FY3464 Quantum Field Theory
Problemset 10

NTNU

Institutt for fysikk

Problem 1
Prove that the free Dirac Lagrangian is invariant under the Lorentz-transformation:

$$x \to x',\ \psi(x) \to \psi'(x'), \tag{1}$$

where we showed in the lectures that $\psi'(x') = U_\gamma(\Lambda)\psi(x)$.

Problem 2
The following transformation of the field alone:

$$\psi(x) \to \psi'(x) \tag{2}$$

is also a symmetry transformation of the free Dirac Lagrangian. In effect, it causes

$$\mathcal{L} \to \mathcal{L} + \delta\mathcal{L} \tag{3}$$

where $\delta\mathcal{L}$ can be written as a total divergence.

Derive the conserved Noether-current that exists due to the symmetry transformation in Eq. (2).

Hint: the derivation of the Noether-current is quite long. An intermediate result, to check that you're on the right track, is the following. Upon transforming $\psi(x) \to \psi'(x)$ so that $\mathcal{L}(x) \to \mathcal{L}(x')$, and using that $\psi(Ux) = U\psi(x)$ for an infinitesimal Lorentz-transformation $U(\Lambda)$ in order to cancel several factors, you should get:

$$\begin{aligned}\mathcal{L}'(x) &= \mathrm{i}\bar{\psi}\partial\!\!\!/\psi - \bar{\psi}m\psi \\ &+ [-\frac{\mathrm{i}}{2}\varepsilon_{\mu\nu}(x^\mu\partial^\nu - x^\nu\partial^\mu)\bar{\psi}\delta^\lambda_\sigma\gamma^\sigma\partial_\lambda\psi + \mathrm{i}\bar{\psi}\varepsilon_\nu{}^\mu\gamma^\nu\partial_\mu\psi \\ &+ \mathrm{i}\bar{\psi}\delta^\lambda_\sigma\gamma^\sigma\partial_\lambda(-\frac{1}{2})\varepsilon_{\mu\nu}(x^\mu\partial^\nu - x^\nu\partial^\mu)\psi + \frac{m}{2}\varepsilon_{\mu\nu}(x^\mu\partial^\nu - x^\nu\partial^\mu)\bar{\psi}\cdot\psi + \frac{m}{2}\bar{\psi}\varepsilon_{\mu\nu}(x^\mu\partial^\nu - x^\nu\partial^\mu)\psi].\end{aligned} \tag{4}$$

This expression can then be cleaned up (a lot) to arrive at the desired result.