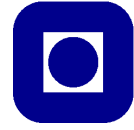


FY3464 Quantum Field Theory

Problemset 10

NTNU



Institutt for fysikk

Problem 1

Prove that the free Dirac Lagrangian is invariant under the Lorentz-transformation:

$$x \rightarrow x', \quad \psi(x) \rightarrow \psi'(x'), \quad (1)$$

where we showed in the lectures that $\psi'(x') = U_\gamma(\Lambda)\psi(x)$.

Problem 2

The following transformation of the field alone:

$$\psi(x) \rightarrow \psi'(x) \quad (2)$$

is also a symmetry transformation of the free Dirac Lagrangian. In effect, it causes

$$\mathcal{L} \rightarrow \mathcal{L} + \delta\mathcal{L} \quad (3)$$

where $\delta\mathcal{L}$ can be written as a total divergence.

Derive the conserved Noether-current that exists due to the symmetry transformation in Eq. (2).

Hint: the derivation of the Noether-current is quite long. An intermediate result, to check that you're on the right track, is the following. Upon transforming $\psi(x) \rightarrow \psi'(x)$ so that $\mathcal{L}(x) \rightarrow \mathcal{L}(x')$, and using that $\psi(Ux) = U\psi(x)$ for an infinitesimal Lorentz-transformation $U(\Lambda)$ in order to cancel several factors, you should get:

$$\begin{aligned} \mathcal{L}'(x) = & i\bar{\psi}\partial\psi - \bar{\psi}m\psi \\ & + \left[-\frac{i}{2}\epsilon_{\mu\nu}(x^\mu\partial^\nu - x^\nu\partial^\mu)\bar{\psi}\delta_\sigma^\lambda\gamma^\sigma\partial_\lambda\psi + i\bar{\psi}\epsilon_\nu{}^\mu\gamma^\nu\partial_\mu\psi \right. \\ & \left. + i\bar{\psi}\delta_\sigma^\lambda\gamma^\sigma\partial_\lambda\left(-\frac{1}{2}\right)\epsilon_{\mu\nu}(x^\mu\partial^\nu - x^\nu\partial^\mu)\psi + \frac{m}{2}\epsilon_{\mu\nu}(x^\mu\partial^\nu - x^\nu\partial^\mu)\bar{\psi}\cdot\psi + \frac{m}{2}\bar{\psi}\epsilon_{\mu\nu}(x^\mu\partial^\nu - x^\nu\partial^\mu)\psi\right]. \quad (4) \end{aligned}$$

This expression can then be cleaned up (a lot) to arrive at the desired result.