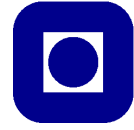


# FY3464 Quantum Field Theory

## Problemset 4

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### Problem 1

In the lectures, we derived that the amplitude for a particle starting at  $\mathbf{x}_1, t_1$  and ending up at  $\mathbf{x}_2, t_2$  is:

$$\langle \mathbf{x}_2, t_2 | \mathbf{x}_1, t_1 \rangle = \left( \frac{m}{2\pi i \hbar \Delta t} \right)^{3N/2} \int \prod_{t_1 < t < t_2} d\mathbf{x}(t) e^{iS/\hbar}. \quad (1)$$

The path integral formalism can also be used to compute other transition amplitudes besides going from one position state to another, such as transitions between different states of the system  $|\psi, t\rangle$ . More specifically, to find the system in a state  $|\psi_2\rangle$  at  $t_2$  when we know that it is in state  $|\psi_1\rangle$  at  $t_1$ , we need to compute the overlap amplitude  $\langle \psi_2 | \Psi, t_2 \rangle$  where  $|\Psi, t_1\rangle = |\psi_1\rangle$ .

Now, defining  $|\psi_j, t_j\rangle \equiv e^{iHt_j} |\psi_j\rangle$ , derive that

$$\langle \Psi_2, t_2 | \Psi_1, t_1 \rangle = \left( \frac{m}{2\pi i \hbar \Delta t} \right)^{3N/2} \int \prod_{t_1 \leq t \leq t_2} d\mathbf{x}(t) \left( \Psi_2[\mathbf{x}(t_2)] \right)^* \Psi_1[\mathbf{x}(t_1)] e^{iS/\hbar}. \quad (2)$$

Note in particular the inclusion of  $t_1$  and  $t_2$  in the product over states.

*Hint:* a completeness relation is often handy.

### Problem 2

Prove that the partition function (set the overall prefactor  $C = 1$ )

$$Z = \int \mathcal{D}\phi e^{iS} = \int \mathcal{D}\phi \exp \left[ i \int dt \frac{1}{2} (\partial_0 \phi)^2 - \frac{1}{2} \omega_0^2 \phi^2 + i\epsilon \phi^2 + J\phi \right] \quad (3)$$

can be written as

$$Z = Z(J=0) \times e^{-\frac{1}{2} \int \int dt dt' J(t) G_F(t-t') J(t')} \quad (4)$$

where  $G_F(t-t')$  is the scalar field Green function in the 0+1 dimensional case.