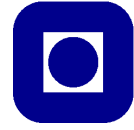


FY3464 Quantum Field Theory

NTNU

Problemset 1



Institutt for fysikk

Problem 1

Show that the requirement of invariant length squared of a 4-vector leads to the requirement

$$\eta_{\alpha\beta} = \eta_{\mu\nu} \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta. \quad (1)$$

for the Lorentz-transformation Λ . Then, use this to show that

$$(\Lambda^{-1})^\sigma{}_\rho = \Lambda_\rho{}^\sigma \quad (2)$$

by making use of the fact that the inverse of a tensor A is given by:

$$(A^{-1})^\sigma{}_\rho A^\rho{}_\mu = \delta^\sigma{}_\mu. \quad (3)$$

Problem 2

Show that

$$H = \int \frac{d^3k}{(2\pi)^3} \omega(\mathbf{k}) a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \quad (4)$$

acting on the Fock space states

$$|\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n\rangle = \prod_{i=1}^n \sqrt{2\omega(\mathbf{k}_i)} a_{\mathbf{k}_i}^\dagger |0\rangle \quad (5)$$

produces

$$H |\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n\rangle = \sum_{i=1}^n \omega(\mathbf{k}_i) |\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n\rangle. \quad (6)$$

Problem 3

Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\nu \phi \partial^\nu \phi - \zeta \phi^2 \quad (7)$$

where the canonical momentum $\Pi(x)$ to the field $\phi(x)$ is

$$\Pi(x) = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)}. \quad (8)$$

The equations of motion for the field ϕ can be derived from the Hamiltonian of the system

$$H = \int d^3x (\Pi \dot{\phi} - \mathcal{L}) \quad (9)$$

by using the Poisson brackets (omitting the x^0 -dependence in the fields):

$$\dot{\phi}(\mathbf{x}) = \{\phi(\mathbf{x}), H\}, \quad \dot{\Pi}(\mathbf{x}) = \{\Pi(\mathbf{x}), H\}. \quad (10)$$

The Poisson brackets are defined via

$$\{A, B\} = \int d\mathbf{x}' \left(\frac{\delta A}{\delta \phi(\mathbf{x}')} \frac{\delta B}{\delta \Pi(\mathbf{x}')} - \frac{\delta A}{\delta \Pi(\mathbf{x}')} \frac{\delta B}{\delta \phi(\mathbf{x}')} \right) \quad (11)$$

and the functional derivatives satisfy

$$\begin{aligned} \frac{\delta \phi(\mathbf{y})}{\delta \phi(\mathbf{x})} &= \delta^3(\mathbf{x} - \mathbf{y}), \\ \frac{\delta \Pi(\mathbf{y})}{\delta \Pi(\mathbf{x})} &= \delta^3(\mathbf{x} - \mathbf{y}). \end{aligned} \quad (12)$$

Derive the equation of motion for ϕ by using the Poisson brackets.