FY3464 Quantum Field Theory

Problemset 2

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Problem 1

Let $f(z) = \sqrt{z^2+1}$. Use the technique we showed in the lectures to identify the branch-points of this function. Is $z \to \infty$ a branch point? If so, why?

Problem 2

The correlator $\langle \phi(x)\phi(y)\rangle$ for a scalar field is Lorentz-invariant:

$$\langle \phi(x)\phi(y)\rangle = \int \frac{d^3k}{(2\pi)^3 2\omega(\boldsymbol{k})} \mathrm{e}^{-\mathrm{i}k(x-y)}. \tag{1}$$

Consider a timelike separation $(x-y)^2 > 0$, choose a frame $\boldsymbol{x} = \boldsymbol{y}$ and assume that $x^0 - y^0 > 0$.

Taking all the above into account, use the Lorentz-invariance to prove that in the limit $m|x-y| \gg 1$ (corresponding to a heavy scalar field):

$$\langle \phi(x)\phi(y)\rangle \simeq \frac{1}{4\pi^2}\left(\frac{m\pi\mathrm{i}}{2|x-y|^3}\right)^{1/2} \mathrm{e}^{-\mathrm{i}m|x-y|} \tag{2}$$