# FY3464 Quantum Field Theory
# Problemset 1

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**Problem 1**
Show that the requirement of invariant length squared of a 4-vector leads to the requirement

$$\eta_{\alpha\beta} = \eta_{\mu\nu}\Lambda^{\mu}{}_{\alpha}\Lambda^{\nu}{}_{\beta}. \tag{1}$$

for the Lorentz-transformation $\Lambda$. Then, use this to show that

$$(\Lambda^{-1})^{\sigma}{}_{\rho} = \Lambda_{\rho}{}^{\sigma} \tag{2}$$

by making use of the fact that the inverse of a tensor $A$ is given by:

$$(A^{-1})^{\sigma}{}_{\rho}A^{\rho}{}_{\mu} = \delta^{\sigma}_{\mu}. \tag{3}$$

**Problem 2**
Show that

$$H = \int \frac{d^3k}{(2\pi)^3}\omega(\boldsymbol{k})a^{\dagger}_{\boldsymbol{k}}a_{\boldsymbol{k}} \tag{4}$$

acting on the Fock space states

$$|\boldsymbol{k}_1, \boldsymbol{k}_2, ...\boldsymbol{k}_n\rangle = \prod_{i=1}^{n}\sqrt{2\omega(\boldsymbol{k}_i)}a^{\dagger}_{\boldsymbol{k}_i}|0\rangle \tag{5}$$

produces

$$H|\boldsymbol{k}_1, \boldsymbol{k}_2, ...\boldsymbol{k}_n\rangle = \sum_{i=1}^{n}\omega(\boldsymbol{k}_i)|\boldsymbol{k}_1, \boldsymbol{k}_2, ...\boldsymbol{k}_n\rangle. \tag{6}$$

**Problem 3**
Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_{\nu}\phi\partial^{\nu}\phi - \zeta\phi^2 \tag{7}$$

where the canonical momentum $\Pi(x)$ to the field $\phi(x)$ is

$$\Pi(x) = \frac{\partial\mathcal{L}}{\partial(\partial_0\phi)}. \tag{8}$$

The equations of motion for the field $\phi$ can be derived from the Hamiltonian of the system

$$H = \int d^3x(\Pi\dot{\phi} - \mathcal{L}) \tag{9}$$

by using the Poisson brackets (omitting the $x^0$-dependence in the fields):

$$\dot{\phi}(\boldsymbol{x}) = \{\phi(\boldsymbol{x}), H\},\ \dot{\Pi}(\boldsymbol{x}) = \{\Pi(\boldsymbol{x}), H\}. \tag{10}$$

The Poisson brackets are defined via

$$\{A,B\} = \int d\boldsymbol{x}' \Big( \frac{\delta A}{\delta\phi(\boldsymbol{x}')} \frac{\delta B}{\delta\Pi(\boldsymbol{x}')} - \frac{\delta A}{\delta\Pi(\boldsymbol{x}')} \frac{\delta B}{\delta\phi(\boldsymbol{x}')} \Big) \tag{11}$$

and the functional derivatives satisfy

$$\frac{\delta\phi(\boldsymbol{y})}{\delta\phi(\boldsymbol{x})} = \delta^3(\boldsymbol{x}-\boldsymbol{y}),$$
$$\frac{\delta\Pi(\boldsymbol{y})}{\delta\Pi(\boldsymbol{x})} = \delta^3(\boldsymbol{x}-\boldsymbol{y}). \tag{12}$$

Derive the equation of motion for $\phi$ by using the Poisson brackets.