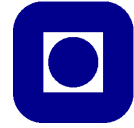


FY3464 Quantum Field Theory

Problemset 8

NTNU



Institutt for fysikk

Problem 1

By applying N successive infinitesimal Lorentz transformations, letting $N \rightarrow \infty$, and using

$$\frac{i}{2}\epsilon_{\nu\lambda}[S^{\nu\lambda}, \gamma^\mu] = -\epsilon_{\nu}^{\mu}\gamma^\nu, \quad (1)$$

show that

$$\Lambda_{\nu}^{\mu}\gamma^\nu = U_{\gamma}^{-1}(\Lambda)\gamma^\mu U_{\gamma}(\Lambda), \quad (2)$$

with $U_{\gamma}(\Lambda) = e^{i\omega_{\mu\nu}S^{\mu\nu}/2}$ and where Λ_{ν}^{μ} now represents a finite Lorentz transformation. How is $\omega_{\mu\nu}$ related to the N and the infinitesimal Lorentz transformation?

Problem 2

In the lectures, we showed that the solution for the Dirac equation for $\mathbf{k} \neq 0$ can be obtained via a boost matrix $U_{\gamma}(\Lambda)$. Assuming that the boost-direction was $+\hat{z}$, we found that

$$u(\mathbf{k}) = U_{\gamma}(\Lambda)\sqrt{m} \begin{pmatrix} \xi \\ \xi \end{pmatrix} \quad (3)$$

where ξ is an arbitrary 2×1 spinor and

$$U_{\gamma}(\Lambda) = e^{-i\eta S^{03}} = \exp\left[-\frac{1}{2}\eta \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}\right] \quad (4)$$

Show that the solution

$$u(\mathbf{k}) = \begin{pmatrix} \sqrt{k \cdot \sigma} \xi \\ \sqrt{k \cdot \bar{\sigma}} \xi \end{pmatrix} \quad (5)$$

follows from the above equations and that it is valid for any boost direction, in effect any direction of \mathbf{k} . Above, \sqrt{A} should be understood as a matrix whose eigenvalues are the square root of the eigenvalues of A .

Hint: $U_{\gamma}(\Lambda)$ is a diagonal matrix. Square its blocks to see what it produces. It is also useful to note that $k \cdot \sigma$ is a Hermitian (self-adjoint) matrix. By the way, this problem is tough - don't despair if you can't solve it entirely. A key purpose with this problem is to make you think hard about how you can approach it.