result if we can prove that the first term (the integral) is zero. We solve this integral by shifting measure from $d^4y_1d^4y_2$ to $d^4yd^4y_2$ with $y = y_1 - y_2$ being the relative coordinate. The Jacobian of this transformation is

$$\det \begin{vmatrix} \partial y/\partial y_1 & \partial y/\partial y_2 \\ \partial y_2/\partial y_1 & \partial y_2/\partial y_1 \end{vmatrix} = \det \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1, \tag{5}$$

so $d^4y_1d^4y_2 = d^4yd^4y_2$. We thus obtain that the first term in Eq. (3) is

$$\begin{aligned}
&\int\int d^4yd^4y_2 \frac{k_1^2 - m^2 + i\varepsilon}{i} \frac{k_2^2 - m^2 + i\varepsilon}{i} G_F(y) e^{-ik_1y_1 - ik_2y_2} G_F(x_1 - x_2) \\
&= \int\int d^4yd^4y_2 \frac{k_1^2 - m^2 + i\varepsilon}{i} \frac{k_2^2 - m^2 + i\varepsilon}{i} G_F(y) e^{-ik_1(y_1 - y_2) - iy_2(k_1 + k_2)} G_F(x_1 - x_2) \\
&= [\int d^4y_2 \tilde{G}_F(-k_1) e^{-iy_2(k_1 + k_2)}] G_F(x_1 - x_2) \\
&= (2\pi)^4 \delta(k_1 + k_2) G_F(x_1 - x_2) \tilde{G}_F(-k_1).
\end{aligned} \tag{6}$$

This is zero since our initial assumption was that $k_1 \neq -k_2$, which completes the proof.