FY3464 Quantum Field Theory
Problemset 4

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Problem 1
In the lectures, we derived that the amplitude for a particle starting at $\boldsymbol{x}, t_1$ and ending up at $\boldsymbol{x}_2, t_2$ is:

$$\langle \boldsymbol{x}_2, t_2 | \boldsymbol{x}_1, t_1 \rangle = \left(\frac{m}{2\pi i \hbar \Delta t}\right)^{3N/2} \int \prod_{t_1 < t < t_2} d\boldsymbol{x}(t) e^{iS/\hbar}. \tag{1}$$

The path integral formalism can also be used to compute other transition amplitudes besides going from one position state to another, such as transitions between different states of the system $|\psi, t\rangle$. More specifically, to find the system in a state $|\psi_2\rangle$ at t_2 when we know that it is in state $|\psi_1\rangle$ at t_1, we need to compute the overlap amplitude $\langle \psi_2 | \Psi, t_2 \rangle$ where $|\Psi, t_1\rangle = |\psi_1\rangle$.

Now, defining $|\psi_j, t_j\rangle \equiv e^{iHt_j} |\psi_j\rangle$, derive that

$$\langle \psi_2, t_2 | \psi_1, t_1 \rangle = \left(\frac{m}{2\pi i \hbar \Delta t}\right)^{3N/2} \int \prod_{t_1 \leq t \leq t_2} d\boldsymbol{x}(t) \Big(\psi_2[\boldsymbol{x}(t_2)]\Big)^* \psi_1[\boldsymbol{x}(t_1)] e^{iS/\hbar}. \tag{2}$$

Note in particular the inclusion of t_1 and t_2 in the product over states.

Hint: a completeness relation is often handy.

Problem 2
Prove that the partition function (set the overall prefactor $C = 1$)

$$Z = \int \mathcal{D}\phi e^{iS} = \int \mathcal{D}\phi \exp\left[i \int dt \frac{1}{2}(\partial_0 \phi)^2 - \frac{1}{2}\omega_0^2 \phi^2 + i\varepsilon\phi^2 + J\phi\right] \tag{3}$$

can be written as

$$Z = Z(J = 0) \times e^{-\frac{1}{2}\int\int dt dt' J(t) G_F(t-t') J(t')} \tag{4}$$

where $G_F(t - t')$ is the scalar field Green function in the 0+1 dimensional case.