FY3464 Quantum Field Theory
Problemset 7

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Problem 1
a) Prove that for a general, infinitesimal Lorentz-transformation

$$\Delta^{\mu}{}_{\nu} = \delta^{\mu}_{\nu} - \varepsilon^{\mu}{}_{\nu} \tag{1}$$

the tensor ε has to be antisymmetric: $\varepsilon_{\mu\nu} = -\varepsilon_{\nu\mu}$.

b) Prove that one can use an infinitesimal version of a Lorentz transformation:

$$x^{\mu} \to (x')^{\mu} = x^{\mu} + \varepsilon_{\nu}{}^{\mu} x^{\nu} = x^{\mu} - \frac{i}{2}\varepsilon_{\nu\lambda} J^{\nu\lambda} x^{\mu}, \tag{2}$$

to write a *finite* Lorentz transformation in exponential form as

$$x^{\mu} \to x^{\mu'} = U^{-1}(\Lambda) x^{\mu}, \tag{3}$$

where

$$U(\Lambda) = e^{i\omega_{\mu\nu} J^{\mu\nu}/2}. \tag{4}$$

Above, we have defined

$$J^{\mu\nu} = \mathrm{i}(x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu}). \tag{5}$$

c) Identify what $\omega_{\mu\nu}$ is in terms of ε^{μ}_{ν}.

Problem 2
Prove that

$$S^{\mu\nu} = \frac{\mathrm{i}}{4}(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}) \tag{6}$$

satisfies the Lorentz algebra:

$$[S^{\mu\lambda}, S^{\nu\sigma}] = \mathrm{i}(\eta^{\lambda\nu} S^{\mu\sigma} - \eta^{\mu\nu} S^{\lambda\sigma} - \eta^{\lambda\sigma} S^{\mu\nu} + \eta^{\mu\sigma} S^{\lambda\nu}). \tag{7}$$