FY3464 Quantum Field Theory
Problemset 3

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Problem 1
The Feynman propagator for free scalar field theory is

$$G_F(x-y) = \langle T\{\phi(x)\phi(y)\}\rangle \tag{1}$$

where

$$\langle\phi(x)\phi(y)\rangle = \int \frac{d^3k}{(2\pi)^3 2\omega(\boldsymbol{k})} \mathrm{e}^{-\mathrm{i}k(x-y)}. \tag{2}$$

Prove that

$$(\partial^2 + m^2) G_F(x-y) = -\mathrm{i}\delta^4(x-y). \tag{3}$$

Problem 2
Consider a Lagrangian $\mathcal{L} = \mathcal{L}(\{\phi_\alpha\})$ which depends on a number of fields ϕ_α. Assume that $\mathcal{L}$ has no explicit dependence on the 4-position vector:

$$\partial\mathcal{L}/\partial x_\nu = 0. \tag{4}$$

It is then possible to derive a stress-energy tensor $T^{\mu\nu}$ which is divergence-free:

$$d_\mu T^{\mu\nu} = 0 \tag{5}$$

by combining Eq. (4) and the Euler-Lagrange equations. In fact, the quantity $T^{\mu\nu}$ is the conserved current that follows from Noether's theorem for space-time translational symmetry. Do this to identify an expression for the stress-energy tensor in terms of the Lagrangian and derivatives of it.

Note that an alternative way of deriving $T^{\mu\nu}$ is to use Noether's theorem and consider the transformation of the fields ϕ_α and $\mathcal{L}$ under space-time translation when the system has space-time translational symmetry. The conserved tensor (vector of Noether currents) arising is then precisely $T^{\mu\nu}$.