FY3464 Quantum Field Theory

Problemset 6

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Problem 1

We have discussed that some Lagrangians have global symmetries, such as a U(1) symmetry $\phi \to \phi e^{i\theta}$ where θ is a real number. If the Lagrangian includes a gauge-field (such as the magnetic vector potential), it can also have a gauge symmetry where it is invariant under $\boldsymbol{A} \to \boldsymbol{A} + \nabla\chi$ where χ is a scalar function of position and time. Is there any difference in the physical meaning of global symmetries and gauge symmetries? *Hint:* how do the equations of motion and the physical properties of the system change after a global vs. gauge symmetry transformation?

Problem 2

In the lectures, we considered in great detail the two-loop sunset Feynman diagram in scalar theory. Our first approach was to do a double Wick rotation and then use Feynman parametrization. This gaves us the expression:

$$\frac{i\lambda^2}{6}\int\int\frac{d^4l_{1E}d^4l_{2E}}{(2\pi)^4(2\pi)^4}\int_0^\infty d\rho\rho^2\int_0^1\int_0^1\int_0^1 dx_1dx_2dx_3$$
$$\times\,\delta(1-x_1-x_2-x_3)\exp[-\rho(x_1l_{1E}^2+x_2l_{2E}^2+x_3(k_E-l_{1E}-l_{2E})^2+m^2]. \tag{1}$$

before doing the dimensional regularization. Then, we did a new derivation (which was without any leaps of faith in the form of assuming that we could do the Wick rotations to begin with) by instead *first* using the Feynman parametrization and aiming to apply the Wick rotations *afterwards*. In this case, we arrived at

$$\frac{i\lambda^2}{6}\int_0^1\int_0^1\int_0^1 dxdydz\delta(x+y+z-1)\int\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\frac{2}{(\alpha k_1^2+\beta k_2^2+\gamma k^2-m^2)^3}. \tag{2}$$

after doing the Feynman parametrization, but before doing the Wick rotations. In both the above expressions, we've absorbed a $-i\varepsilon$ into m^2.

Prove that it is permitted to do Wick rotations in Eq. (2), perform the rotations, and show that the resulting equation can in fact be rewritten to exactly Eq. (1), thus justifying *a posteriori* our approach where the Wick rotations were done before the Feynman parametrization.