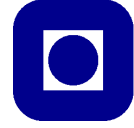


# FY3464 Quantum Field Theory

## Problemset 7

NTNU



Institutt for fysikk

### Problem 1

a) Prove that for a general, infinitesimal Lorentz-transformation

$$\Delta_{\nu}^{\mu} = \delta_{\nu}^{\mu} - \epsilon_{\nu}^{\mu} \quad (1)$$

the tensor  $\epsilon$  has to be antisymmetric:  $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}$ .

b) Prove that one can use an infinitesimal version of a Lorentz transformation:

$$x^{\mu} \rightarrow (x')^{\mu} = x^{\mu} + \epsilon_{\nu}^{\mu} x^{\nu} = x^{\mu} - \frac{i}{2} \epsilon_{\nu\lambda} J^{\nu\lambda} x^{\mu}, \quad (2)$$

to write a *finite* Lorentz transformation in exponential form as

$$x^{\mu} \rightarrow x'^{\mu} = U^{-1}(\Lambda) x^{\mu}, \quad (3)$$

where

$$U(\Lambda) = e^{i\omega_{\mu\nu} J^{\mu\nu}/2}. \quad (4)$$

Above, we have defined

$$J^{\mu\nu} = i(x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu}). \quad (5)$$

c) Identify what  $\omega_{\mu\nu}$  is in terms of  $\epsilon_{\nu}^{\mu}$ .

### Problem 2

Prove that

$$S^{\mu\nu} = \frac{i}{4}(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}) \quad (6)$$

satisfies the Lorentz algebra:

$$[S^{\mu\lambda}, S^{\nu\sigma}] = i(\eta^{\lambda\nu} S^{\mu\sigma} - \eta^{\mu\nu} S^{\lambda\sigma} - \eta^{\lambda\sigma} S^{\mu\nu} + \eta^{\mu\sigma} S^{\lambda\nu}). \quad (7)$$