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PHASE ABERRATION CORRECTION IN MEDICAL ULTRASOUND IMAGING
Phase aberration correction in medical ultrasound imaging

by

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Preface

The subject of this dissertation is distortion and improvement of medical ultrasound images. The main issue of the thesis is to explore and improve the focusing through inhomogeneous human tissue.

The work is done at the Department of Mathematical Sciences, Norwegian Institute of Technology, University of Trondheim and at the Dept. of Physiology and Biomedical Engineering, University of Trondheim. The work is funded by a scholarship from the Department of Mathematical Sciences.

The academical advisers were senior scientific officer John Tyssedal at the Department of Mathematical Sciences, Ph.D. Hans Torp at the Dept. of Physiology and Biomedical Engineering and professor Sverre Holm at Dept. of Informatics, University of Oslo.

The content of the thesis is mainly medical ultrasound. The plan was to use statistical methods for medical ultrasound imaging, but after a while I realized that the problem could not be solved by statistical methods only. Instead of looking for another problem that would require statistical methods, I chose to look for other methods to improve medical ultrasound imaging. It has thus not been easy for John Tyssedal to give me academical advise, but I want to thank him for practical help during the work on this thesis.

Sverre Holm encouraged me to start this work, and helped me while he worked at the Norwegian Institute of Technology. I have also benefited from his work on the simulation program Ultrasim. Frode Teigen and Vebjørn Berre have also given important contributions to Ultrasim. It has been a pleasure working with you all.

I want to thank Torgrim Lie for preparing the equipment for the ultrasound experiments, and Jørn Kværness for helping me taking the MR-images presented in the thesis.

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Summary

Waves back-scattered from inhomogeneities in human tissue are used to make ultrasound images. To perform the focusing required to make images of high quality, the speed of sound in tissue is assumed to be 1540 m/s. This is almost true, but the speed of sound fluctuates with almost 10 percent. The speed of sound is for example 1440 m/s in fat and 1580 m/s in muscles. Variation in the speed of sound will disturb the focusing. The validity of the approximation of constant speed of sound is explored in the thesis. Methods for focusing through tissue with spatially varying speed of sound is reviewed, suggested and explored.

In this work, a simple wave equation which includes absorption and allow relatively rough spatial fluctuations in the mass density is deduced. The deduced wave-equation is solved for a layered medium by combining ray-tracing and the Helmholtz-Kirchhoff integral formula. The boundaries are allowed to be relatively rough, and the solution is valid both for pulsed and continuous waves. For a homogeneous medium, the deduced integral solution simplifies to the Rayleigh integral.

A simulation program is written to explore the focusing through layered media. It is based on the deduced integral formula. The simulation program calculates two - and three dimensional refraction due to Snell's law. Attenuation factors such as absorption, reflection and attenuation due to the broadening of the ray-tube are included. Measurements verify the simulation program.

Observations show that the quality of ultrasound images of obese patients is poorer than images of thin patients. MR-images are used to build models of the outer tissue layers of a thin and an obese patient. The models are explored by the simulation program. The models contain an annular transducer, dome fluid, a spherical dome and an outer layer of fat. The combination of the spherical dome and the outer layer of fat causes a lens effect. The lens effect is largest for the obese patient. The algorithm for electronic focusing will not specify optimal delays, and the result is a defocused image. The lens effect is shown to be removed by tuning the parameters of the algorithm for electronic focusing. The fat lens tuning (FLT) is done by modifying the radius of curvature (ROC) parameter used in the focusing algorithm. The ROC parameter must be set to the focal depth of the transducer-medium system. The tuning gives best results if the transducer diameter is tuned first. To correct for refraction caused by the dome, the transducer diameter should be tuned down. The tuning of the transducer diameter can be done in a water bath, while the ROC must be tuned for each patient.

To make good ultrasound images of an obese person, correctly tuned parameters is a necessary, but not sufficient condition. A further study is made to explore the poor quality of the ultrasound image of an obese patient. One has observed that the muscle layer between the outer layer of fat and the heart has high fat content. Because of the difference in speed of sound in fat and the speed of sound in muscles, this layer will cause phase aberrations. Experiments that show the distortion caused by a layer of muscle with a high content of fat are presented. These distortions are
so large that they can explain the poor image quality of the obese patient.

About 75 articles have been published on phase aberration correction and related subjects during the last 10 years. A review is presented in the thesis. To perform corrections, 1.5D or 2D arrays are needed. The correction methods can be classified according to the tools that are used to do the focusing:

1. Both the correlation method and the maximum speckle brightness method regulate the time delays on the elements to do the focusing.

2. The time reversal mirror (TRM) method regulates the delays, the apodization and the pulse form.

Experiments presented in literature show that the TRM-method corrects better than the time - delay focusing methods if the phase aberrations are generated at a distance from the transducer.

The TRM-method needs a point reflector to focus on. Point reflectors are not common in the human body. The van Cittert Zernike theorem shows that the waves back-scattered from diffuse scatterers and observed by adjacent elements are correlated. The correlation is found to be high enough to estimate the time delays needed to perform time - delay focusing.

The phase aberrations due to the fat-containing muscle layer of an obese person are generated at a distance from the transducer. There has yet not been presented a method that can use waves back-scattered by diffuse inhomogeneities to correct these phase aberrations well. In this work, simulations have been done to find out what information is needed to focus through an aberrator at a distance from the transducer. One has shown that a rough wave front generates amplitude aberration by propagation, while the distortion of the pulse form is small. The amplitude fluctuations of the received wave is found to contain more information about the aberrator than the pulse form.

In this thesis, it is suggested to use both time - delay control and apodization to focus optimally through an aberrator at a distance from the transducer. The van Cittert Zernike theorem shows that the waves back-scattered from diffuse scatterers and received by adjacent elements are correlated. The phase and amplitude information is therefore available. To focus optimally through the aberrator, the time - delay corrections are set to the opposite of the phase aberrations of the received wave. The apodization is set equal to the amplitude aberrations in the received wave.

Simulations show that the suggested delay and amplitude focusing (DAF) does better focusing through the aberrator than time - delay focusing. DAF is also an improvement compared to the TRM method since DAF can be used to focus on areas with diffuse scatterers. The suggested delay and amplitude focusing is therefore an improved method for phase aberration correction in medical ultrasound imaging.
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Nomenclature

List of Symbols

\( \rho \) = mass density
\( u \) = particle velocity
\( p \) = pressure
\( r \) = position = \((x,y,z)\)
\( t \) = time
\( c \) = speed of sound \((c=c(\ell))\)
\( c_0 \) = reference speed of sound
\( b \) = gravity force
\( \mu_0 \) = absorption coefficient
\( \mu \) = absorption coefficient \(= \mu_0 K \frac{\omega^2}{c} \)
\( \omega \) = angular frequency
\( K \) = compressibility
\( \mathbf{n} \) = the normal vector to a surface
\( n \) = refraction index \(= \frac{c_0}{c} \)
\( J \) = surface area
\( g \) = acceleration due to gravity \(= 9.81 m/s^2 \)
\( f \) = frequency
\( F(t) \) = The shape of the emitted pulse
\( \lambda \) = wavelength
\( \Phi \) = \( \nabla^{-1} (\rho u) \)
\( A \) = amplitude of the wave
\( S \) = acoustical distance \((k_c s = \text{the phase of the wave})\)
\( k_c \) = \( \frac{c}{c_0} \)
\( k_{c0} \) = \( \frac{c}{c_0} \)
\( \mathbf{s} \) = a normalized vector for the direction of the ray
\( I \) = intensity
\( \alpha \) = the angle between \( \mathbf{s} \) and \( \mathbf{n} \)
\( \sigma \) = geometrical distance along the ray
\( V \) = volume
\( dB \) = the area of the cross section of the ray tube
\( Z \) = \( \rho c \)
\( G \) = Greens function
\( D \) = Transducer diameter
Abbreviations

ROC = Radius Of Curvature of the transducer, i.e. the distance between the transducer and the geometrical focal point of the transducer.

rms = root mean square
std = standard deviation
ROI = region of interest
ASF = average sidelobe floor
PSF = peak sidelobe floor
PCM = phase conjugate mirror
TRM = time reversal mirror
DAF = delay and amplitude focusing
SAD = sum of absolute differences
FLT = fat lens tuning
CW = continuous waves
PW = pulsed waves
FEM = finite element method

Div.

focal depth = distance from the transducer to the focal point.
focal length = The length where the beam has intensity higher than a specified dB-value.
focal-point = The point where the signals emitted from the transducer arrive at the same time if the medium is homogeneous.
f-number = focal depth/ transducer diameter
acoustical noise = Noise/ artifacts caused by the side-lobes and by multiple reflections.
reverberations = Noise/ artifacts caused by multiple reflections.
correlation length = the distance between half-maximum amplitude points of the autocorrelation function.
jitter = displacement of the true peak of the cross-correlation function.
on-axis = The focal axis, i.e. the axis between the center element of the transducer and the focal point.
off-axis = An axis normal to the on-axis
elevation = One of the off-axes
azimuth = One of the off-axes. azimuth is normal to elevation.
1 Introduction

The quality of medical ultrasound images vary from patient to patient. Fig. 1 shows images of the heart of two persons; one that gives ultrasound images with high quality and one that gives ultrasound images with low quality. The fundamental question is why the quality of the images changes from patient to patient. If we can expose the factors which determine the quality of an ultrasound image, it may be possible to improve the images. In this thesis, the author explores why the image quality changes from patient to patient. The answers are used to suggest methods of improving the quality of poor ultrasound images.

Figure 1: Ultrasound images of the heart of two different persons. The thin person to the left gives ultrasound images with high quality, while the obese person to the right gives ultrasound images with low quality.
1.1 Principles of medical ultrasound imaging

When an ultrasound pulse encounters a boundary between two human tissue structures, for instance fat and muscular tissue, the ultrasound will be partially reflected and partially transmitted. The reflection depends on the difference in the acoustic impedance of the two materials. The acoustic impedance is given by the materials' mass density multiplied with its speed of sound. On the surfaces between different soft tissues, the reflection is weak, and most of the energy is transmitted. The transmitted pulse is then partially reflected and transmitted at the second boundary so that we now have two reflected pulses propagating back to the transducer. The back-scattered signals are used to image the inhomogeneities in the body.

Medical ultrasound images are mapped out line by line. For each excitation of the transducer, one line in the ultrasound image is mapped. To map one line, the emitted pulse is focussed as a narrow beam. Inhomogeneities illuminated by the beam will scatter some of the waves, and the envelope of the back-scattered signal will be presented as a line in the image. The time from a pulse is transmitted and to the reflected wave arrives the transducer is used to determine the position (depth) of the different inhomogeneities. It is assumed that the speed of sound in human tissue is \(1540\, \text{m/s}\).

1.2 Factors affecting image quality

The image quality can be described by two main factors [1]:

1. The spatial resolution, which is limited by the spatial smearing in the image of a small target. It is determined laterally by the width of the main-lobe of the beam and radially by the length of the transmitted pulse.

2. The contrast resolution, which describes the ability to detect small variations in the intensity of the back-scattered signal from targets that are close to one another. The main limitation of contrast resolution is acoustical noise, which is caused by side-lobes of the beam and multiple reflections of the ultrasound pulse (reverberations). Electronic noise in the receiver will also limit the contrast resolution.

The radial resolution is proportional to the length of the signal back-scattered from a point reflector. A long pulse emitted from the transducer will therefore smear the point out, while a short pulse emitted from the transducer will give high radial resolution. The radial resolution is almost the same for all observation depths and it is usually higher than the lateral resolution in medical ultrasound images.

The lateral resolution is given by the width of the beams' main-lobe, which mainly depends on the transducer and the frequency of the emitted signal. The lateral resolution in the image is increased by using a transducer with smaller f-number and by emitting a pulse with higher frequency. The f-number is the focal depth of the transducer divided by the diameter of the transducer. Beam-profiles measured
for different transducers and excitation signals are shown in section 5.2.2. Rules of thumb are compared with the measurements in the same section. The experiments also demonstrate how the lateral resolution changes with the observation depth.

The lateral resolution will be reduced by frequency-dependent attenuation. Measurements on human tissue show that waves with high frequencies are more attenuated than waves with low frequencies [8]. The low frequencies of the pulse will therefore be more dominant. The result is beams with a broader main-lobe, which will reduce the lateral resolution.

![Figure 2: Examples of noise and artifacts that limit the contrast resolution.](image)

The definition of contrast resolution can be restated as the ability to show as black a region in the image which has no scatterers. Examples of different distortions that limit the contrast resolution are shown in fig. 2. The upper left image shows a good ultrasound image of a specimen with bacon in water. In the other images, the gain is manipulated to show noise and artifacts.
It is used an extremely high gain at depth 3-6 cm in the image up to the left to show electronic noise. Electronic noise are back-ground noise caused by the amplifier. The attenuation of an emitted wave increases with the propagation distance in tissue. This has to be compensated by increasing the gain. At increased gain, the power of the electronic noise will increase. Electronic noise observed in different positions in the image is uncorrelated and has zero mean.

Reverberations are effects caused by multiple reflections. Multiple reflections between inhomogeneities in the tissue will be referred to as internal reverberations. There may also be multiple reflections between weak scatterers in the tissue and strong scatterers as the transducer, the ribs or the lungs. Reverberations generate artifacts, i.e. apparently objects in the image. The artifacts are placed deeper in the image than the scatterer that caused them. An example of internal reverberations are shown in the image down to the left in fig. 2.

A good transducer will emit a narrow beam, i.e. a beam with a strong and narrow main-lobe and weak side-lobes. If the side-lobes are to high, strong reflectors irradiated by the signals in the side-lobes may scatter more sound than weak reflectors irradiated by the signals in the main-lobe. If the signals in the main-lobe of the beam irradiate water and the signals in the side-lobes of the beam irradiate inhomogeneous tissue, the signals in the side-lobes will reflect stronger signals than the signals in the main-lobe. The water that should have been imaged as black in the ultrasound image will therefore be imaged as there is some weak scatterer there. The effect is demonstrated in the image down to the right in fig. 2.

1.3 Phase aberration

Phase aberrations arise when the speed of sound in the propagation medium fluctuates. A transducer is focused physically or electronically to a focal point. If the medium is homogeneous, the signals emitted from different points on the transducer will use the same propagation time to the focal point. The signals will be added constructively at the focal point, while the signals are added destructively in the area on the side of the focal point. The transducer will therefore focus well in a homogeneous medium.

Human tissue is almost homogeneous, but the speed of sound fluctuates with about 10 percent. The propagation time between different points on the transducer and the focal point will therefore vary. The different propagation times cause phase aberrations between the signals added in the focal point. The phase errors reduce the constructive interference in the focal point. The result is beams with a decreased main-lobe.

The destructive interference of the signals beside the focal point is also affected by phase aberrations. The phase aberrations will alternately do local focusing and defocusing of the wave-front. The signals will therefore not interfere completely destructively in the area on the side of the focal point. The result is increased side-lobes.
Figure 3: An example of distortions caused by phase aberrations. A specimen of bacon is imaged through water in the upper image, and the same specimen is imaged through 15 mm inhomogeneous bacon and water in the lower image. The phase aberrations generated by propagation through the inhomogeneous specimen reduced the lateral resolution of the lower image compared with the upper image.
The decreased main-lobe and the increased side-lobes reduce the lateral contrast resolution. The effect of large phase-aberrations is illustrated by the images shown in fig. 3. The upper image shows a specimen of bacon in water at 37°C. The white object shows the specimen, while the grey shown under the tissue is reverberations. The grey dots around in the image are caused by waves reflected from small air bubbles in the water. The wave has propagated through water, and the image is therefore not disturbed by phase aberrations.

In the lower image, a bacon-specimen of meat with much fat spread around inside the meat is kept between the transducer and the object shown in the upper image. The speed of sound in the fat was measured to 1437 m/s, while the speed of sound in the meat was measured to 1594 m/s. This is close to the values measured in fat and muscle in human tissue [8]. The differences in the speed of sound in the propagation medium cause phase aberrations which defocus the beam. The consequence of the decreased main-lobe and the increased side-lobes caused by the phase aberrations can be seen by comparing the image of the object shown in the lower, left part of the images. The gain is equal in the two images.

The reduced brightness of the imaged object and the fact that the small air bubbles are no longer visible in the image, show that the intensity of the main-lobe are reduced by propagation through an inhomogeneous specimen of tissue. The reduced intensity of the signals in the main-lobe is due to attenuation and phase aberration caused by propagation through the bacon specimen.

The radial resolution in the lower image is still good, but the lateral resolution is reduced dramatically. It is difficult to see the vertical boundary between the object and the water. The reduced lateral resolution is due to increased side-lobes. The increased side-lobes are caused by the phase aberrations caused by propagation through the inhomogeneous specimen. Note that the phase aberrations cause side-lobes with increased intensity in spite of the additional attenuation caused by absorption and reflection loss due to propagation through the inhomogeneous specimen of bacon.

The images presented in fig. 3 are taken on a 3.25 MHz transducer, which is a typical frequency used in medical ultrasound imaging. The phase aberrations will increase if we use higher frequencies. In general, the possibility to use higher frequencies to increase the lateral resolution is limited by phase aberrations.

### 1.4 Outline of the thesis

The size and effects of phase aberrations have been studied the last 25 years, and methods to do corrections have been suggested. A review of published articles are presented in section 2.

Section 5.1 and 5.2 present experiments that are done to detect and explore the factors that disturb ultrasound images. The reason to the different image quality of a thin and an obese patient is especially explored.

The observations from these experiments are used as motivation to suggested two
new methods for phase aberration correction. The Fat Lens Tuning (FLT) method is presented in section 6, and the Delay and Amplitude Focusing (DAF) method is presented in section 7.

To evaluate the correction methods, a simulation program for wave propagation through a layered medium is developed. The theory behind the simulation program is deduced in section 3, and a description of the simulation program and its possibilities are given in section 4. The simulations are verified with experiments in section 5.3.

Section 8 discuss the work presented in the thesis.
2 A review of phase aberration correction in medical ultrasound imaging

2.1 Introduction

The design of phased-array diagnostic ultrasonic imaging systems is based on the assumption that the acoustic velocity in human tissue is constant in space, typically 1540 m/s. The assumed acoustic velocity is used to calculate delays, which ensure that signals from all elements are in phase at the focal point. These calculations are based on geometric path length and the speed of sound. The velocity in soft tissue varies between 1440 m/s in fat and 1580 m/s in muscle. These variations will generate phase aberrations, and the focusing will not be optimal. The size and effects of phase aberrations have been studied in the literature and methods to do corrections have been suggested. A review of articles published before 1995 is presented here.

2.2 Beam distortion due to human tissue

In 1966 Thurstone and McKinney [82] predicted that ultrasonic wave fluctuations generated by propagation through tissue would limit the performance of imaging systems. White et al. (1969) [96] were the first to investigate how ultrasonic beams are distorted by biological tissues. They studied the deformation of the ultrasonic field due to transmission through the skull. The speed of sound in the skull is twice as high as in soft tissue, and a wave which propagate through the skull is highly attenuated. The deviation and distortion of the beam measured in this and subsequent work explained why ultrasonic imaging of the brain in the intact adult skull is so disappointing. Halliwell and Mountford (1973) [36] [56] were the first to investigate the distortion of ultrasonic beams as they travel through living soft tissues. They compared the shape of the beam cross-section after passing through the human calf and compared this with the beam shape at the same range in water. The beam due to human calf was disturbed compared with the beam in water, but the deviation and distortions were not as large as in the skull.

2.2.1 Distortion of the main-lobe

Halliwell (1978) [37] studied beam transmission in the normal human female breast. Using an 18 mm diameter 2.25 MHz transducer and a hydrophone, he found that the beam in breast was steered off-axis at random as much as 10 mm in some cases. He also measured a slight broadening of the beam.

To evaluate how a focused ultrasonic beam is defocused by a propagating through human tissue, the -6 dB width of the beam was chosen as criterion for optimum focusing. Foster and Hunt (1979) [32] measured the one-way main-lobe broadening of focused ultrasonic beams as a result of passage through human tissue in water.
The -6 dB width of the beam was smallest in water, some broader in liver, even broader in tumour tissue and much broader in the breast. An experiment with liver tissue showed that the broadening increased with the tissue thickness. It was found that some of the broadening is due to frequency dependent attenuation. In tissue, high frequencies are attenuated more than low frequencies. This down shift of the mean frequency gives a broadening of the beam. Frequency dependent attenuation explained the broadening in human brain, but not in the breast. Phase distortions due to variations of speed of sound was suggested to explain the beam distortion in the breast. In spite of distortions due to an attenuating and inhomogeneous medium, Foster and Hunt found it to be possible to reduce the -6dB beam width in the focal zone by decreasing the f-number (focal depth/ transducer diameter). Even for breast the optimum f-number was below 3 for a 4.1 MHz transducer.

Moshfeghi and Waag (1988) [55] measured similar effects. They studied the two-way beam for a small spherical reflector and a transmitted pulse with center frequency 3 MHz. The -6dB width was about 15 % larger in breast than in water for the 19 mm transducer, and the broadening for the 50 mm transducer was approximately 35 %. The -6dB width was about 10 % larger in liver than in water for the 19 mm transducer and the broadening for the 50 mm transducer was approximately 25 %. Observation depth was 50 mm. Despite of this broadening, the -6dB width of the beam decreased by decreasing f-number from 2.6 to 1 for both liver and breast tissue.

2.2.2 Distortion of the side-lobes

The focusing properties of the transducer judged by the width of the main-lobe at the -6 dB level determines point or detail resolution. To evaluate the contrast resolution we have to look at the side-lobe level. Moshfeghi and Waag (1988) [55] observed that the side-lobe amplitudes increase in tissue. In many of the breast subjects the side-lobe level was as high as -10 dB, while in liver tissue with similar thickness it was -30 dB or less.

2.2.3 Why is the beam disturbed?

O'Donnell and Flax (1988) [66] have examined in vivo images of the human liver for the effect of wave front distortion. Measured phase aberrations were small in all subjects exhibiting high quality images. In contrast, large phase aberrations were measured in subjects producing low quality images. There were small amplitude variations across the array for all subjects studied. These results suggested that the absence of significant phase aberrations is a necessary condition for high quality phased array imaging.

Trahey et.al. (1991) [85] simulated beam-profiles based on phase aberrations measured in 22 female breasts. The simulations showed that phase aberrations typical for the breast, significantly degrade breast image quality for typical transducer
frequencies and sizes. (See also [101], [33], [79].)

2.3 Size and effect of wavefront aberrations

A pilot study to measure the ultrasonic amplitude and phase fluctuations occurring after a plane wave has passed through tissue was published by Aindow and Chivers in 1988 [19]. The tissue was circa. 10 mm thick specimens of fresh beef liver from two different animals. The experiments were done while the specimen of tissue was immersed in a liquid with speed of sound similar to the speed of sound in the specimen. This was done to reduce the effects of phase aberrations caused by nonplane boundaries between the tissue and the liquid. But even then both phase and amplitude variations were measured. The measurements were done with a 0.55 mm hydrophone close to one of the specimens. The measurements were done in one dimension. The normalized amplitude varied in the interval 0.6-1 and the phase fluctuations varied from 0 to 70° (f=948 kHz).

2.3.1 Size of wavefront aberrations in the abdomen

Krüümmer and Hassler (1987) [48] measured the time delay fluctuations caused by propagation through samples from the human abdominal wall in front of the liver. They determined that the level of wave front distortion varies significantly between individual specimens. Normal liver introduced only minor phase distortions. Samples consisting of skin and a fat layer gave phase aberration with std≈ 20ns, while samples consisting of skin, fat and a muscle layer gave standard deviations from 10 ns and up to 117 ns. They did not discussed whether the phase aberrations were due to a rough boundary between fat and muscle or due to inhomogeneous tissue inside the muscle. How much nonplane boundaries between the tissue samples and water influenced on the measurements was not commented.

Sumino and Waag (1991) [81] also measured phase aberrations after passage through human abdominal wall. The specimens were put into water with room temperature during the measurements.

A major improvement was done by Hinkelman et al. [39]. They did the measurements at body temperature. This was motivated by experimental results showing that the speed of sound in mammalian tissue changes with temperature and that the change depends on tissue type [20]. The speed of sound in fat and muscle change in opposite directions as temperature increases. The fourteen different abdominal specimens used consisted of skin, a layer of fat, a lower layer of muscle, and the peritoneal membrane. They ranged in thickness from 10 to 35 mm. The outer layers of the abdominal wall are skin on one side and the peritoneal membrane on the other, both of which are smooth. Wavefront distortion due to surface irregularities against water is therefore assumed to be negligible in these measurements. Specimens were stored frozen from their receipt until shortly before their use in measurements. The
donors were mainly older adults ranging from 54 to 96 years. The aberrated signal was measured in a 2D plane immediately after passing through an abdominal specimen. For the 14 different abdominal wall specimens, the standard deviation (std) of the time delay fluctuations and of the energy level differences have average values of 43.0 ns and 3.3 dB, respectively. The associated correlation lengths of the time delay fluctuations and of the energy level differences are 7.9 mm and 2.28 mm, respectively. Correlation length is defined as the distance between half-maximum amplitude points of the autocorrelation functions of the phase aberrations. These values offer information to choose an optimal transducer geometry and to decide which frequency that gives optimal focusing through the human abdominal wall. Higher frequencies give improved focusing, but only if the time delay fluctuations are small compared with a wave period.

2.3.2 Size of wavefront aberrations in the breast

Trahey et al. (1991) [85] measured one-dimensional phase aberrations encountered by ultrasonic pulses propagating through breast tissue in twenty-two female volunteers. The results indicate that phase aberrations significantly degrade breast image quality for typical transducer frequencies and sizes. The standard deviation of the phase errors caused by propagation through female breasts ranged from 17.3 ns to 50.0 ns with an overall average value of 36 ns. The average full-width half-maximum of the autocorrelation function (correlation length) for all patients was 2.1 mm (std=0.74 mm).

The direction of the emitted beam was also disturbed. The average magnitude of the steering error was 0.76 deg with a maximum of 4.1 deg. Significant change in the shape of the pulses received across the array was not observed in water or in tissue. The correlation coefficient measured between the signals received at neighbour elements are high in both water and tissue.

Wavefront amplitude distribution in the female breast was measured by Zhu and Steinberg (1994) [104]. Their measurements showed that the wavefront amplitude distribution generated by propagation through the breast is close to the Rayleigh distribution. The amplitude is normalized by dividing with the amplitude through oil. The mean amplitude value was 0.89 and the standard deviation was 0.52. The measurements were not done with a small hydrophone, but with an element of size 4λ × 26λ. Experiments with a short pulse showed that delays of more than 2μs exist. They concluded that strong scattering and/or refraction are the dominant wavefront distortion sources in the breast. (See also [79], [33], [101])

2.3.3 Beam profiles caused by typical phase aberrations

To evaluate how much of the beam-distortions that can be explained by phase aberration, Trahey et al. (1991) [85] used phase profiles measured after propagation through human breast tissue as input in a simulation program. The program calculated one-way beam profiles. They found that the distortions are largest for
the side-lobe level. An example with a 13.2 mm transducer with f=5.0 MHz and f-number 3.8 gave 13 % / 19 % / 86 % increase in the -6 dB / -10 dB / -20 dB main lobe width by introducing phase aberrations typical for the breast. The maximal loss of the amplitude of the main-lobe was 45 %.

2.3.4 Beam profiles caused by typical amplitude aberrations

The theoretical relationship between the average side-lobe floor (ASF) in the image and the medium-induced amplitude variance of the wavefront was developed by Zhu and Steinberg (1993) [102]. The companion paper [103], described in vivo measurements of the rise of the side-lobe level in a single-source image obtained through the female breast as a function of the distortion of the wavefront amplitude. By theory and by experiments, the ASF is shown to be proportional to the variance of the amplitude of the wavefront normalized to the square of its mean value.

Ødegaard (1995) [64] simulated different one-way beam-profiles based on typical phase and amplitude aberrations measured in the abdominal wall [39]. Simulations were done for a 128 elements 50 mm phased array focused to 100 mm. A pulse with center frequency 3.5 MHz was used. The sidelobe level after adding both phase and amplitude errors to the transducer gave sidelobe level 10-15 dB higher than in the undisturbed case. After perfect phase aberration correction, i.e. only amplitude errors left, the sidelobe level were reduced 5-10 dB. But the sidelobes were still about 5 dB higher than in the ideal case.

2.4 Phase aberration correction

Phase aberrations due to propagation through human tissue disturb the beam-profile. The phase aberrations increase by using a pulse with a shorter wave period, i.e. by using a pulse with increased frequency. This will limit the possibility to improve the image quality by using higher frequencies. The main effect is increased side-lobe level, which will reduce the contrast in the image. Amplitude aberrations give a similar but smaller effect, which indicate that it is more important to correct for phase- than amplitude aberrations. To do phase aberration correction, several methods have been suggested.

Smith et al. (1986) [77] studied the imaging of an organ of interest through an intervening planar tissue layer, such as liver through fat in the abdomen or brain through skull bone in the adult head. They concluded that fat/liver (1470 m/s / 1600 m/s) planar interfaces do not degrade image quality significantly, while a skull/brain (3200 m/s / 1600 m/s) planar interface degrades resolution. Based on this intervening planar layer model and snell's law correction delays were found by iterations. This correction technique improve focusing through a planar skull.

An improved method developed for annular array transducers was suggested by Ødegaard et. al. in 1993 [62]. To understand why image quality depend on the patient, ultrasound and Magnetic Resonance (MR) images of two patients who
gave ultrasound images with different quality were recorded. The MR-images were used to make models of the two patients. The tissue, including probe fluid and cap, were modeled as layers with smooth, nonplane boundaries. Ray-tracing [63] on the models showed large/small focus displacement in depth for the patient producing ultrasound images with low/high quality. Such displacements will play the algorithm for electronic focusing a trick. It was demonstrated by simulations [62] that the focusing can be optimized for both patients if we modify two parameters in the algorithm. The parameters are transducer diameter and Radius Of Curvature (ROC). Rays from the outer transducer ring are refracted more through the dome than rays from the center element, such that the transducer looks smaller than it is. The focusing will thus work better if we do not use the physical transducer diameter, but a smaller one, as parameter in the focusing algorithm. This effect is independent of the patient and can be corrected once for all. The focus displacement in depth depends on the patient and has to be corrected for each patient. This is done by tuning the ROC parameter. In an example with a 5 MHz transducer the two-way side-lobe level was reduced from -21 dB to -34 dB by this correction method. The method is easy to use, but it correct only for some of the phase aberrations.

To correct for arbitrary phase aberration, more advanced methods are necessary. The main idea is to adaptively correct phase aberration by using information in the received signals. A review of phase aberration correction methods suggested for medical ultrasound imaging is presented here.

2.4.1 Phase aberration correction using correlation methods

Time delay estimation via cross correlation to correct for phase aberrations induced in the near-field was studied by Flax and O'Donnell in 1988 [30]. They showed that an iterative phase correction procedure can be used to obtain accurate estimates of arrival time differences using signals from point and diffuse scatterers. Each step is based on cross correlation between signals received at neighbour elements, and for each iteration step they used the estimated phase aberration as correction for both transmit and receive for the next pulse. It is not necessary to iterate if a point reflector exists.

The method was tested experimentally [67] for phase aberrations induced by a 1.5 mm thick plate with grooves close to the transducer. The observation object was a graphite-gel AIUM resolution phantom. A 20-mm long region of diffuse scatterers observed in the center of the image was used to estimate the correction. The correction was applied to all beams used for creating the image. Three iterations gave good improvements. The experiment showed that iterative phase correction using signals from diffuse scatterers can greatly improve the quality of images corrupted by near-field velocity inhomogeneities.

Alternative methods to estimate phase aberrations exists. Kanda et.al (1991) [44] used 'average' phase information from the received signal that is quadrature detected. The improvements obtained by the cross correlation method and by the
'averaged' phase method were almost the same under normal conditions, i.e. for signal to noise ratio (SNR) larger than one and for phase aberrations with standard deviation below 60 ns.

A computationally efficient method for phase aberration estimations in ultrasound imaging is presented by Karaman et al. in 1993 [45]. The method is based on time delay estimation via minimization of the sum of absolute differences (SAD) between radio frequency samples of adjacent array elements. They expected the estimation performance to be similar as the cross correlation method.

Inoperable elements are a problem in correlation methods. Dead elements and elements receiving unwanted signals as strong reverberations are defined as inoperable elements. Trahey et al. [86] demonstrated in 1991 that large errors can result if these inoperable elements are allowed to contribute to estimation of phase aberration using the correlation approach. O'Donnell and Engeler (1992) [70] showed that this problem can be avoided by first detecting the inoperable elements, and then estimating relative phase aberration between the nearest active elements. In each element, the envelope amplitude of the received signal is summed over a set of ranges. By computing mean and standard deviation of the accumulated envelope amplitude in the elements, "dead" channels and channels contaminated by strong reverberant signals can be identified as statistical outliers. Both kinds of element are considered inactive. The inoperable elements may also be detected by comparing the root mean square of the signal received at each channel. Karaman et al (1994) [46] compared normalized cross correlation and sum of absolute differences in the presence of inoperable elements. They concluded that the methods show comparable performance.

Two types of errors in delay estimates are: 1) Noise and signal decorrelation may increase the amplitude of a secondary correlation peak above the primary correlation peak. 2) Noise, signal decorrelation and sampling effects may cause a slight displacement (jitter) of the true peak of the cross-correlation function. Walker and Trahey applied the "Cramer-Rao Lower Bound" to derive an analytical expression which predicts the size of jitter errors incurred when estimating delays using radio frequency (RF) data. In 1994 [91], they presented an expression which predict the magnitude of jitter due to white noise. The analytical expression was extended to data from speckle targets in 1995 [92]. The results presented in the last paper provide insight into the relationship between jitter magnitude and center frequency, bandwidth, window length, signal to noise ratio (SNR) and the correlation coefficient. The analytical expression fitted well with simulations. An example with signals with center frequency of 5.0 MHz, a 50% fractional bandwidth, a window length of 4 $\mu$s, SNR=30dB and a correlation coefficient of 0.8 gave a standard deviation of jitter of almost 6 ns. If the correlation coefficient is decreased to 0.5 the standard deviation increase to 13 ns.

Signals received at neighbour elements have to look similar, in order for the cross correlation between the two signals to have a distinct peak. If not, the cross correlation peak will give inaccurate or even incorrect delay estimates. A point
scatterer will give a distinct peak in the cross correlation. Diffuse scatterers will not. The correlation structure in speckle data can be studied by the Van Cittert-Zernike theorem, that was applied to pulse echo ultrasound by Mallart and Fink in 1991 [52]. The Van Cittert-Zernike theorem was originally developed for incoherent sources, but a random medium irradiated with ultrasound behaves as an incoherent source. The spatial covariance of the scattered field from diffuse scatterers in the focal zone of a linear array is a triangular function whose base is twice as long as the linear array. It means that the signals received at the first and the last element are uncorrelated, while signals at two neighbour elements are highly correlated. The degree of correlation depends on the length between two elements compared with the length of the whole transducer. Therefore, the array element should be small enough to produce high correlation between neighbours. This was shown both theoretically and experimentally [52]. The result does not depend on f-number or frequency. Note that this correlation structure between neighbour signals refer to waves backscattered from diffuse scatterer in the focal zone of the transducer. If we are outside the focal zone or phase aberrations are introduced, the correlation between signals in neighbour elements will decrease. This is due to the broadening of the beam emitted from the transducer.

2.4.2 Phase aberration correction using a quality factor

A quality factor is an image parameter that increases monotonically as the image improves.

Phase aberration correction using a quality factor has been used successfully in optics. Buffington et.al (1977) [21] ([57]) built and tested an optical telescope which used six movable mirrors to compensate for atmospherically induced phase distortions. A feedback system adjusted the mirrors in real time to maximize the intensity of light passing through a narrow slit in the image plane. In this diffraction limited experiment the algorithm operated successfully. To use a similar method in ultrasonic imaging we need a quality factor that works for human tissue.

Hirama et.al (1982) [40] proposed an adaptive method for imaging through an inhomogeneous layer located close to the transducer. The inhomogeneity was estimated before the image was reconstructed. The estimation was done by including one by one element. For each added element the phase was modified until the focusing criterion was maximized. The focusing criterion was the degree of correlation between the transmitted signals, and the signals reflected from the object under observation. This focusing criterion will work for a point scatterer, but not for diffuse scatterers.

In medical ultrasound images there is much speckle and few strong point scatterers, so it is important to find a quality factor that can be used on a region of interest (ROI) in an image which is dominated by speckle. Speckle generating tissue is modeled as a collection of independent scatterers which are so numerous that there are many within one resolution cell of the scanner [22]. Wagner et.al. (1983)
[90] calculated that the speckle carries information only about the transducer and its focusing pattern. The speckle cell size is comparable to the resolution cell size so that in the axial direction the speckle cell size is proportional to the pulse length, and in the lateral direction it is proportional to the transducer beam width. Trahey and Smith (1988) [83] studied speckle in the presence of phase aberrations. They found that the mean brightness of speckle patterns was sensitive to even mild phase aberrations. Brightness is defined as the envelope of the total received RF-signal. Ultrasound images display brightness. Both simulations and measurements showed that the mean speckle brightness decreased with increasing phase aberrations.

Nock, Trahey and Smith (1989) [60] suggested speckle brightness as a quality factor for phase aberration correction. The delays of a phased array transducer are modified, element by element, to maximize the mean speckle brightness in a region of interest (ROI) in the image. The algorithm consist of:

1. Measure the brightness in a ROI
2. Choose an uncorrected element
3. Add phase delays on the element at both transmit and receive, and use all elements to do a new measurement of the brightness in the ROI.
4. Check if the quality factor is maximized. If yes, go to 2. If no, go to 3.

Trahey et al. (1990) [84] tested the viability of the technique by experiments. In order to assess the viability of the technique with clinical targets, an electronic phase aberration profile was introduced to the normal receive and transmit phasing of the system. The liver of a normal 24 years old male volunteer was imaged. The estimated delay corrections for one ROI were used to correct phase aberrations for all lines in the image. The correction did well for the whole image. This shows that the quality factor, mean speckle brightness, works on human tissue. Another experiment, now with a phantom, but with a physical aberrator close to the transducer was also done. The aberrated image was corrected well in the range close to the ROI, but not in the area far away from the ROI used in the estimation of the phase aberrations. This effect was also observed in [60]. This shows that we have to do corrections for several ROI for a physical aberrator.

If there is speckle in the ROI, the algorithm prefers large ROI, but for a point scatterer the algorithm prefers a ROI as small as possible in the lateral direction [84]. Experiments [84] also indicate that a single line of data is sufficient large for a ROI. A theoretical comparison of image quality factors for phase aberration correction with diffuse and point targets was presented by Zhao and Trahey in 1991 [99]. The mean of different orders of the echo magnitude in a region of interest (ROI) are evaluated. For diffuse targets, all of the quality factors studied performed similarly. They concluded that the mean of any power of the speckle brightness is maximized when the phase aberrations is absent. However, the theory, simulations and experiments show that for point targets, the mean of the echo magnitude is considerably worse than the other quality factors for large lateral ROI dimensions. A point reflector will only work if we use small lateral ROI dimensions. Their conclusion was that
the mean of the echo magnitude squared or higher powers of the signal will function as quality factors.

There are several advantages to the method proposed. No point target is required, and it is not necessary to predict the acoustic velocity in the aberrating medium. The quality factor is simply an integral of a parameter (brightness) that is readily available in any ultrasonic imaging system. We do not need to save the RF-signal received in each element.

There are some problems with the phase correction technique proposed. Discontinuities of one wavelength occur in the corrected phase profiles because the average speckle brightness as a function of element phasing has local maxima at multiples of one wavelength. Discontinuities in the phase correction may be detected by adding the phase at each element by one wavelength to find the true maximum in speckle brightness. It may also be possible to detect discontinuities in the corrected phase profile. A strong reflector off-axis may give steering error. If a linear slope were removed from the correction profile, the steering error might be minimized. A larger problem is that the algorithm is too slow to perform continuous corrections on a real time scan. The most time consuming part is the need to emit a new beam for each step in the iterations.

The maximum brightness algorithm is developed for iterative correction on both transmit and receive. The theoretical argumentations behind is based on an optimization of the two-way beam-profile, but the method will even work if we use it on receive only. That can be explained from the fact that speckle targets give correlation between signals received in adjacent elements.

The correlation structure in speckle data can be studied by the Van Cittert-Zernike theorem, that was applied to pulse echo ultrasound by Mallart and Fink in 1991 [52]. Their analytic expression for the correlation between signals received in different elements refers to waves backscattered from diffuse scatterers in the focal zone of the transducer. If we are outside the focal zone or phase aberrations are introduced, the correlation between signals in neighbour elements will decrease. This is due to the broadening of the emitted beam. Mallart and Fink (1994) [54] suggested a way to use this as a criterion for optimal focusing. They suggested a focusing criterion based on average intensity of the total received signal compared with the sum of the average intensity in each element. Based on Cauchy-Schwartz inequality the factor was shown to vary between 0 and 1. They concluded that for improved beams, the correlation between signals received at adjacent elements will increase and thus also the focusing criterion. Notice that for a perfect focus, the focusing criterion value may be smaller than 1. The focusing criterion takes the optimal value 1 only when signals in all elements are equal. This will happen only when the pressure field emanates from an unique pointlike target and not from diffuse scatterers. The optimal value for a diffuse scatterer in the focal zone of a linear transducer is for instance 2/3 [54].
2.4.3 Other Methods

Hirama et al. (1984) [41] proposed an adaptive method for imaging through an inhomogeneous layer located close to the transducer. In this method a set of data is first acquired by repeating transmission and reception for all possible combinations of pairs of elements on the transducer, then the spatial frequency components of the object and the structure of the inhomogeneous layer are estimated from these data by means of least-mean-square error fitting. The image is reconstructed from the estimated spatial frequency components through inverse Fourier transform. The effectiveness of the method was verified both by numerical analyses and experiments. They concluded that the special features of the method were that: 1) the effectiveness of the method is not object dependent, and 2) the method is fairly robust to noise such as measurement errors. The approximations/ limitations are: 1) The aberrating layer (phase aberr. only) must be close to the transducer. 2) The object must be on-axis or in the far-field. 3) The attenuation factor due to propagation distance is omitted. 4) The object must not be moved when the data is collected.

Synthetic receive aperture (SRA) with phase aberration correction for motion and for tissue inhomogeneities was suggested by Nock and Trahey in 1992 [61],[87]. For a single transmit pulse, from all transmit elements, the sum of the radio frequency (RF) data for one receive subaperture is formed and stored in memory. Then a second identical pulse is transmitted in the same direction and the sum of the received signals of another subaperture is formed and stored. After the signals have been acquired from all receive subapertures, the total RF data is formed by coherently summing together the signals from the various subapertures. This algorithm enables an imaging system to address a large number of transducer receive elements without a correspondingly large number of parallel receive channels. If phase correction for tissue inhomogeneities, compensation for target motion, and receive subaperture data combination are correctly implemented, then the diffraction-limited resolution corresponding to the large number of receive elements may be realized with such a system. The result presented in [87] show that uncorrected axial motion in an SRA imaging system can degrade image quality, and that this degradation will be more severe at higher frequencies. However, adaptive motion correction can be used to form viable SRA images, by removing the effect of axial motion. The performance of such a motion correction algorithm is directly related to the correlation between the signals received at different subapertures.

Wang et al. (1994) [93] used acoustic sensors implanted inside the treatment volume to do phase aberration correction for ultrasonic hyperthermia. The method has been experimentally verified on a phantom where it showed good results. Since invasive thermocouples are usually implanted inside the treatment volume to monitor temperature for ultrasonic hyperthermia, acoustic sensors could be integrated together with thermal sensors adding little complexity to the clinical situation. The algorithm starts by measuring phase and amplitude of the acoustic pressure at the focal points produced by each individual transducer element after an aberrator is...
placed. Then an inverse iterative algorithm calculates phase and amplitude for each transducer element to satisfy specified pressure in the focal points.

2.4.4 Comparison between Max Correlation and Max Brightness

The cross correlation technique and a speckle brightness maximization technique are explored and compared for the two dimensional geometry of a sectored annular array system by Gambetti and Foster (1993) [34]. The experiments were done via a rotating 5 element transducer. The total aperture was a 65-element sectored annular array with 51 mm diameter and geometric focus of 120 mm (f=4MHz). To reduce system complexity, the central element was used to transmit, while all beam-forming and phase corrections were performed on receive. Tissue equivalent materials were moulded into a double layer aberrating medium to generate phase distortions introduced by the rectus abdominis muscle. This phantom was placed 5 cm from the transducer and it generated smooth phase aberrations. When using signals returned by the point target, both techniques considerably improved the focus of the beam. Speckle brightness maximization improved the beam-profile somewhat more than the cross correlation when speckle signals were used, but both methods gave improvements.

It should be noted that this comparison does not compare the optimal use of the two methods. First, they correct only on receive. It is possible to get better results by correct iteratively on transmit and receive. Note also that the cross correlation technique used on speckle targets requires a narrow transmit beam to offer necessary correlation between signals received on neighbour elements [52]. That is not satisfied in this experiment where only the inner element is used to make a transmit beam. The reason for why it works anyway is that the speckle target in the homogeneous phantom had limited size. The speckle target, a cylinder with diameter 4 mm, gives the same effect as a beam of width 4 mm. This is approximately twice as broad as the optimal focused transducer, but narrow enough to give correlation between signals received on neighbour elements.

Trahey and Freiburger (1991) [86] used a 2D phased array to compare maximum correlation and maximum speckle brightness. The corrections were done on receive, with simulations and a speckle target. They commented that the difference in these techniques mainly results from the signal pairs selected for phase difference measurements. In the cross-correlation technique [30], phase difference measurements are made on the data from each pair of adjacent elements. The phase profile is formed by summing the pair-wise phase differences across the array. The errors accumulate then across the array. In the speckle brightness technique [60] the phase differences are calculated between each element and the sum of echos from all remaining elements, which forms the reference signal for the element being corrected. In an alternative speckle brightness method evaluated, they restricted the contributors to this sum to a selectable size group of elements surrounding the element being corrected. They found this reference group to be better than using the sum of all
remaining elements. They noticed that the speckle brightness techniques performed better than cross correlation for r.m.s. aberration magnitudes of less than 0.16 of a wavelength. Examination of the phase correction profiles indicates that the speckle brightness technique generates large residual errors beyond this point because of one wavelength jumps. Shifts that are a multiple of a wavelength do not seriously degrade the beam profile. The performance of the cross-correlation technique is independent of the size of the phase aberrations. It depends on the decorrelation between signals on adjacent elements and added noise. The simulations also found the speckle brightness method to be more stable than the cross correlation method against noise and inoperable elements. This was explained with the difference in the reference signal. A further study to find the optimal reference signal was done by Ng et.al (1994) [58].

2.4.5 Equipment for phase aberration correction

Since significant aberration exists both in elevation and azimuth [39] [33], phase correction should be performed in both dimensions to be most effective. Kanda et.al (1991) [44] and Trahey and Freiburger (1991) [86] used simulations to show that beam patterns distorted by two-dimensional phase aberration need two-dimensional correction. O’Donnell and Li (1991,1995) [69] [49] extended the cross correlation technique from 1-D arrays to 2-D arrays.

The size of the transducer elements must be smaller than the correlation length of the aberrations. The correlation length is defined as the full width at half maximum of the phase profile’s autocorrelation. The correlation length of the phase aberrations measured through the abdominal wall is 7.9mm [39] and 2.1mm [85] through the female breast. The correlation length of amplitude aberrations in the abdomen is 2.3 mm [39].

The size of the transducer elements must also be so small that signals received in adjacent elements are correlated, even when the received signal is scattered by diffuse scatterer and propagated through an aberrating medium. The correlation between speckle signals received in different points on the transducer can be found from the van Cittert-Zernike theorem that is applied to pulse echo ultrasound by Mallart and Fink in 1991 [52]. The theorem predicts that for two points in an nonapodized, rectangular array with common transmit and receive elements and with no phase aberrations, the correlation coefficient of the speckle echo data received at the two elements is given by $\rho(x) = 1 - \frac{x}{L}$. $L$ is the length of the array and $x$ is the distance between the two array points. Note that the speckle echo data must be received from the electronic focus of the array. By using speckle echo outside the focal zone or if phase aberration is introduced, the transmitted beam is broadened. A broader emitted beam will give a lower correlation between back-scattered signals received on adjacent elements than predicted by the formula. The distance between the transducer elements must be small compared with the size of the transducer to ensure high correlation between neighbour elements even in the aberrated case.
Trahey and Freiburger (1991) [86] and Ng et al. (1994) [58] evaluated the effects of interelement correlation, missing elements, and system noise on phase correction algorithms in two dimensions. They concluded that the important issue in phase correction is not the method used in measuring arrival time differences between two signals, but rather the geometry of the reference used for this measurement. A geometry should be chosen such that the reference signal from it is stable and highly correlated with the signal being corrected. A large reference region would be highly stable and insensitive to noise and missing elements, but for a diffuse scatterer the reference signal would have poor correlation with the signal to be corrected. A small reference region would produce a highly correlated signal, but this signal would be easily affected by noise and missing elements. In the presence of mild aberrations, larger reference regions would be desirable since neighbouring elements are more correlated.

Ng et al. (1994) [58] simulated cross correlation corrections of phase aberrations typical for the breast. They used a speckle target and a 5 × 80 element array (10.5mm × 12.32mm). The correlation was 0.99 between adjacent elements in azimuth and 0.8 in elevation in the unaberrated case. With phase aberration typical for the female breast the correlation was simulated to decrease to 0.96 and 0.4, respectively. Another simulation showed that the estimated time delay between two signals with correlation coefficient 0.4 had a standard deviation of 10 ns. A correlation coefficient of 0.96 gave a std≈1 ns. To do phase corrections, different reference signals were used. The method using a reference signal based on the 6 or 12 last corrected elements gave best results. Using the last 6 corrected elements means to find a reference signal from a transducer area of 1mm × 2.1mm, while 12 elements cover an array area of 2mm × 2.1mm. This method decreased the standard deviation of the phase aberrations from approximately 40 ns to 13 ns in the first iteration and to approximately 7 ns after the second iteration. Using the signal in the neighbour element as a reference, the phase error was reduced to about 20 and 12 ns, after first and second iterations, respectively.

The main conclusion is that it is possible to do phase aberration correction with only five elements in the elevation direction.

2.5 Limitations and improvements of phase aberration correction

In the previous section it is concluded that we can use signals from diffuse scatterer to estimate phase aberrations. It does not matter if we use 'max correlation' or 'max speckle brightness' as optimization criterion. It is the size of the reference group used under the estimation that is the fundamental question. To do the corrections, we need a 2D array where the elements are smaller than the correlation length in the phase aberrations. The elements must also be small compared to the correlation structure in the back scattered signal. If we use data from the focal zone, which gives highest correlation, it is enough with five elements in the elevation direction.
The next question is how much that can be corrected by phase aberration correction only, and how we can do even better.

2.5.1 Beam-profile improvements by phase aberration correction

Zhu and Steinberg (1993) [103] did some experiments to explore the effect of phase aberration correction in female breasts. They used a single source of width 1.49 mm to emit a pulse with frequency 3 MHz. The emitted wave propagated through baby oil (c=1430 m/s) and a female breast, totally 12 cm. The active receiving aperture was 96 mm with 64 elements. The element spacing was 1.5 mm and the height of the element was 10 mm. Beam-Profiles through oil, through breast without phase correction and through breast with phase correction were compared. In a typical example the average sidelobe floor (ASF) was -37.4dB, -9.6dB and -19.8dB, respectively. The peak side lobe floor (PSF) was -31.3dB, -2.4dB and -15.2dB, respectively. These results indicate that we can do important improvements in breast imaging by phase aberration correction. But, even when all phase errors are corrected the side-lobes are higher than in the oil example. Zhu and Steinberg concluded that these high side-lobe levels are due to the amplitude distortion of the wavefront.

Liu and Waag (1994) [50] used measurements [39] to evaluate the effect of phase aberration correction in the abdominal wall. A hemispheric transducer that emulates a point source was placed in a water tank and the transmitted pulse was measured in a two-dimensional aperture by mechanically translating a one-dimensional, 128-element array over 32 elevations. Different human abdominal wall specimens were placed in front of the receiving aperture. The -10 and -20 dB width in the array, the elevation and the depth (time) direction of the beam-profiles in the focal zone were compared. The average -10 dB width in the array direction was 2.0/4.4/2.2 mm for ideal/aberrated/phase corrected beam-profile, respectively. The average -20 dB width in the array direction was 2.9/13.6/4.6 mm for ideal/aberrated/phase corrected beam-profile, respectively. Similar results were obtained in the elevation direction. This indicates that phase aberration correction alone can remove the broadening at the -10 dB width. It even work well at the -20 dB width, but the width of the corrected beam is here not as small as in the homogeneous case. Note also that the -20 dB width in the time direction is not influenced by propagation through abdominal wall specimens.

2.5.2 Wavefront corrections using Time Reversal Mirror (TRM)

Optical phase conjugation is a phase distortion compensation method applied to monochromatic fields. A wave distorted by the propagation through an inhomogeneous medium is reflected by the phase-conjugate mirror (PCM), leading to a second wave that is the conjugate of the first. The distortions of the second wave are cancelled after the back propagation through the medium. This process can be
used to focus on a reflective target that may behave as a source after being illuminated. However, this method ignores the shape modifications of the signal due to propagation in an inhomogeneous medium. The time reversal mirror (TRM) is a generalization of the optical phase-conjugated mirror (PCM) in the sense that the TRM applies to pulsed broad-band signals rather than to monochromatic ones. TRM take care of shape distortions of the pulse. Basic principles, theory and experimental verification of the time reversal mirror has been presented by Fink et al (1991-1993) [72] [29] [98] [23] [24].

Time reversal mirror can be used to focus through inhomogeneous media on a reflective target that behaves as an acoustic point source after being irradiated. The first wave transmitted from the transducer is reflected by the target. The divergent pressure field reflected from the acoustic point source is received and stored in shift registers at the transducer. One signal for each transducer element is stored. The signals are reversed in time and then retransmitted. The pressure field is now converted into a convergent wave focusing on the source. If the medium contains several reflectors, the TRM can be iterated. As this process is iterated, the ultrasonic beam will select the target with the highest reflectivity. This is useful in lithotripsy in order to do automatic focusing to the stone position through inhomogeneous tissue. They show experimentally that the iterative TRM process is able to select one of the kidney stones and to focus on a small portion of it. The method can even allow small movements in the stone position. Prada et al (1993) [73] have shown how it is possible to focus to weaker point reflectors than the most reflective one.

The reciprocity theorem says that the position of a point source and an observer can be reversed without altering the observed acoustic pressure. The reciprocity theorem is valid in homogeneous as well as in lossless inhomogeneous media (Landau L.D. 1959, Fluid mechanics). This is the fundamental part used to prove that time reversal mirror optimizes the focusing. Let \( E_i \) be an element on the transducer and let \( A \) be a point reflector. The reciprocity theorem states that the transfer function from \( E_i \) to \( A \) is equal to the transfer function from \( A \) to \( E_i \). According to the matched filter theory an optimal signal at \( A \) is made if we from \( E_i \) transmit the time reversed signal received from \( A \). Another point in the argumentation was that \( p(r, -t) \) is a solution of the wave equation if \( p(r, t) \) is a solution. This is not true if absorption is included, and the effect of TRM will in that case be reduced.

In theory the time reversal method needs a closed surface (cavity), where the waves are time reversed. The closed surface must contain the medium of interest and the initial source. The method is simplified by replacing the closed cavity by a plane time reversal mirror of finite size. This will limit the performance of the method since it is not possible to time reverse all signals scattered from the point source. The consequences of replacing the closed surface with a mirror get worse for strong inhomogeneities. First, it is necessary to measure the pressure field on a very long time interval to take into account the multiple scattered waves that decay slowly; Secondly, the scattered field is radiated in all directions and a mirror cannot measure the complete information needed to optimize the true reversal. In
comparison, in a weakly inhomogeneous medium that induces only single scattering
(1.order Born approx) the measurement of the distorted pressure field requires a
shorter time duration.

Cassereau and Fink (1994) [25] presented a theoretical formulation and numerical
results for focusing with a plane time-reversal mirror through a plane interface
separating two homogeneous fluids. In a further study [26] they demonstrated that
the TRM method can be used in ultrasonic non destructive testing to detect small
defects in solid media in water. By experiments and simulations they showed that
the TRM can be used to focus on defects through a liquid-solid interface. The
technique is self adaptive and only needs the presence of a defect in the sample of
interest. They concluded that the TRM focused better than phase correction only.

2.5.3 Wavefront corrections using Back-propagation
Liu and Waag (1994) [51] used a model where amplitude and shape distortions de­
velop as the wavefront propagates in a uniform medium after passing through a
phase screen that only causes time shifts. They presented a method to correct such
aberrations on receive using waveforms emitted by a pointlike source. The wave is
distorted by propagation through specimens from the abdominal wall in water. The
distortions are compensated by back-propagation of the received wavefront using the
angular spectrum method. The waveforms were first corrected for geometric path
and then back-propagated over a sequence of increasing distances. At each distance,
a waveform similarity factor was calculated. The back-propagation of waveforms to
the distance of maximum waveform similarity is followed by time-shift compensa­
tion. The similarity function measures the shape-similarity and is not influenced by
the amplitude. Another criterion is necessary to measure the amplitude similarity.
The time-shift compensation after the back-propagation is done with an adaptive
reference signal. The estimated phase aberrations are smoothed to remove outliers.
The final beam-profiles on receive are compared. Time-shift compensation in the
2D receiving aperture decreases the sidelobe level compared to the uncompensated
data. However, back-propagation followed by time-shift compensation decreased the
sidelobe level even more. Even after back-propagation the sidelobe level was not as
low as in the ideal case. This may be due to the fact that even though the amplitude
aberrations were reduced by the back-propagation they were not removed.

2.5.4 Time Reversal Focusing compared with Time Delay Focusing
Wu et. al (1991) [97] ([53]) did an experimental comparison between time reversal
focusing and time delay focusing. The beam profile in the focal plane of a 3 MHz
linear array was measured with a hydrophone. The distance between the transducer
and the hydrophone was constant, while the depth of an aberrating layer between
them was changed. The measured defocusing was largest for an aberrating layer close
to the transducer. To do corrections, they used information in the backscattered
signal from the hydrophone, i.e a point scatterer. The adaptive time delay focusing
decreased the side-lobes well when the aberrator was close to the transducer. The effect of the method is reduced for increasing distances. The time reversal focusing decreased the side-lobes well for all distances between the transducer and the aberrator.

Ødegaard (1995) [64] presented simulations that explain why the time reversal method works better than the time delay method. Ødegaard simulated a phase aberrated wavefront under propagation. He demonstrated that a phase aberrated wavefront generate amplitude aberrations, that increase with the propagation distance. Such amplitude errors will increase the sidelobe level. A phase aberrating layer close to the transducer will generate phase errors only and may then be completely corrected by time delay correction. A phase aberrating layer far away from the transducer will generate both phase and amplitude error and will then not be completely corrected by time delay correction only. The time reversal method correct for phase, amplitude and even shape aberrations, and will thus do corrections wherever the aberrator is. However, note that the TRM needs a point reflector while the time delay methods also work for diffuse scatterer. The TRM is constructed for optimal transmit focusing, but a back-propagation algorithm is a possible method to use the TRM principle at receive. The time delay methods can be used to do corrections on both transmit and receive.
3 Wave propagation in a layered medium

A theoretical description of wave propagation in an inhomogeneous medium is an important tool to explore phase aberrations. An analytic solution that is valid for arbitrary inhomogeneities is hard to find. Some limitations are therefore necessary. A study of the anatomy in the human body (fig. 21 and 31) shows that the human tissue has a layered structure. The dome used with annular transducers is also layered, and the theory section is therefore focused on wave propagation in a layered medium. The deduced theory is valid for homogeneous layers with relatively smooth boundaries. The theory may also be extended to allow smooth variations within each layer. Since the main purpose of the theory is to study phase aberrations, multiple reflections are neglected. Anyway, the energy reflected at a boundary between fat and muscle is only 1% of the transmitted energy, and the effect of multiple reflections between soft tissue layers will therefore be weak in human tissue.

The wave equation is deduced in section 3.1. The wave equation allows smooth variations in the medium parameters, but no discontinuities. Discontinuities will be taken care of by boundary conditions. Absorption is included in the wave equation.

To solve the deduced wave equation, several techniques can be used. Since we want to study the phase of the wave, the Rytov approximation [1] may be a natural choice. Its weakness is that it assumes smooth inhomogeneities, and it can thus not be used on media with discontinuous tissue parameters. Another possibility is the Finite Element Method (FEM). This method can handle all kinds of discontinuities. The problem is that the method requires much computer memory to study interesting cases. Here, it is chosen to use the ray-tracing approximation. It allows a direct study of the phase aberrations, and it is found to handle relative rough boundaries well.

First, the wave equation is solved for a point source in a layered medium. These calculations can mainly be found in acoustical text books. One special point here is that absorption is included. It is also shown that smooth spatial mass density variations within each layer can be allowed.

Finally, an integral equation to calculate the properties of an emitted beam after it has propagated through a layered medium is deduced. This is done by following the known development of the Kirchhoff-Helmholtz integral theorem for a homogeneous medium. The special point in this case is that the deduced solution for a point scatterer in a layered medium was chosen as the Greens function.

The deduced integral equation is first solved for continuous waves, but by using the Fourier transform it is extended to be valid for pulsed waves also.

Finally, the integral equation is discretized to be implemented in a computer.
3.1 Development of the wave equation

3.1.1 The basic equations

The derivation of the wave equation is based on three equations:

1. Mass is neither created nor destroyed. The differential form of the law of conservation of mass, also known as the continuity equation is exactly deduced in [18] or [7]:
\[ \frac{\partial}{\partial t} \rho(r, t) + \nabla[\rho(r, t)u(r, t)] = 0 \]  \text{(1)}
Here, \( \rho \) is the mass density, \( u \) is the particle velocity, \( r = (x, y, z) \) is the position and \( t \) is the time.

2. The rate of change for momentum of a portion of the medium equals the force applied to it (Newton’s second law). Let \( J \) be the surface of a portion of the medium. The total force on this portion is assumed to be the integral of \( p(r, t)n \) over the surface \( J \). The function \( p(r, t) \) is the pressure and \( n \) is the vector normal to the surface. Note that the force is in the direction \( n \) and that the force acts perpendicular to the surface \( J \), i.e., there are no tangential forces. This implies neglecting viscosity. The differential form of the law of balance of momentum for a nonviscosity medium is developed exactly in [18] or [7]:
\[ \rho(r, t) \frac{\partial}{\partial t} u(r, t) + \rho(r, t)[u(r, t)\nabla]u(r, t) = -\nabla p(r, t) + \rho(r, t)b \]  \text{(2)}
The term \( \rho(r, t)b \) is the effect of gravity.

3. The third equation models the relationship between mass density and pressure. An equation by Stokes,
\[ p(r, t) = \frac{1}{K(r)} \left( \frac{\rho(r, t) - \rho(r)}{\rho(r)} \right) + \mu_0(r, \omega) \frac{\partial}{\partial t} \left( \frac{\rho(r, t) - \rho(r)}{\rho(r)} \right), \]  \text{(3)}
is useful for this purpose ( [14] p220). Experience has shown that eq. 3 is valid for fluids. The equation models how efficient the pressure field compresses the material. The first term models the normalized mass density changing to be proportional to the pressure \( p(r, t) \). The proportionality is given by the compressibility; \( K(r) \). The second term is a correction term to correct for sound absorption. When the mass is compressed or decompressed, energy will transform to heat. This sound absorption is assumed to be proportional to the time derivative of the normalized mass density. The proportionality constant is \( \mu_0(r) \), and may be a function of frequency. Eq. 3 may be rewritten as:
\[ \rho(r, t) = \rho(r) + \rho(r)K(r)p(r, t) - \mu_0(r, \omega)K(r)\frac{\partial}{\partial t}\rho(r, t) \]  \text{(4)}
3.1.2 Approximations

To simplify the further calculations three approximations on eq.2 is done.

1. The term $\rho(r, t)\partial u(r, t)/\partial t$ in eq. 2 is the effect of gravity. Although gravity is always present, it can be neglected if the frequency is larger than $\frac{g}{c}$ ( [18] p.9). $g = 9.8 m/s^2$ is acceleration due to gravity and $c$ is the speed of sound. In human tissue, $c$ is approximately 1540 $m/s$. Normally frequencies between 1-10 MHz is used in medical ultrasound imaging and the approximation is therefore good.

2. The second approximation is to assume that:

$$\rho(r, t)\frac{\partial}{\partial t} u(r, t) + \rho(r, t)[u(r, t)\nabla u(r, t)] \approx \rho(r, t)\frac{\partial}{\partial t} u(r, t)$$  

(5)

The approximation is good if $u << c$. It is reasonable to use this approximation in medical ultrasound, since the speed of sound, $c$, is considerably larger than the particle velocity $u$ in liquids and solids ( [17] p229).

3. The first two approximations are common, while this one is suggested by the author. This approximation gives a homogeneous wave equation where the mass density and the compressibility are allowed to vary spatially. The suggested approximation is:

$$\frac{\partial}{\partial t} \rho(r, t) u(r, t) = \rho(r, t)\frac{\partial}{\partial t} u(r, t) + u(r, t)\frac{\partial}{\partial t} \rho(r, t) \approx \rho(r, t)\frac{\partial}{\partial t} u(r, t)$$  

(6)

The evaluation of this approximation is done by assuming plane waves and by using eq. 1. Eq. 6 is valid if $u << c$, i.e. if $u << c$ and the mass density $\rho$ do not vary to rapidly compared to a wave length $\lambda$.

3.1.3 A wave equation with absorption included

The three basic equations can now be combined in two equations:

$$0 = \frac{\partial}{\partial t} [\rho(r, t)u(r, t)] + \nabla p(r, t)$$  

(7)

$$0 = \nabla[\rho(r, t)u(r, t)] + \rho(r)K(\tau)\frac{\partial}{\partial t} p(r, t) + \mu_0(\tau, \omega) K(\tau) \nabla [\frac{\partial}{\partial t} [\rho(r, t)u(r, t)]]$$  

(8)

The first one is eq. 2 modified by using the three approximations. The second equation is the time derivative of eq. 4 combined with eq. 1. Eq. 7 and 8 will give the wave equation. $\Phi = \Phi(r, t)$ is defined by:

$$\nabla \Phi(r, t) = \rho(r, t)u(r, t)$$  

(9)
It is possible to find the pressure $p$ and the particle velocity $\mathbf{u}$ from $\Phi$. Eq. 7 and 9 give:

$$\mathbf{u}(\mathbf{r}, t) = \frac{1}{\rho(\mathbf{r}, t)} \nabla \Phi(\mathbf{r}, t) \approx \frac{1}{\rho(\mathbf{r})} \nabla \Phi(\mathbf{r}, t)$$

(10)

$$p(\mathbf{r}, t) = p_0 - \frac{\partial}{\partial t} \Phi(\mathbf{r}, t) = -\frac{\partial}{\partial t} \Phi(\mathbf{r}, t)$$

(11)

Eq. 9 and 11 into eq. 8 give a wave equation with absorption:

$$\nabla^2 \Phi(\mathbf{r}, t) - \frac{1}{c^2(\mathbf{r})} \frac{\partial^2}{\partial t^2} \Phi(\mathbf{r}, t) = -\mu_0(\mathbf{r}, \omega)K(\mathbf{r}) \frac{\partial}{\partial t} \nabla^2 \Phi(\mathbf{r}, t), \quad c^{-2}(\mathbf{r}) = \rho(\mathbf{r})K(\mathbf{r})$$

(12)

The term at the right side is due to absorption. Note that both the mass density and the compressibility are functions of space so that the speed of sound $c$ also is a function of space. The allowed variations are given by the validity of the approximation in eq. 6. The approximation is not valid if the variations in mass density are discontinuous, but will be good for smooth variations. In the further calculations eq. 12 will be used. To account for discontinuous tissue parameters, the boundary problem will be solved.

### 3.1.4 The Helmholtz equation with absorption

Eq. 12 is a wave equation with absorption. The wave equation is linear so if a continuous wave is a solution, then a sum of continuous waves is also a solution. From Fourier analysis we know that it is possible to make all kinds of pulses from continuous waves, so assuming continuous waves is not a limitation. By assuming continuous waves, $\Phi(\mathbf{r}, t) = \Phi(\mathbf{r}) e^{i\omega t}$, the wave equation in 12 takes the form:

$$\nabla^2 \Phi(\mathbf{r}) + k^2(\mathbf{r})\Phi(\mathbf{r}) = 0, \quad k^2(\mathbf{r}) = \frac{\omega^2}{c^2(\mathbf{r})} \frac{1}{1 + i\mu_0(\mathbf{r}, \omega)K(\mathbf{r})\omega}$$

(13)

This equation is equal to Helmholtz equation, but $k$ is now complex. In the further calculations it is assume that:

$$[\mu_0(\mathbf{r}, \omega)K(\mathbf{r})\omega]^2 \ll 1$$

(14)

The complex $k$ may thus be approximated to:

$$k^2(\mathbf{r}) \approx k_r^2(\mathbf{r}) - ik_c^2(\mathbf{r}) \quad \text{where}$$

$$k_r(\mathbf{r}) = \frac{\omega}{c(\mathbf{r})} \quad \text{and} \quad k_c(\mathbf{r}) = k_r(\mathbf{r})\sqrt{\mu_0(\mathbf{r}, \omega)K(\mathbf{r})\omega}$$

(15)

The approximation in eq. 14 will be used in further calculations. The validity of the approximation is discussed in section 8.1.2. It is found to be valid for both water and human tissue.
3.2 The ray-tracing approximation

The Helmholtz equation without absorption is solved by using the ray-tracing approximation [18]. Here, the same method is used to solve the Helmholtz equation with absorption. Assume:
\[ \Phi(\mathbf{r}) = A(\mathbf{r})e^{-ik_{\text{r}_0}S(\mathbf{r})} \quad (16) \]
where \( k_{\text{r}_0}S(\mathbf{r}) \) is the phase of the wave, \( k_{\text{r}_0} = \frac{\omega}{c_0} \) and \( c_0 \) is the reference velocity. \( S(\mathbf{r}) \) is the acoustical distance. The acoustical distance is the geometrical distance multiplied by the refraction index \( n(\mathbf{r}) = \frac{c}{c_0} \). By inserting eq. 16 into eq. 13 and separating the real and imaginary terms we get two equations. The real terms give
\[ (\nabla S(\mathbf{r}))^2 = n^2(\mathbf{r}) + \frac{\nabla^2 A(\mathbf{r})}{k_{\text{r}_0}^2 A(\mathbf{r})} \]
and the complex terms give
\[ 2\nabla A(\mathbf{r}) \nabla S(\mathbf{r}) + A(\mathbf{r}) \nabla^2 S(\mathbf{r}) = -\mu_0(\mathbf{r}, \omega)K(\mathbf{r})A(\mathbf{r})\omega k_{\text{r}_0}n^2(\mathbf{r}) \quad (18) \]
The ray-tracing approximation is:
\[ \frac{\nabla^2 A(\mathbf{r})}{k_{\text{r}_0}^2 A(\mathbf{r})} \ll n^2(\mathbf{r}) \quad (19) \]
This approximation reduces eq. 17 to the eikonal equation, i.e. an equation for the phase:
\[ (\nabla S(\mathbf{r}))^2 = n^2(\mathbf{r}) \quad \Rightarrow \quad \nabla S(\mathbf{r}) = n(\mathbf{r})\mathbf{s}(\mathbf{r}) \quad (20) \]
The vector \( \mathbf{s}(\mathbf{r}) \) is a normalized vector for the direction of the wave propagation at a position \( \mathbf{r} \). Eq. 18 is the amplitude equation with absorption included. The solution of the wave equation with absorption is thus:
\[ \Phi(\mathbf{r}, t) = A(\mathbf{r})e^{i(\omega t - k_{\text{r}_0}S(\mathbf{r}))} \quad (21) \]
The acoustical distance, \( S \), is given by the eikonal equation (eq. 20), while the amplitude, \( A \), is given by the amplitude equation (eq. 18).

3.2.1 The pressure and the particle velocity

Putting eq. 21 into eq. 10 and 11 and using the eikonal equation (eq. 20) gives:
\[ p(\mathbf{r}, t) = -\frac{\partial}{\partial t} \Phi(\mathbf{r}, t) = -i\omega \Phi(\mathbf{r}, t) \quad (22) \]
\[ u(\mathbf{r}, t) = \frac{1}{\rho(\mathbf{r})} \nabla \Phi(\mathbf{r}, t) = \frac{1}{\rho(\mathbf{r})} \left[ \nabla A(\mathbf{r}) - i\omega k_{\text{r}_0}n(\mathbf{r}) \mathbf{s}(\mathbf{r}) \right] \Phi(\mathbf{r}, t) \quad (23) \]
By neglecting the term \( \frac{\nabla A(\mathbf{r})}{A(\mathbf{r})} \) in eq. 23 this approximation is given:
\[ u(\mathbf{r}, t) \approx \frac{p(\mathbf{r}, t)}{\rho(\mathbf{r})c(\mathbf{r})} \mathbf{s}(\mathbf{r}) \quad \text{if} \quad \left| \frac{\nabla A(\mathbf{r})}{A(\mathbf{r})} \right| \ll k_{\text{r}_0}n(\mathbf{r}) \quad (24) \]
The vector $\mathbf{s}(r)$ is the direction of the unit vector of the ray. In a homogeneous medium ($A(r) = A/r$) a point scatterer emits a spherical wave. The approximation is then valid if $2\pi r >> \lambda$. The approximated relationship between pressure and particle velocity (eq. 24) may be used for areas several wavelengths from the point scatterer, i.e. when the wave is approximately plane.

### 3.2.2 The intensity

We are interested in the intensity of an acoustical wave. It is given by the time average of pressure multiplied by particle velocity. The unit of intensity is energy per area and time, i.e. power/area. For continuous waves we can use the formula

$$I(r) = \frac{\omega^2}{2\rho c(r)} A^2(r) \mathbf{s}(r)$$

Note that the intensity has the same direction as the ray. Note also that the approximation in eq. 24 gives the same intensity.

### 3.2.3 The ray path

Let us look at the wave propagation through a boundary between two homogeneous materials with different speed of sound. The refraction index $n(s)$ will be discontinuous at the interface between the two materials. This produces refraction and reflection. It can be shown from the eikonal equation that the reflected and refracted rays lays in the same plane as the incident ray and the normal to the interface [2]. The angle of refraction in this plane satisfy Snell’s law:

$$n_{\text{out}} \sin \alpha_{\text{out}} = n_{\text{in}} \sin \alpha_{\text{in}}$$

To calculate the refraction of an arriving ray with direction vector, $\mathbf{s}_{\text{in}}$, let us split $\mathbf{s}_{\text{in}}$ into a component that is parallel to the boundary surface, $\mathbf{s}_{\text{in}}\|$, and a component that is perpendicular to the surface, $\mathbf{s}_{\text{in}}\perp$. Let $|\mathbf{s}_{\text{in}}| = |\mathbf{n}| = 1$.

$$\mathbf{s}_{\text{in}}\perp = \cos(\alpha_{\text{in}}) \mathbf{n} = (\mathbf{s}_{\text{in}} \cdot \mathbf{n}) \mathbf{n}$$

$$\mathbf{s}_{\text{in}}\| = \mathbf{s}_{\text{in}} - \mathbf{s}_{\text{in}}\perp = \mathbf{s}_{\text{in}} - (\mathbf{s}_{\text{in}} \cdot \mathbf{n}) \mathbf{n}$$

The components of the refracted ray, $\mathbf{s}_{\text{out}}$, are:

$$\mathbf{s}_{\text{out}}\perp = \frac{\mathbf{s}_{\text{out}} \cdot \cos(\alpha_{\text{out}}) \mathbf{n}}{\| \mathbf{s}_{\text{out}} \|}$$

$$\mathbf{s}_{\text{out}}\| = \frac{\mathbf{s}_{\text{out}} \cdot \sin(\alpha_{\text{out}}) \mathbf{s}_{\text{in}}\|}{\| \mathbf{s}_{\text{in}}\|}$$

Normalization of the refracted ray gives:

$$\frac{\mathbf{s}_{\text{out}}}{\| \mathbf{s}_{\text{out}} \|} = \frac{\mathbf{s}_{\text{out}}\perp + \mathbf{s}_{\text{out}}\|}{\| \mathbf{s}_{\text{out}} \|} = \cos(\alpha_{\text{out}}) \mathbf{n} + \sin(\alpha_{\text{out}}) \frac{\mathbf{s}_{\text{in}}\|}{\| \mathbf{s}_{\text{in}}\|}$$

The angle $\alpha_{\text{out}}$ is given by eq. 26 and $\mathbf{s}_{\text{in}}\|$ is given by eq. 28.
3.2.4 The amplitude

The amplitude of the ray is given by eq. 18, which can be rewritten as:

\[ \nabla (A^2 \nabla S) = -\mu A^2 n \quad \text{where} \quad \mu = \mu_0 K \frac{\omega^2}{c} \]  

(32)

Let us combine this equation with the expression for the intensity in eq. 25 and the eikonal equation, i.e. with \( I = \frac{\omega^2}{2pc} A^2 S \) and \( \nabla S = n_S \). Note that smooth spatial variations of \( \rho \) (eq. 6) is allowed. Eq. 32 can then be written as:

\[ \nabla (\rho I) = -\mu \rho I \quad \iff \quad \nabla (I) = -(\mu + \frac{s \nabla \rho}{\rho}) I \]  

(33)

Using volume integration over a ray tube at both sides of the equation and using the divergence theorem on the left side gives:

\[ I_2 dB_2 - I_1 dB_1 = - \int_V (\mu + \frac{s \nabla \rho}{\rho}) I dV \]  

(34)

It means that power coming out of a ray tube minus power coming in is equal to the loss along the ray tube. The parameter \( \mu \) gives the absorption loss, while \( \frac{s \nabla \rho}{\rho} \) is due to energy fluctuations caused by smooth spatial variations in the mass density. The solution for a medium with the constants \( \mu \) and \( \rho \) is given by:

\[ IdB = I_1 dB_1 e^{-\mu \sigma} \]  

(35)

The variable \( \sigma \) is the geometrical distance along the ray and \( dB \) is the area of the cross section of the ray tube. At the interface between two homogeneous materials, some of the energy is reflected and some is transmitted. By requiring continuity...
of the pressure and the normal velocity, and also that the total amount of power entering the boundary must be equal to the total amount of power leaving, we get:

\[
\frac{\text{Transmitted ray power}}{\text{Incident ray power}} = \frac{\text{tr. intensity} \cdot \text{tr. area}}{\text{in. intensity} \cdot \text{in. area}} = 1 - R^2
\]  

(36)

where

\[
R = \frac{Z_{\text{out}}/\cos \alpha_{\text{out}} - Z_{\text{in}}/\cos \alpha_{\text{in}}}{Z_{\text{out}}/\cos \alpha_{\text{out}} + Z_{\text{in}}/\cos \alpha_{\text{in}}}
\]  

(37)

\[Z = \rho c \text{ and } \alpha \text{ is the angle between the normal of the boundary and the ray. The approximated relation between pressure and particle velocity in 24 is used to deduce equation 36 and 37. The combination of eq. 35 and eq. 36 for a medium with } n \text{ layers gives a relation between the power at the start } (I_i dBi) \text{ and the end } (I_f dB_j) \text{ of the ray path:}

\[I_j dB_j = I_i dB_i e^{-\sum_{k=1}^{n_i} \mu_k \sigma_k} \prod_{m=1}^{n-1} (1 - R^2_{m(m+1)})
\]  

(38)

If we combine the equation for intensity (eq. 25) and eq. 38, we get:

\[A_j = A_i \sqrt{\frac{\rho_j c_j dBi}{\rho_i c_i dB_j}} e^{-\sum_{k=1}^{n_i} \mu_k \sigma_k} \prod_{m=1}^{n-1} (1 - R^2_{m(m+1)})
\]  

(39)

The attenuation is divided into three terms. The first, \(dB_i/dB_j\), is due to a broadening of the ray. The second term represents absorption due to transformation of energy into heat, and the last term represents reflection losses. To calculate the amplitude at the end of the ray tube, we need the amplitude at the start of the ray. If the start position of the ray is a point scatterer in a homogeneous medium, it will emit spherical waves. The start amplitude \(A_i\) at a distance, \(\epsilon_i\), from the source is thus given by ( [13] p.159):

\[A_i = -\frac{1}{4\pi \epsilon_i}
\]  

(40)

In the special case without absorption and reflection loss, eq. 39 is reduced to \(A_j = -\frac{1}{4\pi r}\), i.e. the well known spherical attenuation.
3.3 An integral solution for wave propagation in a layered medium

3.3.1 Development of an integral solution for continuous waves

Assume a source in the point \( r_i = (x_i, y_i, z_i) \). The wave-field \( G_i \) in \( r_j = (x_j, y_j, z_j) \), is thus given by:

\[
\nabla^2 G_i(r_i, r_j) + k^2 G_i(r_i, r_j) = \delta(r_j - r_i)
\]

For a layered medium, \( G \) is found to be:

\[
G(t_i, t_j) = A(t_i, t_j) e^{-i\omega s(t_i, t_j)}
\]

\[
\nabla G(t_i, t_j) = \left( \frac{\nabla A(t_i, t_j)}{A(t_i, t_j)} - i \frac{\omega}{c(t_j)} s(t_i, t_j) \right) G(t_i, t_j)
\]

\[
\approx -i \frac{\omega}{c(t_j)} s(t_i, t_j) G(t_i, t_j)
\]

The approximation in eq. 44 is the same as used earlier in eq. 24. \( S(t_i, t_j) \) is the acoustical distance along the ray path from the point \( r_i \) and to the point \( r_j \). The ray path is the path that satisfies Snell's law. The amplitude \( A(t_i, t_j) \) is the wave amplitude after the wave has propagated from point \( r_i \) and to point \( r_j \). The attenuation is due to broadening, absorption and reflection of the ray. The expression for \( A(t_i, t_j) \) is given in eq. 39.

The pressure at a point due to excitation of a transducer, is mathematically equivalent to finding the general solution for \( \Phi \) at any point, when \( \Phi \) and \( \nabla \Phi \) are given on a surface surrounding the observation point. To show this, let us start with the wave equation for \( \Phi \):

\[
\nabla^2 \Phi(t_j) + k^2 \Phi(t_j) = 0
\]

Multiply eq. 45 by \( G \) and subtract the result from the product of eq. 41 with \( \Phi \). By integrating the resulting expression over an arbitrarily volume \( V \), and by applying Gauss's theorem to the integral, we get:

\[
\Phi(t_i) = \int_j [\Phi(t_j) \nabla G(t_i, t_j) - G(t_i, t_j) \nabla \Phi(t_j)] n dJ
\]

\( J \) is any surface enclosing the observation point \( r_i \), and \( n \) is the outward normal from the surface. This result is known as Kirchhoff-Helmholtz's integral theorem. Substituting the solution we have found for \( G \) in eq. 42, we see that we can determine \( \Phi \) at any point if we know \( \Phi \) and \( \nabla \Phi \) on the surrounding surface. By using the approximation for \( \nabla G \) in eq. 44, eq. 46 is modified to:

\[
\Phi(t_i) \approx - \int_j G(t_i, t_j) \left[ i \frac{\omega}{c(t_j)} s(t_i, t_j) \Phi(t_j) + \nabla \Phi(t_j) \right] n dJ
\]

The assumed solution, \( \Phi(t) = A_\Phi(t) e^{-i\omega S_\Phi(t)} \), is used to find an expression for \( \nabla \Phi \) at the transducer surface. The approximation used is \( n \nabla \Phi = (n \frac{\nabla A_\Phi}{A_\Phi} - i k_\alpha n \nabla S_\Phi) \Phi \approx \)
\(-i k_0 n \nabla S_\phi \Phi = -i \frac{\Phi}{c} n S_\phi\). This means assuming that the spatial fluctuations of \(A_\phi\) normal to the transducer surface is much smaller than \(\frac{\Phi}{c}\). Note that the phase and the amplitude of \(\Phi\) are allowed to vary rapidly along the transducer, but not in the direction normal to the transducer. That allow us to calculate the effect of rough apodization and electronic delays. By assuming that the propagation direction of the wave, \(S_\phi\), is normal to the transducer, we get \(n S_\phi = -1\). This is because the normal vector, \(n\), points into the transducer, while the propagation direction is directed out of the transducer. We get \(n \nabla \Phi \approx \frac{i \Phi}{c}\), and eq. 47 is modified to:

\[
\Phi(I, f) = -i \int J \Phi(I, f) G(I, J) \left( \frac{\omega}{c(I)} \right) \left[ s(I, J) n + 1 \right] dJ
\]

Eq. 48 is an equation for continuous waves. For continuous waves Eq. 11 gives \(p(I, J) = -i \omega \Phi(I, J)\), and eq. 48 may therefore be written as:

\[
\Phi(I, f) = \int J \frac{1}{c(I, J)} p(I, J) G(I, J) (1 + s(I, J) n) dJ
\]

### 3.3.2 Development of an integral solution for pulsed waves

Eq. 49 is developed for continuous waves, but by using the Fourier transform, we can develop an integral equation valid for pulsed waves.

\[
h(t) = \int_{-\infty}^{\infty} H(f) e^{i 2 \pi ft} df
\]

\[
H(f) = \int_{-\infty}^{\infty} h(t) e^{-i 2 \pi ft} dt
\]

The Fourier transform \(H(f)\) of a signal \(h(t)\) is a frequency-domain representation of the signal, and it specifies relative amplitudes of the various frequency components of the signal ([9] p.18). The frequency is given by \(f = \frac{n}{2 \pi}\). The signal is uniquely defined by either representation.

To develop an integral equation for pulsed systems, let us start with eq. 49:

\[
\Phi(I, f) = \int J \frac{1}{c(I)} p(I, J, f) G(I, J, f) (1 + s(I, J) n) dJ
\]

Both sides are multiplied by \(e^{i 2 \pi ft}\). The wave equation is a linear equation, so a sum of solutions is also a solution of the wave equation. Let us take solutions for different frequencies and add them together. We can make all kind of signals by adding different continuous waves. We can even make pulses. Let the sum go to the limit, i.e. to the integral over all frequencies and use eq. 42 for \(G\). We get:

\[
\int_{-\infty}^{\infty} \Phi(I, f) e^{i 2 \pi ft} df = \int J \int_{-\infty}^{\infty} \frac{1}{c(I)} p(I, J, f) A(I, J, f, t) e^{i 2 \pi f(t - S(I, J)/c_0)} (1 + s(I, J) n) df dJ
\]
From the Fourier transform we see that:

\[ \Phi(\xi_i, t) = \frac{1}{c(\xi_j)} A(\xi_i, \xi_j, f_0) p(\xi_j, t - S(\xi_i, \xi_j)/c_0)) [1 + s(\xi_i, \xi_j) u] dJ \]  

(54)

Note that if we have frequency depended attenuation, i.e. that the absorption cannot be approximated to the absorption at the center frequency, \( f_0 \), of the pulse, we have to use convolution in the time domain or multiplication in the frequency domain. Time derivation and multiplication with \(-1\) at both sides of the integral equation, and using the relation between \( p \) and \( \Phi \) in eq. 11, give:

\[ p(\xi_i, t) = -\int_J \frac{1}{c(\xi_j)} A(\xi_i, \xi_j, f_0) \dot{p}(\xi_j, t - S(\xi_i, \xi_j)/c_0)) [1 + s(\xi_i, \xi_j) u] dJ \]  

(55)

If we assume that the time derivative of the pressure, \( \dot{p} \), is zero over the integral area, except at the transducer, we can let the area \( J \) be the transducer surface. The pressure at position \( \xi_i \) and time \( t \) is thus given by eq. 55. The solution is valid for arbitrary pulses emitted from a transducer with area \( J \) and for an uneven layered medium. The material in each layer is assumed to be homogeneous, and the interface between two materials must be smooth. The boundaries have to be smooth to avoid rapid variations of the amplitude. Note that \( A \) is the amplitude due to a spherical wave from the observation point. It is not the amplitude of the pressure caused of the transducer. The effect of caustics in the focal point of the transducer is therefore avoided.

### 3.3.3 The final integral solution for pulsed waves

It is deduced an integral solution for pulsed waves in a layered medium. Each layer is assumed to be homogeneous. The boundaries may be uneven, but not rough. The integral solution may be used to calculate how a wave propagates through a layered medium. The source is a transducer with area \( J \). The pressure, \( p \), at position, \( \xi_i \), and time, \( t \), is given by eq. 26, 37, 39 and 55. These four equations may be written as:

\[ p(\xi_i, t) = \int_J \frac{1}{c(\xi_j)} A(\xi_i, \xi_j, f_0) \dot{p}(\xi_j, t - S(\xi_i, \xi_j)/c_0)) \left(1 + \frac{s(\xi_i, \xi_j) u}{2}\right) dJ \]  

(56)

\[ A(\xi_i, \xi_j, f_0) = \frac{1}{2\pi \epsilon_0} \left[ \frac{\rho_j c_j}{\rho_i c_i} dB_i e^{-\sum_{k=1}^{\infty} \mu_k \sigma_k} \prod_{m=1}^{n-1} (1 - R_{m(m+1)}) \right] \]  

(57)

\[ R_{m(m+1)} = \frac{Z_{m+1} \cos \alpha_{m+1} - Z_m \cos \alpha_m}{Z_{m+1} \cos \alpha_{m+1} + Z_m \cos \alpha_m} \]  

(58)

\[ \frac{s\sin \alpha_m}{c_m} = \frac{s\sin \alpha_{m+1}}{c_{m+1}} \]  

(59)

The integral is taken over the surface, \( J \), of the transducer. The main terms of the integral are the time derivative of the pressure, \( \dot{p} \), an amplitude factor, \( A \), and a direction factor.
The arguments of the time derivative of the pressure are the transducer position, \( r_j \), and the time it takes to propagate along the ray path between the specified transducer point and the observation point, \( r_i \). The time is given by, \( S(r_i, r_j)/c_0 \), where \( S \) is the acoustical distance along the ray path and \( c_0 \) is a constant reference velocity. The ray path is specified by Snell's law (eq. 59).

The amplitude factor describes how a wave is attenuated by propagating along the ray paths. The attenuation, \( A \), is divided into three terms. The first, \( dB_i/dB_j \), is due to broadening of the ray tube. The rays are drawn from the observation point \( r_i \), and each ray tube is a cone of a spherical wave emitted from the observation point. The opening angle of the ray tube is small. The cross section area of the ray tube, \( dB_i \), will increase with the distance to \( r_i \). This is clue to spherical broadening and clue to refraction of the rays at the boundaries. The factor, \( dB_i \), is the cross section area of the ray tube at a short distance, \( \varepsilon_i \), from \( r_i \). The factor, \( dB_j \), is the cross section area of the ray tube at the transducer.

The deduced attenuation shows that the broadening of a ray-tube should be calculated by assuming that the ray tube start at the observation point, and that the ray-tube is a cone of a spherical wave emitted by a point source at the observation point. It is more intuitively to assume a point source at each transducer point. This implies a change of the start and the end of the ray-tube. This is allowed due to the reciprocity principle [15]. The integral equation can therefore be interpreted as that each point on the transducer emit a spherical wave.

The second amplitude term represents absorption due to transformation of energy into heat. The variable \( \sigma_k \) is geometrical distance along the ray in layer \( k \), and \( \mu_k \) is the corresponding absorption factor. It is found experimentally that \( \mu_k \) can be written as \( \mu_k = \alpha_k f_0^k \) in human tissue. The absorption factor depends on the frequency, \( f \). So far, the center frequency, \( f_0 \), of the pulse is used to calculate absorption along a ray path.

The last attenuation term represents reflection losses. The reflection coefficient, \( R \), is given by the characteristic impedance, \( Z = \rho c \), and the angle, \( \alpha \), between the normal of the interface and the ray. Both the normal vector of the interface and the ray are directed to the transducer.

The direction factor consist of \( \frac{1 + s(r_i, r_j)^2}{2} \), where \( s(r_i, r_j) \) is the unit vector of the ray on the transducer point, \( r_j \). The rays are drawn from the observation point, \( r_i \), and to the transducer point, \( r_j \). The vector, \( \eta_h \), is the normal vector of the transducer surface. Its direction is from the transducer surface and into the transducer. If the observation point is in a homogeneous medium and on the focus of a circular transducer, the direction factor will be equal to one.

To use the integral solution presented in eq. 56-59 some assumptions and approximations must be valid. First, the fundamental equations and approximation done to deduce the wave equation must be valid (eq. 1-6). The assumptions done to deduce the presented integral equation (eq. 56) are:

\[
1 \gg (\mu_0 K \omega)^2 \quad (60)
\]
The critical approximation is eq. 62, which is discussed in section 8.1.2. The approximation is found to be valid for relatively rough boundaries. Note that $A$ is the amplitude due to a spherical wave from the observation point, while $A_{\Phi}$ is the amplitude of the total wave caused by the transducer.

The area of integration, $J$, is approximated to the transducer surface. This is done by assuming that the time derivative of the pressure, $\dot{p}$, is zero on a surface around the medium, except at the transducer.

The integral equation as presented in this section is valid for a layered medium, where each layer is homogeneous. The energy reflected at the boundaries are treated as loss, i.e. that the effect of multiple reflections are neglected. The neglect of multiple reflections require that the variations of the characteristic impedance are small. For human tissue this is a reasonable approximation.

### 3.4 Discretization

The time derivative of the pressure on the transducer surface, $J$, is assumed to be:

$$\dot{p}(\tau_j, t) = a(\tau_j) F(t - \Delta t(\tau_j)), \quad \tau_j \in J,$$

where $a$ is the apodization and $\Delta t$ is a focusing delay. $F$ is the shape of the emitted pulse. The transducer surface is represented with a mesh of points $(\tau_j)$, where each point represent an area $\Delta J_j$. The integral is represented by a finite sum over the points, $\tau_j$, on the transducer surface. The pressure at the observation point $\tau_i = (x_i, y_i, z_i)$ is then:

$$p_i(t) = K \sum_j A_{ij} a_j F(t - \Delta t_j - t_{ij}) \cdot \cos fact \cdot \Delta J_j,$$

where

- $K$ = constant
- $A_{ij} = A(\tau_i, \tau_j, f_0)$ = attenuation along the ray between $\tau_i$ and $\tau_j$
- $a_j = a(\tau_j)$ = apodization on the transducer
- $F$ = The shape of the emitted pulse
- $t$ = time
- $t_{ij} = S(\tau_i, \tau_j)/c_0$ = propagation time between the observation point $\tau_i$ and the transducer point $\tau_j$
- $\Delta t_j = \text{electronic focusing delay on the transducer}$
- $\Delta J_j = \text{The area to the transducer point } j$
- $\cos fact = (1 + S(\tau_i, \tau_j) \cdot n)/2$ = the angular dependency of the emission from the point on the transducer
If \( a_j \) and \( \Delta t_j \) is given, the algorithm consist of:
1) Calculate \( A_{ij} \) and \( t_{ij} \)
2) For each observation time \( t \) evaluate eq. (65) for all observation points \( i \).

### 3.4.1 Calculation of propagation time

Rays are traced from the observation point to all points on the transducer. The ray paths are found by iteration and by using Snell's law. The propagation time, \( t_{ij} \), is given by the distance along the ray path and the velocity of sound in each layer.

### 3.4.2 Calculation of amplitude

The attenuation for each ray is given by eq. 57. The broadening of the ray is found by calculating two neighboring rays. Both with the same start point as the main ray but with different directions. These rays span out a ray-tube. The area of the cross section of the ray tube, \( dB \), can now be calculated. \( dB_i \) is the area of the cross section close to the start of the ray and \( dB_j \) is the area of the cross section at the end of the ray. The rays are drawn from the observation point and to the transducer.

The second term represents absorption due to transformation of energy into heat. The variable \( \sigma_k \) is geometrical distance along the ray in layer \( k \), and \( \mu_k \) is the absorption factor in layer \( k \). It is found experimentally that \( \mu_k \) can be written as \( \mu_k = \alpha_k f \beta_k \). In water, \( \beta \) is equal to 2, while it is close to 1 in human tissue. The absorption factor depends on the frequency, \( f \). So far, the center frequency of the pulse is used to calculate absorption along a ray path.

The last term represents loss of reflection. Eq. 58 and 59 are used to calculate the loss of reflection at each boundary.
4 The simulation program

The simulation program developed in the thesis is included in a larger program called Ultrasim. Ultrasim is a Matlab computer program for ultrasound wave simulation. The program is a tool to increase the user's understanding of acoustic wave propagation in homogeneous and layered media. Ultrasim is developed by:

- Vingmed Sound, Horten, Norway.
- Department of Biomedical Engineering, University of Trondheim, Norway.
- Dept. of Math. Sciences, Norwegian Inst. of Technology, Trondheim, Norway.
- Department of Informatics, University of Oslo, Norway.

The simulation program has been used for evaluating the effect of time delay quantization in beam-forming [42], and for finding the effect of medium-generated phase aberrations [62]. The program was presented in Cannes in 1994 [63]. A verification of the simulation program is presented in 5.3. My contribution is to extend the program from wave propagation in a homogeneous medium to wave propagation through unevenly layered media.

4.1 The simulation principle

The simulation program is based on the integral in eq. 56. The discretized version presented in section 3.4 is used by the computer.

![Figure 5: Illustration of the simulation principle. The transducer is divided into points and the contribution from each transducer point is added at the obs. point.](image-url)
The surface of the transducer is represented by a mesh of points. The signal received at an observation point is the sum of signals from different transducer points (fig. 5).

To find the contribution from the different transducer points, rays are drawn between the observation point and all the transducer points. The ray paths are found by iteration and by using Snell's law (fig. 6). It is assumed that the rays do not cross each other, so each ray represents the emitted signal from an unique transducer point. The signal emitted from each transducer point is delayed and attenuated before they are added up to find the signal received at the observation point. The phase terms are given by the distance along the ray path and the velocities of sound in each layer. The attenuation along the ray path is given by eq. 57, which describes attenuation caused by ray broadening, reflection, and absorption. The angle between the ray and the normal to the transducer surface can also be taken into the calculation.

The simulation principle can be interpreted as a combination of the Rayleigh integral and ray-tracing techniques. The improvement compared with using only the Rayleigh integral is the possibility to handle wave propagation in a layered medium. The improvement of the traditional ray-tracing technique is this method's handling of caustics. Traditional ray-tracing draws rays from the transducer, where each ray is perpendicular to the transducer surface. The method will therefore not give side-lobes in the focus. Improved ray-tracing methods take care of this by using edge techniques at the boundary of the transducer. The method presented here avoids the problem with caustics, because the rays are drawn from the observation point and to all the transducer points. Another important advantage with the presented
simulation method is its possibility to calculate both the acoustic field and the phase and amplitude aberrations caused by the layered medium.

4.2 Specification of the transducer and the medium

The program uses a window-based user interface with menus. The simulation parameters are set in the flag menu and the configuration menu. The transducer flag must be set to annular array if you want to simulate the acoustic field from an annular array transducer, and to a rectangular and curved rectangular array if you want to simulate the field from 1D, 1.5D or 2D array transducers. The focusing flag may be set to fixed or dynamic focus, while the observation flag may be set to a point, a line or a plane. The medium flag may be set to homogeneous, layered 2D or layered 3D. If you want to do simulations in only two dimensions or the medium is axis symmetric, you are recommended to set the medium flag to layered 2D.

To do a more detailed specification of the parameters, you have to use the configuration menu. It is divided into a transducer, excitation, beam-forming, medium and an observation menu:

- In the **transducer** menu, you may set parameters as number of elements, size and curvature of the transducer. You may also set number of points to use in the discretization of the transducer.

- In the **excitation** menu, the emitted signal is specified by parameters as frequency, pulse length and shape of the envelope. It is also possible to use an experimentally measured pulse as input in the simulation program.

- Electronic focusing and apodization are specified in the **beam-forming** menu. Parameters used by the focusing algorithm can also be modified here. An extra possibility is to add arbitrary phase and amplitude aberrations to the transducer.

- The acoustic properties of each layer and the shape of the interface between them are specified in the **medium** menu. Each layer is specified by the speed of sound, the impedance and the absorption parameters. The interfaces between the layers are specified by analytic functions for the 3D simulations, while the boundaries can be specified as arbitrary functions for the 2D simulations. Note that the simulation program assumes that the boundaries are smooth enough to avoid crossing of rays. It is also assumed that the ray tracing approximation is valid.

- The observation parameters are set in the **observation** menu.

When the problem is configured, it is possible to verify that it looks as specified by using the view menu. The specified transducer, excitation signal, electronic focusing delay, apodization, medium and the observation points may be viewed from this menu.
Table 1: Properties of water, dome and dome fluid at \( T = 23^\circ C \). The absorption along a ray is described by the formula: \( I = I_0 e^{-\alpha f r} \), where \( r \) is the propagation distance, \( f \) is the frequency and \( I \) is the intensity.

### 4.3 Calculation facilities for layered media

The calculation menu offers several calculation algorithms. The layer and the bp modules are made to calculate wave propagation through a layered medium. The layer module offers 2D and 3D axis symmetric simulations, while the bp module offers 3D simulations. The possibilities are described in this section. The results are presented in plots, and both the configuration and the numerical results can be saved for further calculations. More detailed information about the simulation program is given in the user’s manual [10].

#### 4.3.1 Specification of a simulation example

A 5 MHz annular transducer with dome is used to present the main possibilities of the layer- and the bp module. This transducer had 4 equal area rings, \( D=15\text{mm} \), \( \text{ROC}=50\text{mm} \) and a 0.25 mm space between each ring. The emitted signal was a 5 oscillations long cosine-shaped pulse with a center frequency of 5 MHz. The emitted pulse is shown in fig. 7. The plot was generated by choosing excitation in the view menu.

The medium consisted of dome fluid, a dome and water. The shape of the dome was spherical with an inner radius of 9.7 mm and an outer radius of 10.5 mm. The transducer was placed with an offset on 3.7 mm from the center of the dome. The medium is presented in fig. 8. The plot was generated by choosing medium in the view menu. The properties of water, dome and dome fluid are given in tab. 1.

#### 4.3.2 The layer module

**Phase and amplitude aberrations to a single observation point:**

The layer module is a tool to study phase- and amplitude aberrations. A simulation with a single observation point plots the phase- and the amplitude aberrations for each ray, while a simulation with several observation points plots the phase- and the amplitude aberrations for each element.

An example with a single observation point is shown in fig. 9. The example with an annular transducer presented in section 4.3.1 is used, and the observation point

<table>
<thead>
<tr>
<th>medium</th>
<th>speed of sound</th>
<th>impedance</th>
<th>( \alpha )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>1491 m/s</td>
<td>1.49 MRayl</td>
<td>0.05 ( m^{-1}MHz^{-2.0} )</td>
<td>2.0</td>
</tr>
<tr>
<td>dome</td>
<td>1974 m/s</td>
<td>1.84 MRayl</td>
<td>61.5 ( m^{-1}MHz^{-1.4} )</td>
<td>1.4</td>
</tr>
<tr>
<td>dome fluid</td>
<td>1579 m/s</td>
<td>1.69 MRayl</td>
<td>10.8 ( m^{-1}MHz^{-1.0} )</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Figure 7: The pulse emitted from the transducer. The pulse has a center frequency of 5 MHz, is 5 oscillations long and the envelope is cosine-shaped.

Figure 8: The annular transducer with dome in a water medium. Some rays between the ROC of the transducer and the transducer are also drawn.
is set to the radius of curvature of the transducer.

The first of the four plots in fig. 9 shows the medium. Some rays from the observation point to the center of the transducer and to the boundaries between each ring on the transducer are also drawn.

The next plot presents the phase aberrations in the observation point. The horizontal axis shows the position on the transducer where the rays arrive, while the vertical axis shows the arrival time of the rays. The arrival times are normalized to the ray that hits the center of the transducer, i.e. it is the delays compared with the delay of the center ray that are plotted. Positive delays means that the ray arrives at the transducer later than the center ray, while negative delays means that the ray hits the transducer earlier than the center ray. The delays are given in percentage of a wave period. We can see that the delays vary with ±10% of a wave period in this example (f=5MHz). If the medium had been homogeneous, the rays would get a constant delay=0. This plot thus shows the phase aberrations that are caused by the dome.

Both the phase aberrations for each ray and for each element are shown in the plot. The delay for each element is found by first adding the signals from each ray to get the total signal of that element, and then cross correlate the signals of the elements with the signal from the ray that hits the center of the transducer.

The third plot in fig. 9 shows the attenuation of the different rays. The horizontal axis shows the position on the transducer where the rays arrive, while the vertical axis shows the amplitude of the rays. The ray amplitudes are normalized to the maximum amplitude. The maximum amplitude is written above the plot. Both the amplitude aberrations for each ray and for each element are shown in the plot. The simulation shows that the rays that hit the outer element of the transducer are attenuated more than the rays to the inner element.

The attenuation of the different rays includes reflection loss, ray broadening loss, absorption and a cosine factor. It is possible to neglect some of the attenuation factors in the simulation program. By doing four simulations, each with only one of the attenuation factors included, the effects of the four attenuation factors are separated. The result is presented in fig. 10. We see that the effect of the angle between the ray and the normal of the transducer (the cosine factor) may be neglected, while the three other factors seems to be of the same size.

The last plot in fig. 9 shows the sum of all the signals. This signal can be interpreted as the total signal observed at the observation point. It can also be interpreted as the total signal received by the transducer if a spherical wave is emitted from the observation point.

The simulation presented in fig. 9 is mainly two-dimensional, but it is assumed that the transducer and the medium are axis symmetric. The signal at for example transducer position ±5 mm is therefore weighted more than the signal at transducer position ±1 mm. The result presented in fig. 9 is therefore a 3D analysis.
Figure 9: A simulation example on the effect of a dome. The simulation shows the phase aberrations and the attenuation in radius of curvature of the transducer. The transducer is annular with 4 rings, D=15mm and ROC=50 mm.

![Diagram showing phase aberrations and attenuation](image)

Figure 10: A further study of the attenuation in the case studied in the previous figure. Both absorption-, reflection-, ray broadening- and cosine factor attenuation are included in the simulation presented in the previous figure. Here, the effect of the different attenuation factors are separated.

![Diagram showing attenuation factors](image)
The focal point of the transducer-medium system:
In the layer module, there is also included a routine that finds the "focal point" of the transducer-medium system, i.e. it finds the observation point where the phase aberrations are minimized. Two minimizing criteria are implemented: One that minimizes the root mean square of the phase aberrations, and one that finds the point where the inner and the outer transducer ring are in phase.

As an example, the focal point is found for four different media. The example is presented in fig. 11. Each row represents one medium. The plot to the left shows the medium, the plot in the center shows the phase aberrations in the ROC-point of the transducer, while the right plot shows the phase aberrations in the "focal point" of the transducer-medium system.

The first medium consists of a dome and a homogeneous medium with speed of sound 1540 m/s. The "focal point" is moved from depth 50 mm to depth 43 mm.

The second medium is the same as the first, except for a plane layer of fat. The fat-layer is thick, but it has only a minor influence on the focusing.

The two last media show two other possible layers of fat. The first of them shows a fat-layer of a thin person while the last shows the fat-layer of an obese person (see MR-images in fig. 22). These uneven layers of fat introduce larger changes in the delays than the even fat-layer in the previous plot. The layer of fat in the obese patient causes especially large changes in the delay. The simulation shows that the "focal point" of the transducer-medium system is different in a thin and an obese patient. The "focal point" of the thin person is at depth 47 mm, while it is moved to depth 59 mm for the obese person. A further study of this effect and a method to do patient-depended focusing is presented in sec. 6.

Dynamic focusing:
Medical ultrasound imaging is based on back-scattering. It is therefore necessary that both the emitted and the received beams are well focused. The emitted beam can be focused to only one observation depth, while the received beam can be dynamically focused to all depths. The dynamic focusing is based on the propagation time in the medium and the assumption that the medium is homogeneous with a speed of sound of 1540 m/s.

Three possibilities to specify the dynamic focusing of annular array transducers are implemented. In the first one, you may set the number of electronic foci-zones to use. You may also specify the minimum and maximum observation depths. Ultrasim will set the foci between these two depths. This is done in a way that minimizes the phase aberrations. In the second alternative, you may add extra delays to the delays set by the automatic algorithm. You can add one delay for each element. These extra delays are constant for all observation depths. It is thus possible to correct phase aberrations caused by the dome (see sec. 6, fig. 87). The third alternative is a manual specification of the depths of the zones and the delays of each zone, which allow you to set the delays as for instance by the CFM-scanner of Vingmed.

An example with fixed focus is shown in fig. 12, and an example with dynamic
Figure 11: Phase aberrations in some media. The column to the left shows the plot of the media, the next column shows the phase aberrations at the ROC-depth of the transducer, while the last column shows the phase aberrations over the transducer at the depth where the phase aberrations are minimized. f=5MHz, ROC=50mm
focusing is shown in fig. 13. Both simulations are done with the layer module, and the dome in water-example that is described in sec. 4.3.1 is used. It is assumed that the medium is symmetric around the depth axis, and the results are therefore three-dimensional.

Four plots are printed after an on-axis simulation. The first one shows the medium. Rays from the first observation point is drawn to the transducer. In the near field, we see that no ray is drawn between the outer boundary of the transducer and the observation point. The reason is total reflection at the dome, i.e. it is not possible to draw a ray between the outer part of the transducer and the observation point that satisfy Snell's law. The legal ray that hits as close to the outer boundary of the transducer as possible is shown in the plot. The simulation program assumes that the "ray-less" outer part of the transducer does not contribute to the acoustic field at the observation point.

The second plot shows the delay of each element as a function of depth. The delay refers to the arrival time of the ray that hits the center of the transducer, and it is given as percent of a wave period. In the case with fixed focus (fig. 12), the four rings are in phase at depth 50 mm only. The phase aberrations increase with the distance to depth 50 mm. In the case with dynamic focusing (fig. 13), the phase aberrations are relatively small for all depths. If the medium had been homogeneous with c=1540 m/s as assumed, there would not have been any phase aberrations between the rings at all. A simple method to improve the dynamic focusing through a dome is suggested and discussed in sec. 6.

The third of the four plots shows the peak amplitude or the energy of the signal of each ring as a function of depth. The discretized signal of each element can be written as signal(t). The energy to a signal is calculated as $10 \log_{10}(\sum_{i} signal(t)^2)$. The signal of each ring does not depend on the electronic focusing, but of the phase aberrations over each ring and the attenuation in the medium. The lower energy in the outer- than the inner ring is mainly caused by higher attenuation of the rays to the outer- than to the inner ring (see fig. 9).

The fourth of the four plots shows the peak amplitude or the energy of the signal of the whole transducer as a function of depth. The energy is increased considerably by using the dynamic focusing algorithm (fig. 13 ) compared with the fixed focus case (fig. 12).

4.3.3 The bp module

If the transducer and the medium are not axis symmetric around the observation points, the bp module must be used. It is implemented for annular- 1.5D- and 2D array transducers. The boundary between each layer may be specified as an ellipsoid or as a second-order polynomial of x and y. The output from the bp module is the on- or the off-axis beam profile. Both peak and energy calculations are possible. The simulated RF-data is also available.

As an example, the off-axis beam profile of the 5 MHz annular transducer is
Figure 12: A study of the on-axis focusing of an annular transducer. Electronic focusing is not used, i.e. the simulation shows the properties of the transmitted beam. The transducer is annular with 4 rings, D=15mm and ROC=50 mm.

Figure 13: A study of the on-axis focusing of an annular transducer. Dynamic focusing is used, i.e. the simulation shows the properties of the received beam. The electronic delays are set from the assumption that the medium is homogeneous with \( c = 1540 \text{ m/s} \). The transducer is annular with 4 rings, D=15mm and ROC=50 mm.
Figure 14: Off-axis beam profiles with and without a dome. The transducer is annular with 4 rings, D=15mm and ROC=50 mm.

Figure 15: Off-axis beam profiles in water. Gaussian phase aberrations with zero mean and different variance are added. The transducer is annular with 4 rings, D=15mm and ROC=50 mm.
calculated. Two simulations for a water medium is done; with and without a dome. It is calculated for a medium with water only and for the case with dome in water as specified in sec. 4.3.1. The two off-axis beam profiles are compared in fig. 14. We see that the dome changes the beam profile somewhat, but not much. The main effect of the dome is a broadening of the main-lobe, and a decrease of the side-lobes. The effect can be explained by the attenuation through the dome. Fig. 9 shows that the attenuation of the rays increases with the distance to the center of the transducer. The attenuation works as an apodization, which explains the difference between the two beam profiles. The phase aberrations caused by the dome will increase the sidelobes somewhat, but fig. 14 shows that the apodization effect of the dome is stronger.

The bp-module can be used as a tool to get a better understanding of how large phase aberrations that can be accepted. In an example with the annular transducer, phase aberrations of different size are added to the rays. The added phase aberrations were uncorrelated and Gaussian-distributed with zero mean and different variance. The effects on the off-axis beam profile are shown in fig. 15. The effects of the phase aberrations are a decreased main-lobe and increased side-lobes. We can see that the standard deviation of the phase aberrations must be less than 0.4 wave period to get any focusing at all. To get side-lobes below -25 dB, the standard deviation of the phase aberrations must be less than 0.2 wave period.

The simulations presented in fig. 15 are based on uncorrelated phase aberrations. It is also possible to do simulations where the phase aberrations are spatially correlated. Such aberrations are simulated by an AR1 process (see sec. 7.1). Aberrations with the same properties as measured in experiments on human tissue can thus be simulated (see sec. 7.2).

The distortions caused by a phase screen can also be simulated with the bp module. A plane wave that crosses through a phase screen can for example be simulated as the field to a plane transducer with the phase screen as delays. The properties of the aberrated wave front can be simulated for different propagation distances. It is thus possible to see how long the wave front keeps the same shape as the phase screen. Such a study is presented in sec. 7.3.
5 Experiments

A main question is why ultrasound images of obese patients usually have lower quality than images of thin patients. Obvious, the tissue must be different. Magnetic Resonance (MR) images of a thin and an obese person are taken to detect such differences. Further experiments are done to explore whether the observed differences can explain the variations in image quality. How can we for instance explain that reverberations disturb the image of the obese person, while it does not seem to be a problem in the thin person? Is this because the tissue in the obese person make stronger reverberations or is it because the main echo is reduced by phase aberrations?

It is done several experiments to find the answers to these questions. The reverberations and the phase aberrations caused by different tissue as homogeneous fat, homogeneous meat and more mixed tissues are studied in a water tank. The ultrasound images from the experiments are presented in section 5.1, while the beam profiles are presented in section 5.2.

The experiments show that fatty meat disturb the imaging process. The inhomogeneities in fatty meat caused both reverberations and phase aberrations, but the distortions caused by phase aberrations were worst. This observation explain the low image quality of obese patients, because it is probably more fatty meat in an obese patient than in a thin one.

5.1 Patient-depended image quality

A typical image of a thin and an obese patient are shown in fig. 16. Possible explanations to the patient - depended image quality are explored in this section.

The first hypothesis is that a thick outer layer of fat causes distortions. An ultrasound image of the heart of a pig lowered into water is observed through a thick layer of fat to explore the hypothesis. The images are shown in fig. 17. To compensate for the attenuation caused by the layer of fat, the gain is increased. The gain is increased until the electronic noise is visible in the image. But in spite of the high gain, no acoustical noise is observed, and the image is not as disturbed as the image of a fat patient. If the pig fat used in this experiment has similar properties as human fat, we can conclude that a thick layer of fat itself does not explain the large distortions in images of obese patients.

Another hypothesis states that an outer layer of fat will increase the distance between the probe and the ribs. This may be a problem, since we have to observe between the highly attenuating ribs. The ribs are also strong reflectors which may cause strong reverberations. To explore this hypothesis, a thin person was imaged with and without an extra layer of fat. The two images are shown in fig. 18. Acoustical and electronical noise can be observed in the disturbed image, but it is still not as bad as the image taken of the obese person.

Fig. 19 shows a point scatterer imaged through a thick layer of bacon containing
several horizontal layers of fat and meat. The boundaries between fat and meat were smooth and each layer looked homogeneous. The main-lobe is not destroyed, but the side-lobes are increased. Increased side-lobes reduce the contrast in the image.

A larger increase of the side-lobes is observed if a wave propagates through a medium with nonplanar boundaries between the layers. An irregular boundary between two layers with different speed of sound will cause phase aberrations. An example is shown in fig. 20. One has shown that the image of a point scatterer observed through a specimen of outer rib steak of an ox depends on the shape of the boundary between the specimen of meat and the water. When the boundary is not plane, the image of the point scatterer is smeared out as a very long line. The point scatterer is almost imaged as two scatterers. The smeared image of the point scatterer is due to phase aberrations caused by the irregular boundary between the specimen of meat and the water. The speed of sound was approximately 1491 m/s in the water and 1560 m/s in the meat.

A rough boundary between two layers with different speed of sound will cause phase aberrations, and the image will be disturbed. How is the shape of the boundary between different human tissue layers? To answer this question, Magnetic Resonance (MR) images of two different persons were taken. One that gives an ultrasound image of high quality, and one that gives an ultrasound image of low quality. The ultrasound and the MR-images are presented in fig. 21 and 22. The quality of the MR images is higher than the quality of the ultrasound images. In spite of that, the resolution in the MR-images is not high enough to observe rough boundaries between the tissue layers in any of the two persons.

The Magnetic Resonance (MR) images are used in a further study of the differences in image quality. The first observation in fig. 21 shows that the patient who gave ultrasound images of low quality has more fat than the one who gave images of high quality. The obese one has a 2-3 cm thick outer layer of fat and a 1-2 cm thick layer of fat around the heart. The outer fat-layer of the obese patient is so soft that the probe sinks in as seen in the MR-image. The boundary between the fat-layer and the dome is spherical since the dome is spherical. The boundary towards muscle is still plane. The low speed of sound in the fat compared with the higher speed of sound in the dome, will cause a lens effect of the bent fat-layer. The lens will move the focus of the transducer deeper into the body. The same lens effect is not observed in the thin person. In fig. 22, one can see that the fat-layer of the thin person is bent towards both the dome and the layer of muscle. This will not give a lens effect. The MR-images of the thin and the obese person are used to build models. The models are used as input to the simulation program described in section 4 and verified in section 5.3. Section 6 shows the simulations, and a method to correct some of the distortions in the image of the obese person is suggested.

Improvements made possible by the method are shown experimentally in fig. 23. The figure shows ultrasound images of balloons in water. The image to the left shows the balloons in water taken with default focusing. The second image shows the image after changing two of the parameters used in the algorithm for electronic
focusing. The parameters are set to the values that the simulation program found to be optimal for water. It is possible to see significant improvements. In the third image, the parameters are set such that phase aberrations are observed as caused by a fat-lens as observed in the MR-image of the obese patient. The quality of the ultrasound image of the balloons is now significantly reduced. The method suggested in section 6 can correct for the lens effect caused by a bent outer layer of fat. The improvements made possible by the method are given by the different quality of the balloon-images in fig. 23.

The tuning of the diameter and the radius of curvature of the transducer used in the algorithm for electronic focusing can compensate for some of the distortions in the image of the obese person, but the correction cannot alone make ultrasound images of obese patients as good as images of thin persons.

Another observation in the MR-images is that the obese person has a thick layer of fat around the heart. For some directions of the beam, half the beam will propagate through this layer of fat, while the other half of the beam will propagate through parts of the heart. The difference of speed of sound in the heart and the surrounding fat will cause phase aberrations, and the beam will be defocused. The effect can be studied in fig. 24. The upper image shows a point scatterer imaged through a 15 mm specimen of meat without fat. The point scatterer is imaged as a one centimeter long line. The lower image shows a point scatterer imaged through a 15 mm specimen of fat and meat which are seperated vertically. Half the beam propagates through 15 mm fat, while the other half of the beam propagates through 15 mm meat. The phase aberrations caused by the difference in the speed of sound in the fat and the meat are so large that the point scatterer is imaged as a several centimeter long line. The effect is especially large in this example, because the vertical boundary between fat and meat are close to the transducer. Beams emitted in almost all directions will be disturbed. When the vertical boundary is that deep inside the body as shown in the MR-images, only a few beams are disturbed.

Reverberations distroy the image of an obese person, but not of a thin person (fig. 16). An interesting question is then: Is the reverberations stronger in the obese patient, or is the main reflections reduced by phase aberrations?

The size of the reverberations and the side-lobe noise for a well focused beam were studied in an early experiment (fig. 25). Specimens of bacon immersed in water were imaged. Reverberations caused by strong reflectors as the transducer and the water surface were observed. Internal reverberations from fat and homogeneous meat were not observed, but internal reverberations from fatty meat were seen. Artifacts caused by the reverberations and artifacts caused by the side-lobes seemed to be of the same order for the well focused beam used in this experiment.

The experiment was done at room temperature. At room temperature, the speed of sound was measured to 1491 m/s in water, 1500 m/s in fat and 1560 m/s in meat. To do a more realistic comparison between the distortions caused by reverberations and the distortions caused by phase aberrations, the next experiments were done at 37°C. At 37°C, the speed of sound was measured to 1526 m/s in water, 1437
m/s in fat and 1591 m/s in meat. The experiments were done with meat and fat
from a pig, but the speed of sound measured at 37°C fits well with the speed of
sound in human tissue [Duck]. The reverberations from fatty meat increased with
the temperature, while the side-lobe noise were unchanged.

A transducer focus well through homogeneous water. But, if the wave propagate
through inhomogeneous tissue, the beam may be dramatically broadened. Different
specimens of bacon is kept between the transducer and the object to study the
broadening. The specimens are shown in fig. 26. Experiments done at 37°C are
presented in fig. 27-30. The gain is equal for the five images, and it is shown to the
right in fig. 27.

Fig. 27 shows an image of the object. The object is a specimen of almost ho-
monicous meat lowered into the water. The white area in the image shows the
specimen, while the grey area directly below and beside the specimen is acousti-
oise caused by multiple reflections and by the side-lobes. The reverberations and
the artifacts caused by the side-lobes are small.

The distortions caused by keeping a 1 cm thick specimen of fat between the
transducer and the object is shown in fig. 28. Reverberations are seen below the
specimen of fat. The object is still imaged well, but the artifacts caused by the
side-lobes are somewhat stronger. Some small irregularities were observed at the
boundary between the fat and the water. This boundary will cause phase aberrations
that may explain the increased side-lobes.

In fig. 29, a 2 cm thick specimen of tissue is kept between the transducer and
the object. The 10 mm closest to the dome consisted of fat, the next 9 mm of
homogeneous meat, while the last mm consisted of fat. Strong reverberations can
be seen below the 2 cm thick specimen. The boundaries between fat, meat and water
looked smooth. The object was imaged well. It is still possible to see the boundary
between the object and the water. The reason why the object looks somewhat
different from fig. 27 is that the object is moved.

In fig. 30, a 1.7 cm thick specimen of tissue is kept between the transducer and
the object. This specimen consist of 4 mm fat, 9 mm fatty meat and 4 mm fat.
The difference between the medium imaged in fig. 29 and the medium imaged in
fig. 30 is the structure of the layer of meat. The difference in image quality is
great. In fig. 30, the lateral resolution is decreased to a level were it is difficult to
see the vertical boundaries between the object and the water. The decreased lateral
resolution is due to the increased side-lobes caused by propagation through the fatty
meat. The fat (1437 m/s) randomly located around in the meat (1591 m/s) cause
phase aberrations and the side-lobes increase. An image taken without the object
verified that the artifacts beside the object is caused by high side-lobes.

The disturbed beamprofiles were also measured with a hydrophone. The mea-
sured beamprofiles are presented in section 5.2.5.

There is reason to belive that an obese patient has more fatty flesh than a skinny
patient. This will explain why the images of obese patients are of lower quality than
images of thin patients. Fig. 30 shows that a layer of tissue with much fat randomly
located around in the meat cause large phase aberrations. It also shows a cloud of reverberations below the specimen placed close to the transducer. This cloud looks similar to the one that can be seen in the ultrasound image of the obese person shown in fig. 16. The reverberations and the phase aberrations caused by fatty meat can explain the low quality of the images of an obese person. An optical image of the heart and the outer tissue layers of an obese man, shows that the approx. 1 cm muscle layer between the outer layer of fat and the heart consists of much fat. The image is copied from a colour atlas of sectional anatomy [15] and is shown in fig. 31.
Figure 16: The upper ultrasound image shows the 4 chambers of the heart of a thin patient, while the lower image shows the 4 chambers in the heart of an obese patient. The gain is increased three steps in the lower image to compensate for attenuation. When turning off the emitted signal the screen turns almost black. This denotes that the noise in the lower image is mainly acoustic and not electronic noise.
Figure 17: The upper image shows the heart of a young pig lowered into water. The ventricles are filled with water. The lower image shows the heart observed through 5 cm fat taken from a pig. The specimen of fat consists of skin, a thick layer of fat and a thin layer of meat. The specimen is folded. In the lower image the gain is increased to compensate for large attenuation through fat. Only small distortions from reverberations and aberrations are observed. The noise observed in the image of the water and the heart is mainly electronic noise.
Figure 18: The upper image shows the left ventricle of a skinny patient, while the lower image shows the ventricle observed through fat from a pig. The specimen of fat consist of skin and 12 mm homogeneous fat. To compensate for attenuation caused by the external layer of fat, the gain is increased in the lower image. The noise in the lower image is mainly electronic noise.
Figure 19: Both images show a strong scatterer observed through 55 mm bacon lowered into water. The specimen of bacon consists of several horizontal layers of fat and meat. The point scatterer has a diameter of 0.8 mm, and it is shown in the center and lower part of the images. The position of the bacon is not the same in the two images. The side-lobes increase or disappear when the specimen of bacon is moved, while the main-lobe is not effected. The reverberations are weak. The gain is increased until the image of the main-lobe become saturated. The gain is equal in the two images.
Figure 20: Both images show a strong scatterer observed through a specimen of outer rib steak of an ox. The surrounding medium is water. The point scatterer has a diameter of 0.8 mm, and it is shown in the center and lower part of the images. The point scatterer is imaged as a longer line in the lower image than in the upper image. This is due to the phase aberrations caused by the irregular boundary between meat and water. Internal reverberations are observed right underneath the meat. They are caused by small islands of fat. The noise seen in the lower part of the image is caused by specks of dust in the water. The gain is equal for the two images.
Figure 21: 4 ch. Ultrasound and Magnetic Resonance (MR) images of the heart. The patient to the left/right gives ultrasound images with high/low quality.
Figure 22: Magnetic Resonance (MR) images of the heart. The patient to the left/right gives ultrasound images with high/low quality. To the right in the MR-images, it is possible to see the ultrasound probe.

Figure 23: a) Image of balloons in water taken with standard equipment, b) Image after phase aberration correction, c) Image with phase aberrations as caused by an outer bent fat-layer as seen in the MR-images of the obese patient. The gain is equal for the three images.
Figure 24: Both images show a strong scatterer observed through a 15 mm specimen taken from the neck of a pig. The surrounding medium is water. The point scatterer has a diameter of 0.8 mm, and is seen in the center and lower part of the images. The point scatterer is imaged as a longer line in the lower image than in the upper image. The distortions in the lower image are due to phase aberrations caused by inhomogeneous tissue. The specimen in the upper image contain only meat. In the lower image, half the beam propagated through 15 mm fat, while the other half propagated through 15 mm meat. The phase aberrations are caused by the difference in speed of sound in fat and meat. Internal reverberations can be observed right underneath the tissue. They are caused by small islands of fat in the meat. The noise seen in the lower part of the image is caused by specks of dust in the water. Electronic noise can also be seen. The gain is equal for the two images.
Figure 25: The upper image shows a specimen of bacon consisting of skin and 14 mm homogeneous fat lowered into water. Internal reverberations cannot be observed right underneath the fat-layer, but three short horizontal lines can be seen in the center of the image, at 60-70 mm depth. These artifacts are assumed to be caused by reverberations between the transducer and the boundaries water/skin and skin/fat. The artifacts from 75-85 mm are caused by reverberations between the fat-layer and the water surface. The effect of side-lobes is observed directly above the fat-layer. The lower image shows two specimens of meat from a pig lowered into water. The specimen to the left is fatty meat and the specimen to the right is homogeneous meat. There are more artifacts caused by reverberations from the fatty meat than from the homogeneous meat. The artifacts directly below the fatty meat are caused by internal reverberations, while the artifacts seen in the lower left corner of the image are caused by reverberations between the fatty meat and the transducer. Artifacts caused by the side-lobes can be seen between the two specimens. The artifacts caused by reverberations and side-lobes seem to be of the same size. The experiment is done at room temperature.
Figure 26: These four specimens of bacon are used to explore distortions caused by inhomogeneities. The specimen A is imaged through water and through tissue B, C and D in the next four figures. Specimen A is a 9 mm thick layer of meat, B is a 10 mm thick layer of fat, while C is a specimen with 10 mm fat, 9 mm homogeneous meat, and 1 mm fat. Specimen D consist of 4 mm fat, 9 mm fatty meat and 4 mm fat. At 37°C, the speed of sound in the fat and the meat were measured to 1437 m/s and 1591 m/s, respectively. The speed of sound in fat and muscle for human tissue are approximately 1440 m/s and 1580 m/s, respectively.
Figure 27: A specimen of homogeneous meat lowered into water is imaged (specimen A). The temperature of the water was 37°C. The white area in the image shows the specimen, while the grey area directly below and beside the specimen are acoustic noise caused by multiple reflections and the side-lobes.

Figure 28: The image shows the same specimen (A) of meat as in the previous figure. The gain is the same and the temperature in the water is still 37°C. The difference is that a 1 cm thick specimen of fat (specimen B) is kept between the transducer and the object. There were some irregularities in the boundary between the fat and the water.
Figure 29: The image shows the same specimen (A) of meat as in fig. 27. The gain is the same and the temperature in the water is still 37°C. The difference is that a 2 cm thick specimen is kept between the transducer and the object (specimen C). The 10 mm closest to the dome consisted of fat, the next 9 mm consisted of homogeneous meat, while the last 9 mm consisted of fat. The boundaries between fat, meat and water looked smooth.

Figure 30: The difference between this and the previous figure is the structure of the specimen kept between the transducer and the object. Here, the specimen consists of 4 mm fat, 9 mm fatty meat and 4 mm fat (specimen D). The 9 mm layer of meat consisted of a great deal of fat randomly located around in the meat. The result is a dramatical reduction in the lateral resolution.
Figure 31: Optical images of the heart and the outer tissue layers of a human being. The specimen is a horizontal plate from the corpse of an obese middle-aged man. The image is taken from below. The white area in the images are fat. The images are copied from a color atlas of sectional anatomy [Lyons78].
5.2 Measurements of beam-profiles

For each excitation of the transducer, one line in the ultrasound image is mapped. To map one line, the emitted pulse is focused as a narrow beam. Inhomogeneities illuminated by the beam will scatter some of the wave, and the envelope of the back-scattered signal will be presented as a line in the image. A narrow beam will give high resolution, while a beam with low side-lobes will give an image with high contrast. The properties of a beam depend on the f-number, the frequency and the propagation medium.

Beams caused by transducers with different frequencies and f-numbers (focal depth/ transducer diameter) are shown in section 5.2.2, while measurements of beam-distortions caused by propagation through a specimen of tissue are presented in section 5.2.3, 5.2.4 and 5.2.5. The experimental equipment is described in section 5.2.1.

The experiments in section 5.2.2 are performed in water, which is a homogeneous medium with almost neglectable attenuation. A well-known analytic expression for the width of the beam, fits well with the result of the experiments at the focal depth. At depths closer to the transducer, where the Fraunhofer and Fresnel approximations are not valid, a simulation program is required. If the medium has a non-trivial inhomogeneous structure, simulations are needed even for all depths.

A layer of homogeneous tissue with uniform thickness is included in the propagation medium explored in section 5.2.3. A comparison between a wave that has propagated through water and a wave that has propagated through a layer of meat or fat in water, shows that a layer of fat or meat will disturb the focusing. The defocusing is caused by frequency - depended attenuation. The distortions caused by frequency - depended attenuation may be avoided by using a narrow - banded pulse or by using a bandpass filter on the received signal. But even when the measured wave-field is filtered, small distortions are left. The distortions are assumed to be due to inhomogeneities inside the tissue. Inhomogeneities inside the tissue will generate phase aberrations and the incident wave-field will be scattered. Both effects may explain the observed distortions due to a layer of fat or meat.

The wave propagation through several layers of tissue is explored in section 5.2.4. Distortion due to phase aberrations are now a more dominant defocusing effect. The frequency - depended attenuation causes less distortions here, because it is used a more narrow - banded pulse.

In section 5.2.5, the beam profiles are measured through tissue at 37°C. The speed of sound in fat and muscle in these experiments are close to the corresponding values for human tissue. It is found that a wave which propagates through a specimen of fatty meat is dramatically defocused. This tissue specimen looked similar to what is observed in the muscle layer of an obese person (fig. 31). It is concluded that the distortions are caused by phase aberrations.
5.2.1 Method

Figure 32: The principle of the experiments

The focusing properties of a transducer are explored by measuring the beam-profiles. The experiments are executed as presented in fig. 32. An annular transducer was used as a source. The emitted wave propagated through probe fluid, dome, water and a layer of tissue before it was received by a small hydrophone. The hydrophone was moved to measure the emitted wave-field in different positions. The signals received in the hydrophone were stored. They will be denoted as radio frequency data (RF-data). An example of observed RF-data is shown in fig. 35.

A scanner from Vingmed sound was used as pulse generator. The CFM 700 was used as pulse generator in section 5.2.2, while the CFM 800 was used as pulse generator in section 5.2.3-5.2.5. To emit the signals an annular array transducer was used.

The hydrophones used to measure the wave-field were produced by the Force Institutes in Denmark. The diameter of the hydrophone used in the experiments presented in section 5.2.2 and 5.2.5 was 0.4 mm, while a hydrophone with a diameter of 0.6 mm was used in the experiments presented in section 5.2.3 and 5.2.4.

At each hydrophone position, the transducer emitted 16 or 32 equal pulses. The received signals were averaged to reduce the electronic noise. Between each movement of the hydrophone the transducer waited 500 ms before excitation. The hydrophone was moved by "Scanny", which was built at the Department of Biomedical Engineering at the University of Trondheim. "Scanny" is connected to a Macintosh computer and a Philips PM 3323 oscilloscope. The oscilloscope and the movements of the hydrophone are controlled by the computer. The hydrophone is moved by a step size as low as 100\(\mu\)m in the presented experiments, while the smallest step size of "Scanny" is 10\(\mu\)m. It is possible to move "Scanny" in both the x, y and
z directions. The signal received by the hydrophone is amplified before being sent to the oscilloscope controlled by the computer. One has used an amplifier built at the Department of Biomedical Engineering at the University of Trondheim in the experiments presented in section 5.2.2 and 5.2.5, while it is used a Hitachi oscilloscope V-1100A 100 MHz as amplifier in the experiments presented in section 5.2.3 and 5.2.4. The response of the different frequencies in the amplifiers and in the hydrophone is controlled and found to be flat (±1dB) in the range 2-15 MHz.

The received signals are filtered with a 2. or 3. order Butterworth filter to reduce the frequency components that are much lower and higher than the center frequency of the pulse. The filter is weak and does not change the shape of the wanted pulse, but it removes a possible bias and reduces the effect of high frequency noise. The signals are stored with a sampling frequency of four times the upper break of frequency in the Butterworth filter. Typical sampling frequencies are 40 and 60 MHz. The length of the saved signals are 5 μs.

Figure 33: Equipment for the experimental measurements.

RF-data was collected along two axes; an on-axis and an off-axis. The on-axis is the straight line through the center and the focal point of the transducer. A position on the line is denoted by the distance to the transducer, and the line is therefore called the depth-axis, too. The on-axis is parallel to the z-axis in fig. 32. The acoustic field emitted from an annular transducer will be symmetric in the area around the depth-axis if the medium is homogeneous. Measurements along the depth-axis show the degree of focusing at different depths. To measure the transducer's focusing properties along the depth-axis it is necessary to keep the hydrophone on-axis. This is done by moving the hydrophone at the desired depth until the signal is maximized.

Ultrasound images are mapped out line by line. To make good resolution in
ultrasound images, it is necessary that the energy of the emitted wave-field is concentrated around the depth-axis. To measure how well the energy is concentrated, the hydrophone is scanned orthogonal to the depth-axis. This line normal to the on-axis is defined as the off-axis. The off-axis is parallel to the x-axis in fig. 32. Strong signals on the on-axis and rapidly decreased signals with increased distance to the on-axis are wanted. The strong signals around the depth-axis will be denoted as the signals in the main-lobe. The weaker signals, at some distance from the on-axis, will be denoted as signals in the side-lobes. A narrow main-lobe will give high resolution, while low side-lobes will give good contrast in the ultrasound image.

5.2.2 Beam-profiles in water

The section presents measured beam-profiles for a medium with probe fluid, probe cap and water. Water is a non-attenuating and homogeneous medium, which will be used to show optimal beams. Measurements of the width and the length of the focal zone of different transducers are presented. These properties are compared with analytic expressions. Measured beam-profiles at other depths than the focal depth of the transducer are also shown.

Signals with four different annular transducers are emitted. Their properties are listed in tab. 2. Electronic delays are not used. The emitted wave-field is measured with a hydrophone with a diameter of 0.4 mm. Each stored signal is an average of 16 excitations and it is filtered with a 3. order Butterworth filter with break off frequencies as given in tab. 2. The temperature and the sound of speed in the water were $T = 22.7 \pm 1.5^\circ C$ and $c = 1491 \pm 5m/s$.

<table>
<thead>
<tr>
<th>Transducer</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonance frequency, [MHz]</td>
<td>2.35</td>
<td>3.25</td>
<td>5.0</td>
<td>7.5</td>
</tr>
<tr>
<td>Diameter, [mm]</td>
<td>17</td>
<td>14.7</td>
<td>15</td>
<td>11.5</td>
</tr>
<tr>
<td>Radius of curvature (ROC), [mm]</td>
<td>100</td>
<td>78</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>Number of elements</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Trigger freq., [MHz]</td>
<td>3.85</td>
<td>3.45</td>
<td>5.00</td>
<td>7.14</td>
</tr>
<tr>
<td>Lower filter freq., [MHz]</td>
<td>0.5</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Upper filter freq., [MHz]</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 2: Parameters for the four transducers used in the experiments.

The signals observed at the focal points of the transducers are presented in fig. 34. All signals emitted from the transducer are added in phase to the focal point, so the signals shown in fig. 34 are assumed to have the same shape as the emitted signals.

The measured RF-data from one of the transducers are shown in fig. 35. The illustration shows the signals observed at different off-axis positions at the focal depth.
Table 3: Properties of the beam-profiles of the four transducers used in the experiments.

<table>
<thead>
<tr>
<th>Transducer</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured center frequency (f), [MHz]</td>
<td>2.9</td>
<td>3.1</td>
<td>4.4</td>
<td>6.0</td>
</tr>
<tr>
<td>Measured speed of sound (c), [m/s]</td>
<td>1491</td>
<td>1491</td>
<td>1491</td>
<td>1491</td>
</tr>
<tr>
<td>Transducer diameter (D), [mm]</td>
<td>17</td>
<td>14.7</td>
<td>15</td>
<td>11.5</td>
</tr>
<tr>
<td>Observation depth (F), [mm]</td>
<td>100</td>
<td>78</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>Measured -12 dB focal-width (D_F, -12dB), [mm]</td>
<td>6.3</td>
<td>6.3</td>
<td>2.4</td>
<td>1.3</td>
</tr>
<tr>
<td>Analytic: (D_F, -12dB \approx 2\frac{cF}{fD}), [mm]</td>
<td>6.0</td>
<td>5.1</td>
<td>2.3</td>
<td>1.1</td>
</tr>
<tr>
<td>Measured -3 dB focal-length (L_F, -3dB), [mm]</td>
<td>54.5</td>
<td>45.5</td>
<td>26.3</td>
<td>10.1</td>
</tr>
<tr>
<td>Analytic: (L_F, -3dB \approx 7.2\frac{cF^2}{fD^2}), [mm]</td>
<td>128.1</td>
<td>97.5</td>
<td>27.1</td>
<td>8.5</td>
</tr>
</tbody>
</table>

A beam profile is given by the energy in the signal received at each hydrophone position. The energy is calculated as the sum of the square of the observed signal. The off-axis beam-profiles at the focal depth for the four transducers are presented in fig. 36, and the on-axis beam-profiles are shown in fig. 37. The beam profiles are different for the four transducers. A thumb rule says that the -12 dB width of the focal beam is \(D_F, -12dB \approx 2\frac{cF}{fD}\), where \(c\) is the speed of sound of the propagation medium, \(f\) is the frequency of the emitted signal, \(D\) is the transducer diameter and \(F\) is the observation depth and the distance to the focal point [ch.2 Angelsen 95]. The thumb rule is compared with the measured beam-widths in tab. 3, and shown to fit well. A thumb rule for the -3dB focal length is \(L_F, -3dB \approx 7.2\frac{cF^2}{fD^2}\). Comparison to experiments in tab. 3 show that the thumb rule fits well for short focal lengths, but overestimates long focal lengths.

An ideal beam has a long focal-depth and a narrow beam width. The thumb-rules indicate that it is difficult to satisfy both wishes simultaneously. To increase the focal length, it is for instance necessary to allow a broader beam width. It is an advantage to first specify a minimum and maximum observation depth, and then choose a transducer that optimizes the beam in the specified area. An example of how the off-axis beam is for other depths than the focal depth is shown in fig. 38. The beam-profiles for depths close to the focal-depth are good, while those far away from the focal depth are severe. The RF-data to the off-axis beam-profiles shown in fig. 38 are presented in fig. 40, and the envelope of the RF-data is shown in fig. 42.

A further study of off-axis beam-profiles, far away from the focal depth, is presented in fig. 39. The measurements at depth 15 and 32 mm show destructive interference so large that the total beam is worse than the beams to each individual element. Large differences are also observed between the inner and outer elements. The inner element gives a beam with low side-lobes, while the outer element gives an extremely narrow main-lobe.
The emitted pulse can be focused optimally to only one point for each excitation. Theoretically it is possible to shoot several beams, one for each focal depth, and thus get optimal focusing to all depths. This will require a huge amount of time to sample one line in the image. We do not have that much time when we want to make real-time images. On the other hand, the received wave can be focussed optimally to all depths without waisting unnecessary time. It must then be used that \( c \approx 1540m/s \) in tissue, and that the back-scattered signals observed at the transducer at time \( t \) from an excitation at time \( t_0 \) are caused by scatterer at depth \( s(t) = (t - t_0)c/2 \). Electronic focusing may thus be done by focussing to depth \( s(t_1) \) at time \( t_1 \) and to depth \( s(t_2) \) at time \( t_2 \). This technique is referred to as dynamic focusing.

Today, it is common practice to make one line in the ultrasound image for each excitation of the transducer. The emitted beam will thus be focused as for instance in fig. 38, while the receiving beam is focused optimally to all depths. The width of the receiving beam is given approximately by the formula; \( D_{F-12dB} \approx 2\sqrt{\frac{F}{D}} \). It is assumed that the elements on the transducer are small enough to avoid phase aberrations over each element. It is also assumed that the quantization error of the delays are small compared to a wave period.
Figure 34: Normalized signals and their spectra. Each signal is observed at the focal point of the corresponding transducer.

Figure 35: Signals (RF-data) observed by the hydrophone at different off-axis positions at the focal depth of the transducer, $f=3.25 \text{MHz}$ (transducer b), Media=water.
Figure 36: The normalized off-axis beam-profiles at the focal depth of the four transducers. The energy is $10 \times \log_{10}(\sum \text{signal}^2)$, and is normalized to 0 dB at the on-axis. The noise level is below -40 dB.

Figure 37: The normalized on-axis energy of the four transducers. The energy is normalized to 0 dB at the focal depth.
Figure 38: The off-axis beam-profiles at different depths, \( f=3.25\text{MHz} \) (trans. b). The energy is \( 10 \times \log_{10}(\sum \text{signal}^2) \), and the noise level is below -40 dB.

Figure 39: The off-axis beam-profiles for different elements and different depths. El.1 is the inner ring. \( f=3.25\text{MHz} \) (transducer b). The noise level is below -40 dB.
Figure 40: The signals (RF-data) observed by the hydrophone at depth 22, 26, 50, 78, 120 and 160 mm, scaling 1, f=3.25 MHz (transducer b)

Figure 41: Color scaling 1
Figure 42: The envelope of the signals (RF-data) observed by the hydrophone at depth 22, 26, 50, 78, 120 and 160 mm, scaling 3, f=3.25 MHz (transducer b)

Figure 43: Color scaling 3
5.2.3 Beam distortion due to one layer of fat or meat

The section presents how a transducer focus through: 1) water, 2) a 12 mm layer of bacon - meat in water, 3) a 9 mm layer of bacon-fat in water. The layers of tissue had uniform thickness and looked homogeneous, except for some small islands of fat in the meat.

A complete description of the principle of the experiments are given in section 5.2.1 (fig. 32). An annular transducer with center frequency 7.5 MHz, diameter 11.5 mm and focal-depth=25mm was used as a source. The emitted wave propagated through probe fluid, dome, water, a layer of tissue, and water before it was received by a hydrophone at the focal-depth of the transducer. The hydrophone was moved orthogonally to the depth axis to measure off-axis beam-profiles. Vingmed's CFM 800 scanner was used as pulse generator. Gain was set to 5 step over minimum, and M-mode and range 0-4 cm was used to get one fixed beam with no electronic delays during the excitation. In each hydrophone position, 32 signals were averaged to reduce electronic noise. The saved signals are filtered through a 2.order Butterworth filter, which reduces the frequency components below 0.5 MHz and higher than 15 MHz. The signals are stored with a sampling frequency of 60 MHz.

The experimental results are shown in fig. 44 - 51. Fig. 44 shows the signals observed at the focal point of the transducer. We note that the pulse is delayed and attenuated differently by the three media. The speed of sound in water was measured to 1491 ± 5 m/s. The time delay caused by keeping a plane layer of tissue between the transducer and the hydrophone, and the thickness of the tissue layer gave an estimated speed of sound of approximately 1508 ± 10 m/s in fat and 1557 ± 10 m/s in meat. The measurements were made at room temperature (≈ 22°C).

Fig. 44 shows that a wave is attenuated more by propagation through tissue than through water, and that fat attenuates more than meat. The attenuation can also be studied in fig. 45, which presents the spectra of the signals in fig. 44. The spectra show that the difference in attenuation increases with increased frequencies. The normalized attenuation in the lower plot of fig. 45 presents the frequency dependence of the attenuation. The three spectra are normalized against the spectrum of water. The absorption in water at room temperature is 0.002 dB cm⁻¹ MHz⁻² [ref.Duck90 p95], which is almost zero. The frequency dependence of the attenuation in fat and meat is thus approximately given by the slope of the curves in the lower part of fig. 45. If we approximate the curves with straight lines, the attenuation coefficient become 0.75 dB MHz⁻¹ cm⁻¹ in fat and 2.3 dB MHz⁻¹ cm⁻¹ in fat. The values are useful to calculate how the attenuation increases with frequency and tissue thickness. The measured attenuation coefficient is not due to spherical attenuation, because all measurements are made at the same position. It is not due to reflection either, because the reflection coefficient at a plane boundary is independent of frequency. Thus, the measured attenuation coefficient must be due to absorption or scattering loss of small scatterers within the tissue.

Fig. 44 and 45 presents the properties of the signals at the focal point. The shape
of the observed signal changes if we move the hydrophone normal to the depth-axis. The signals (RF-data) observed at different off-axis positions at the focal depth are presented in fig. 46.

The left plot of fig. 47 presents the off-axis beam profiles for the three different media. The sampling frequency and the length of each signal are constant, to make it possible to compare the energy of different signals. We observe that the difference between the beam profiles is almost the same in the main- and the side-lobes. This indicates that the difference is not caused by phase aberrations, which redirect energy from the main-lobe to the side-lobes. The plot to the right in fig. 47 shows the normalized beam profiles. This figure shows that the main effect of propagation through the meat or the fat compared with water is a broadening of the main-lobe. The distortion caused by a layer with uniform thickness of homogeneous fat or meat is thus mainly due to frequency - depended attenuation. The effect is a result of higher attenuation of high than low frequencies. The low frequencies in the wave will thus be more dominant. The result is a broader beam, which will reduce the resolution.

A further study of the frequency - depended attenuation is presented in fig. 48 and 49. Fig. 48 is a combination and extension of fig. 45 and the left plot in fig. 47: Each vertical line shows the frequency spectrum of the signal observed at the respective position. Each horizontal line in fig. 48 shows thus the beam-profile for a specific frequency. Observe that the attenuation through meat and especially through fat is so large for high frequencies that the wave almost disappears below the noise.

A normalized version of fig. 48 is presented in fig. 49. The normalization is done line by line, so that the maximum of each horizontal line (beam-profile) is 0 dB. The plot shows which frequencies make the lowest side-lobes relative to the main-lobe. We observe optimal beam-profiles for frequencies as high as 14-19 MHz when the medium is water. The good beam-profiles at these frequencies are destroyed if the wave also propagates through a layer of tissue. The main explanation is that high frequencies are attenuated so much that the signal to noise ratio is dramatically reduced. Good beam profiles for all the three tested media are observed in the frequency range from 4.5 to 9 MHz.

Fig. 49 indicate thus possible improvements by using a band-pass filter on the received signals. The envelope of the original RF-data is presented in fig. 50, while the envelope of the filtered RF-data are presented in fig. 51. It is used a 3.ord. 4.5-9MHz Butterworth filter. The envelope of each signal is calculated as the absolute value of the sampled signal, smoothed with a hamming window of length 1.5 wave-period. Improved lateral resolution is observed by the filtering.

The off-axis beam profile of the filtered data is presented in fig. 52. The width of the main-lobes are equal after the filtering, i.e we have removed the effect of the frequency - depended attenuation. Note that the width of the beam-profile through water alone also is decreased by the band-pass filtering. This may be explained by the use of a broad-banded pulse.
Higher side-lobes are observed by propagation through meat and especially through fat compared with water after the filtering. These side-lobes cannot be due to phase aberration caused by a layer of uniform thickness and constant speed of sound, because there are larger difference in the speed of sound between meat and water than between fat and water. There should thus have been higher side-lobes in the meat than in the fat case, but the experiments show highest side-lobes caused by fat.

The side-lobes may be caused by first order scattering of inhomogeneities within the tissue. First order scattering will be seen beside the main-lobe, while second order scattering will be seen as a tail on the pulse. If there are inhomogeneities inside the tissue, phase and amplitude aberrations will also be introduced, which will also increase the side-lobes. The first explanation will dominate if there are small strong point scatterers, while phase aberrations will dominate if there are slow variations inside the tissue. If there are large and strong inhomogeneities, both factors will influence on the side-lobe level. The strength of the inhomogeneities may be tested by measuring the size of the tail caused by the second order scattering. A such tail was not observed when changing propagation medium from water to tissue. The length and size of the tails were the same through the three tested media, while the side-lobe level has increased to a higher level (fig. 51). The weak second order scattering indicates that the inhomogeneities within the tissue are weak, and thus mainly generate phase-aberrations.
Figure 44: Signals observed by a hydrophone at the focal point of the transducer. The three different propagation media are: 1) Water, 2) A 12 mm layer of bacon-meat in water, 3) A 9 mm layer of bacon-fat in water.

Figure 45: Frequency spectra of the signals observed at the focal point of the transducer. The three different propagation media are: 1) Water, 2) A 12 mm layer of bacon-meat in water, 3) A 9 mm layer of bacon-fat in water. The lower plot shows the spectra normalized against water.
Figure 46: Signals (RF-data) observed by the hydrophone at different off-axis positions at the focal depth, $f=7.5\text{MHz}$, medium=water only.

Figure 47: Off-axis beam-profiles at the focal depth. The data is collected by moving the hydrophone normal to the depth-axis. The energy is calculated as $10 \ast \log_{10}(\sum \text{signal}^2)$. The three different propagation media are: 1) Water, 2) A 12 mm layer of bacon-meat in water, 3) A 9 mm layer of bacon-fat in water.
Figure 48: Each vertical line shows the frequency spectrum of the signal observed in one specific position. Each horizontal line is thus the off-axis beam profile for one specific frequency. The signal is emitted with an annular transducer with center frequency 7.5MHz. The three different propagation media are: 1) Water, 2) A 12 mm layer of bacon-meat in water, 3) A 9 mm layer of bacon-fat in water.
Figure 49: The figure is a normalized version of the previous figure. The normalization is done line by line, so that the maximum of each horizontal line (beam-profile) is 0 dB. The three different propagation media are: 1) Water, 2) A 12 mm layer of bacon-meat in water, 3) A 9 mm layer of bacon-fat in water.
Figure 50: The envelope of the signals (RF-data) observed at the focal depth. The envelope is calculated by smoothing the absolute value of the signals with a hamming window with length 1.5 wave-period. $c = 1491 m/s$.

Figure 51: The envelope of the signals (RF-data) observed at the focal depth. The envelope is calculated by smoothing the absolute value of the signals with a hamming window with length 1.5 wave-period. The RF-data are filtered with a 3.ord. 4.5-9MHz Butterworth filter before the smoothing. $c = 1491 m/s$
Figure 52: Normalized off-axis beam-profiles at the focal depth. The signals are filtered with a 3.ord. 4.5-9MHz Butterworth filter before the energy is calculated. The energy is calculated as $10 \cdot \log_{10}(\sum \text{signal}^2)$. The three different propagation media are: 1) Water, 2) A 12 mm layer of bacon-meat in water, 3) A 9 mm layer of bacon-fat in water.
5.2.4 Beam distortion due to several layers of fat and meat

The section presents measurements on how a beam is disturbed by several layers of tissue from a pig. The pig was slaughtered the same day. An annular transducer with center frequency 3.25 MHz, diameter 14.7 mm and focus at 78 mm depth was used as a source. Each off-axis beam profile was measured with a hydrophone at 78 mm depth. The tissue specimen was placed between the transducer and the hydrophone at a depth of 25 mm from the transducer.

Fig. 53 presents the signal observed in the focal point for water, 1.5 mm skin in water, 9mm fat in water, 10 mm meat in water and a medium consisting of all the three layers in water. The corresponding frequency spectra are shown in fig. 54. These observations are used to estimate the speed of sound and attenuation in the different layers. The speed of sound in water was measured to 1493 ± 5m/s. The time delay caused by keeping a plane layer of tissue between the transducer and the hydrophone, and the thickness of the tissue layer gave an estimated speed of sound of approximately 1516 ± 10m/s in skin, 1498 ± 10m/s in fat and 1556 ± 10m/s in meat. The measurements were done at room temperature (≈ 22°C). The normalized attenuation in the lower plot of fig. 54 presents the frequency dependence of the attenuation. The spectra are normalized against the spectrum of water. The absorption in water at room temperature is 0.002dBcm⁻¹MHz⁻² [ref.Duck90 p95], which is almost zero. The frequency dependence of the attenuation in fat and meat is thus approximately given by the slope of the curves in the lower part of fig. 54. If we approximate the curves with straight lines, the attenuation coefficient becomes 0.40dBMHz⁻¹cm⁻¹ in meat and 2.1dBMHz⁻¹cm⁻¹ in fat. The values tell us how the attenuation increases with frequency and tissue thickness. The attenuation coefficient for skin was measured to 0.5dBMHz⁻¹

The envelope of the measured RF-data for different media are presented in fig. 55. The beam profile is not disturbed by propagation through a separate layer of skin, fat or meat, but we see some increase in the side-lobe level if the propagation medium consists of all the three layers. The envelope of the RF-data caused by propagation through two other media is also presented in fig. 55. These media did not consist of separate plane layers with fat and meat, but of one specimen with several horizontal layers of tissue. The side-lobes increased by propagation through both these specimens.

To explore why a beam is disturbed by propagation through a specimen of layered tissue, let us first study the effect of the frequency - depended attenuation. The effect can be studied in fig. 56, where each horizontal line shows the normalized off-axis beam profile for a specific frequency. The difference between beam-profiles at one specific frequency cannot be explained by frequency - depended attenuation. The off-axis beam-profile at frequency 3MHz shows increased side-lobes by propagation through layered tissue compared to propagation through water alone, and we can thus conclude that other distortions exist in addition to the frequency - depended attenuation.
The distortions caused by the frequency-dependent attenuation, can be reduced by using a band-pass filter, as successfully demonstrated in the previous section. Fig. 56 shows that the best beam profiles lie between 2.5MHz-4.5MHz. The poor side-lobe level at 5MHz is caused by a low signal to noise ratio at that frequency. Fig. 57 and fig. 59 shows the off-axis beam-profile of the originally measured signals, while fig. 58 and fig. 60 shows the off-axis beam-profiles if the RF-data is filtered with a 3. order Butterworth filter with break-off frequencies 2.5 and 4.5 MHz. Only a small improvement is obtained through filtering, which shows that frequency-dependent attenuation is a minor problem in this experiment. The consequence of frequency-dependent attenuation is less for the 3.25 MHz than the 7.5 MHz transducer, because the 7.5 MHz pulse was more broad-banded than the 3.25MHz pulse. A broad-banded pulse is more sensitive to frequency-dependent attenuation, because the attenuation of the different frequency components in a such pulse varies more than in a narrow-banded pulse.

In fig. 52 in the previous section, the beam profile was disturbed by propagation through a layer of fat. The side-lobe level in the filtered case was higher in the fat case than the water case, because of phase aberrations and/or first order scattering from inhomogeneities within the tissue. A similar increase in the side-lobes is not observed in fig. 58 in this section. The reason is probably that a transducer with center frequency 3.25 MHz is used in this section, while a 7.5 MHz transducer was used in the previous section. Both the phase aberrations and the scattering from small inhomogeneities will increase when higher frequencies are used.

By using the 3.25 MHz transducer, one layer of skin, fat or meat, is not enough to disturb the beam, but a specimen of several layers increases the side-lobes. That can be seen in fig. 60. How can we explain these side-lobes? Multiple reflections between the layers will only cause a tail to the signal and cannot explain the increased side-lobes. Phase aberrations caused by a layer of uniform thickness was found to cause minor distortions in the previous section, and there is thus no reason to assume that phase aberrations caused by several layers with uniform thickness is a problem. The increase in the side-lobes from propagation through a layered tissue must thus be caused by inhomogeneities inside each layer or by irregular boundaries between them. First order scattering from the inhomogeneities may cause side-lobes, but since the inhomogeneities do not make reverberations strong enough to make a tail on the signal, it is more probable that the inhomogeneities are weak and mainly generate phase-aberrations. Small irregularities are observed at the boundaries in the two specimens which consist of several layers of fat and meat. The irregular boundaries between fat and meat can explain why these specimens generate the largest side-lobes, because a rough boundary between tissue with different speed of sound will cause phase aberrations.
Figure 53: Signals observed at the focal point of the transducer. The different propagation media are: 1) Water, 2) A skin layer from a pig in water, 3) A 9 mm layer of fat in water, 4) A 10 mm layer of meat in water, 5) Skin, 9mm fat and 10 mm meat in water. The tissue was taken from a pig that was slaughtered the same day as the experiments were performed.

Figure 54: Frequency spectra of the signals observed at the focal point. The lower plot shows the spectra normalized against water.
Figure 55: The normalized envelope of the signals (RF-data) observed at the focal depth. The envelope is calculated by smoothing the absolute value of the signals with a hamming window with length 1.5 wave-period. The RF-data are filtered with a 2.ord. 0.2-10MHz Butterworth filter before the smoothing. $c=1493 \text{ m/s}$. 
Figure 56: Normalized beam-profiles at different frequencies. The three different propagation media are: 1) Water, 2) Skin, 9mm fat and 10mm meat in water (3 specimens), 3) skin, 5mm fat, 5mm meat, 2mm fat and 10mm meat in water (1 specimen)
Figure 57: Normalized beam-profiles of the measured signals (RF-data). The different propagation media are: 1) Water, 2) A skin layer from a pig in water, 3) A 9 mm layer of fat in water, 4) A 10 mm layer of meat in water, 5) Skin, 9mm fat and 10 mm meat in water.

Figure 58: The normalized beam-profiles of the measurements presented in the previous figure, if the RF-data are filtered with a 3. order 2.5-4.5 MHz Butterworth filter.
Figure 59: Normalized beam-profiles of the measured signals (RF-data). The different propagation media are: 1) Water, 2) Skin, 9mm fat and 10 mm meat in water, 3) skin, 5mm fat, 5mm meat, 2mm fat, 10 mm meat, 4) 4mm fat, 6 mm meat, 2mm fat, 11mm meat.

Figure 60: The normalized beam profiles of the measurements presented in the previous figure, if the RF-data is filtered with a 3rd order 2.5-4.5 MHz Butterworth filter.
5.2.5 **Beam-profiles measured at 37°C**

The section presents measurements on how well a transducer focuses through different specimens of bacon lowered into water at 37°C. The specimens are shown in fig. reffig/measuredimages37C/bacondocbilder. The reason for increasing the temperature in the water from room temperature to 37°C, is that the speed of sound and the absorption in fat and meat depends on the temperature. Table 4 shows the measured speed of sound and the measured frequency - depended attenuation coefficient for the tissue used in section 5.2.3, 5.2.4 and 5.2.5. The table shows that the tissue properties vary significantly with the temperature. The speed of sound in fat decreased considerably when the temperature was increased to 37°C. The speed of sound in meat increased somewhat. The attenuation coefficient in fat decreased much when the temperature was increased, while the attenuation coefficient in meat increased somewhat. No significant differences were observed when salt and preserving agent were added to the tissue. The properties measured at 37°C fit well with similar measurements on human tissue [ref:Duck].

The first figures presents how a transducer focuses through a separate layer of fat and a separate layer of meat. To avoid phase aberrations caused by irregularities between the tissue and the water, the tissue was put between two thin plates of hard plastic. The measurements presented in fig. 61- 64 show that the distortions caused by the plates of plastic were small. These measurements also show that a layer of fat or meat with uniform thickness attenuates the wave and increases the side-lobes. The distortions were relatively small.

Side-lobes caused by a layer of fat with and without plates of plastic at the boundaries are shown in fig. 65 and 66. The side-lobes were highest when we did not use plates of plastic at the boundaries. The reason was an irregular boundary between fat and water which caused phase aberrations. To avoid phase aberrations from irregular boundaries between tissue and water, all the other experiments were done with plates of plastic between tissue and water. The irregular boundaries between fat and water did not cause phase aberrations in the previous sections, because these experiments were done at room temperature where the speed of sound in fat and water are almost equal.

The last experiment explored the focusing properties through two different specimens of mixed fat and meat. The two specimens used are shown as specimen C and D in figure 26. The thickness of the specimens were almost equal, and they consisted of equal portions of fat and meat. The difference between the specimens was the structure of the meat. Specimen C was relatively homogeneous, while the meat in specimen D contained of several islands of fat. The islands had an irregular form. The size was approximately 0.1-2 mm. The fat containing meat looked similar to the muscle layer of the obese person imaged in fig. 31.

The beam profiles measured after the wave had propagated through the two specimens are shown in fig. 67 and 68. The envelope of the RF-data is presented in fig. 69. Distortions caused by the homogeneous meat were minor, while distortions...
Table 4: Properties of the pig tissue used in the experiments. The bacon specimens were bought in a normal food store, i.e. salt and preserving agent (E-250 and E-450) were added to the tissue. In another experiment, it was used a pig that was slaughtered the same day as the experiment was done, i.e. no salt and preserving agent were added. The particulars of the measured speed of sounds are ±10 m/s, while the particulars of the measured attenuation coefficients are ±0.2 dB cm⁻¹ MHz⁻¹. The uncertainty is mainly due to the inaccurate measurements of the tissue thickness.

<table>
<thead>
<tr>
<th>tissue</th>
<th>section</th>
<th>temp. [°C]</th>
<th>speed of sound [m/s]</th>
<th>attenuation. coeff. [dB cm⁻¹ MHz⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat; bacon</td>
<td>5.2.3</td>
<td>≈ 22</td>
<td>1508</td>
<td>2.3</td>
</tr>
<tr>
<td>Fat; New slaughtered pork</td>
<td>5.2.4</td>
<td>≈ 22</td>
<td>1498</td>
<td>2.1</td>
</tr>
<tr>
<td>Fat; bacon</td>
<td>5.2.5</td>
<td>37</td>
<td>1437</td>
<td>1.0</td>
</tr>
<tr>
<td>Meat; bacon</td>
<td>5.2.3</td>
<td>≈ 22</td>
<td>1557</td>
<td>0.75</td>
</tr>
<tr>
<td>Meat; New slaughtered pork</td>
<td>5.2.4</td>
<td>≈ 22</td>
<td>1556</td>
<td>0.4</td>
</tr>
<tr>
<td>Meat; bacon</td>
<td>5.2.5</td>
<td>37</td>
<td>1591</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 5: Measured properties of water and of thin plates of hard plastic. The plates of plastic are used at the boundaries between tissue and water in the experiments performed at 37°C. The values were measured by the author, except for the attenuation coefficient in water. The attenuation coefficient for water is found in diagram 4.8 in [Duck90].

<table>
<thead>
<tr>
<th>medium</th>
<th>temp. [°C]</th>
<th>speed of sound [m/s]</th>
<th>attenuation. coeff. [dB cm⁻¹ MHz⁻¹]</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>≈ 22</td>
<td>1491 ±5</td>
<td>0.0020</td>
<td>2</td>
</tr>
<tr>
<td>water</td>
<td>37</td>
<td>1526 ±5</td>
<td>0.0014</td>
<td>2</td>
</tr>
<tr>
<td>plastic</td>
<td>37</td>
<td>2090 ±50</td>
<td>3.7 ±0.2</td>
<td>1</td>
</tr>
</tbody>
</table>
caused by the fat containing meat were large. The two specimens had almost equal thickness and are therefore assumed to cause the same attenuation. The decreased main-lobe and the increased side-lobes due to propagation through fatty meat compared with homogeneous meat are concluded to be caused by phase aberrations. The distortions in the ultrasound image caused by the two specimens are shown in fig. 29 and 30.

The difference in speed of sound in the fat (1437 m/s) and the meat (1591 m/s) cause larger phase aberrations at increased frequencies. A study of this effect is presented in fig. 70, 71 and 72. Fig. 70 shows the off-axis beam profiles for the different frequency components. The case with fatty meat clearly has highest side-lobes for all the frequency components. Frequency depended attenuation can therefore not explain the distortions caused by fatty meat. The beam profiles for the 2MHz frequency component of the two specimens are presented in fig. 71, while the beam profiles for the 4 MHz frequency component are shown in fig. 72. The focusing through homogeneous meat is largely improved by increasing the frequency from 2 to 4 MHz, while the focusing through fat containing meat deteriorates by increasing the frequency. The increase of the side-lobes by using higher frequencies is a typical effect of phase aberrations.
Figure 61: Signals observed at the electronic focal point of the transducer. The different propagation media are: 1) Dome, water. 2) Dome, 0.34 mm plastic, 5 mm water, 0.45 mm plastic, water. 3) Dome, 0.34 mm plastic, 10 mm fat, 0.45 mm plastic, water. 4) Dome, 0.34 mm plastic, 9 mm meat, 0.45 mm plastic, water.

Figure 62: Frequency spectra of the signals presented in the previous figure. The lower plot shows the spectra normalized against the plastic in water case.
Figure 63: Off-axis beam-profiles at the electronic focal depth. The data is collected by moving the hydrophone normal to the depth-axis. The energy is calculated as $10 \times \log_{10}(\sum \text{signal}^2)$. The propagation media are: 1) Dome, water. 2) Dome, 0.34 mm plastic, 5 mm water, 0.45 mm plastic, water. 3) Dome, 0.34 mm plastic, 10 mm fat, 0.45 mm plastic, water. 4) Dome, 0.34 mm plastic, 9 mm meat, 0.45 mm plastic, water.

Figure 64: Normalized version of the previous figure
Figure 65: Off-axis beam-profiles at the electronic focal depth. The data is collected by moving the hydrophone normal to the depth-axis. The energy is calculated as $10 \cdot \log_{10}(\sum signal^2)$. The propagation media are: 1) Dome, 10 mm fat, water. 2) Dome, 0.34 mm plastic, 10 mm fat, 0.45 mm plastic, water.

Figure 66: Normalized version of the previous figure
Figure 67: Off-axis beam-profiles at the electronic focal depth. The data is collected by moving the hydrophone normal to the depth-axis. The energy is calculated as $10 \times \log_{10}(\sum \text{signal}^2)$. The propagation media are: 1) Dome, 0.34 mm plastic, 5 mm water, 0.45 mm plastic, water. 2) Dome, 0.34 mm plastic, specimen C (homogeneous meat), 0.45 mm plastic, water. 3) Dome, 0.34 mm plastic, specimen D (fatty meat), 0.45 mm plastic, water.

Figure 68: Normalized version of the previous figure
Figure 69: The envelope of the signals (RF-data) observed at the electronic focal depth. The different media are described above each plot. The envelope is calculated by smoothing the absolute value of the signals with a hamming window with length 1.5 wave-period. $c=1526$ m/s.
Figure 70: Each horizontal line represents the normalized beam profiles at one specific frequency. The different media are described above each plot. The plots are made by: 1) Each vertical line is the frequency spectrum of the signal observed at that hydrophone position. 2) The figure is then normalized for each horizontal line, so that the maximum of each horizontal line (beam-profile) is 0 dB.
Figure 71: The beam-profile for the 2 MHz frequency component. The two media are: 1) Dome, 0.34 mm plastic, specimen C (hom. meat), 0.45 mm plastic, water. 2) Dome, 0.34 mm plastic, specimen D (fatty meat), 0.45 mm plastic, water.

Figure 72: The beam profiles for the 4 MHz frequency component. The media are the same as in the previous figure.
5.3 Verification of the simulation program

A theory for wave-propagation through a layered medium is developed in section 3, and a simulation program based on this theory has been written. The program is described in section 4, and an experimental evaluation of the simulations is presented here. The purpose of the evaluation is to verify the implementation of the program, and confirm that the theory is sufficient to describe focusing through a layered medium.

5.3.1 The test media

An annular transducer with two rings is used to verify the simulation program. The rings have equal areas and there is a 0.25 mm space between them. The diameter of the transducer is 14.7 mm, and the Radius Of Curvature (ROC) is 73 mm. The ROC-value was specified to 78 mm [76], but the experiments in water showed that the signal to the rings was in phase at depth 73 mm. It is therefore assumed that the ROC of the used transducer is 73 mm.

The verification is done for two different media: One that consists of water only, and one that includes the transducer dome (fig. 73). The same transducer was used in all the experiments and the temperature was 21°C. The purpose of the experiments was to show that the beam profiles may be significantly changed by a layered medium, and that the simulation program can predict the difference.

The effect of the dome depends on the fluid inside it. A dome with the usual fluid (c=1579 m/s) gives only a minor effect on the beam profiles. This is good for the imaging, but the medium is not suitable to verify the simulation program. The effect of the dome increases if we use water (c=1486 m/s) inside the dome. The water-dome-water medium is therefore used to evaluate the simulation program.

The dome is spherical with an inner radius of 9.7 mm and an outer radius of 10.5 mm. The dome was split in two after the experiments to verify that the thickness was 0.8 mm. The transducer was placed with an offset 3.7 mm from the center of the dome. Other properties of the water [8] and the dome [76] are given in tab. 6.

The resonance frequency of the transducer is 3 MHz, but since the frequency response of the hydrophone is (+/- 1dB) in the range 2-15 MHz only, the excitation of the transducer was increased to 4 MHz.

<table>
<thead>
<tr>
<th>medium</th>
<th>speed of sound</th>
<th>impedance</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>1486 m/s</td>
<td>1.49 MRayl</td>
<td>0.05 $m^{-1}MHz^{-2.0}$</td>
<td>2.0</td>
</tr>
<tr>
<td>dome</td>
<td>1996 m/s</td>
<td>1.86 MRayl</td>
<td>61.5 $m^{-1}MHz^{-1.4}$</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Table 6: Properties of water and the dome at $T = 21^\circ C$. The absorption along a ray is described by the formula: $I = I_0e^{-\alpha fr}$, where $r$ is the propagation distance, $f$ is the frequency and $I$ is the intensity.
Figure 73: The two media used to evaluate the simulation program: One that consists of water only, and one that consists of water, dome and water (T=21°C). Some of the rays between the focal point and the transducer are also drawn.

Figure 74: The figure shows the signal measured at the focal point (z=73 mm) of the transducer. The medium was water only. The pulse is used as input to the simulation program.
Fig. 74 shows the pulse measured at the focal point of the transducer. The medium was water only, i.e. the signals from different transducer positions were added together without phase aberrations. The frequency-dependent attenuation in water is weak, and it is therefore assumed that the signal observed in the focal point gives a good approximation to the signal emitted from the transducer. The experimental pulse shown in fig. 74 is therefore used as input to the simulation program.

5.3.2 Comparison between simulations and experiments

On-axis and off-axis beam profiles are measured with a 0.4 mm hydrophone as described in section 5.2.1. The same beam profiles are simulated with Ultrasim. The two media shown in fig. 73 are used.

The simulation program takes care of phase aberrations and of the attenuation factors absorption-, reflection- and ray broadening loss. Attenuation of a ray because it does not arrive normally to the transducer is also included in the simulations.

The calculation of the phase aberrations can be evaluated by fig. 75. The delay between the signal from the inner and the outer element is plotted for increasing distances to the transducer. For the case with water only, the experiments and the simulations show equal delay between inner and outer ring. For the medium with dome, the experimentally measured delay between the two rings is shifted. The focal point is moved towards the transducer. The shift in the focal point from 73 mm to 53 mm shows that it is important to take care of the dome. Fig. 75 shows that the phase aberrations from the dome are well predicted by the simulation program.

The measured and simulated on-axis energy is shown in fig. 76. The simulations fit very well with the experiments in water. There is a somewhat larger difference between the simulations and the experiments when the dome is included in the medium. However, this difference is much less than the difference caused by the dome. The simulation program therefore predicts the change in the on-axis beam profile by including the dome in the medium well.

Both the experimental and the simulated curves for the water medium (fig. 76) are normalized to 0 dB at depth=73 mm. The same normalization is used for the dome-curves, i.e. the difference between the curves for water and the curves for the dome shows the effect of the dome. The main effect is that the focal point is moved to the transducer, and that the signal is attenuated by propagation through the dome.

The on-axis beam profiles for each element are plotted in fig. 77 and 78. The dome gives greatest effect on the outer ring. The simulations fit very well when the medium is water only. When the dome is included in the medium, the difference between simulations and experiments increases somewhat (≈ ±1dB). This is a notable difference, but the difference is small compared to the effect of the dome.

The simulation of off-axis beam profiles is also verified. Fig. 79 shows the beam profiles at depth 68 mm. The depth of the hydrophone is given by the propaga-
Figure 75: The delay between the inner and the outer ring is measured for the two test media. Medium 1 consists of water only, while medium 2 consists of water, a dome and water. The measurements are compared with simulations to verify that the developed simulation program can predict the phase aberrations caused by a dome.

Figure 76: The figure shows the on-axis energy of a transducer for the two test media. The figure shows both simulations and experiments. energy = 10 log10(Σ_i signal(i)^2)
Figure 77: A comparison of simulations and measurements. The figure shows the on-axis energy of the inner ring for the two test media. Medium 1 consists of water only, while medium 2 consists of water, a dome and water.

Figure 78: A comparison of simulations and measurements. The figure shows the on-axis energy of the outer ring for the two test media. Medium 1 consists of water only, while medium 2 consists of water, a dome and water.
Figure 79: A comparison of simulations and measurements. The figure shows the off-axis energy of a transducer for the two test media at depth=68 mm. Medium 1 consists of water only, while medium 2 consists of water, a dome and water.

Figure 80: A comparison of simulations and measurements. The figure shows the off-axis energy of a transducer for the two test media at depth=22 mm. Medium 1 consist of water only, while medium 2 consist of water, a dome and water.
Figure 81: The figure shows two signals measured at the focal point of the transducer. The solid line shows the pulse measured at the focal point ($z=73$ mm) in water, while the dash-dotted line shows the pulse measured at the focal point ($z=53$ mm) when the dome is included in the medium.

Figure 82: The figure shows two simulations of the off-axis beam profile. The dome is included in the medium. The only difference between the simulations is the excitation signal. The two signals shown in the previous figure have been used.
tion time from the transducer and the speed of sound in water (1486 m/s). The simulations fit well with the measurements for both media.

An off-axis comparison in the near-field (depth=22 cm) is shown in fig. 80. The simulation program predicts the effect of the dome well.

The main conclusion is that the errors in the simulations and the experiments are small compared with the differences between the beam profiles for the two media. The simulation program therefore predicts the effect of the dome well. Thus, the simulation program seems to be a good tool to describe the effect of a layered medium.

The small difference between the simulations and the experiments when the dome is included in the medium can be caused by errors in the measurements. But, since the difference is notable only when the dome is included in the medium, it is more probable that the dome is not sufficiently modelled. One possible explanation can be shear waves in the dome. Such waves are not included in the simulation program. Another possible explanation may be that the parameters used for the dome are inaccurate. Small errors in the input variables to the simulation program can explain the differences in the beam profiles. There may for instance be errors in the speed of sound, the absorption or the shape of the dome. The position of the transducer compared with the dome will also influence the beam profile.

5.3.3 Frequency-depended attenuation

In the simulation program, the attenuation of the different frequency components of the pulse is approximated to the attenuation of the center frequency of the pulse. The simulated pulse in the focal point of the medium will therefore get the same shape as the emitted pulse.

In fig. 81, we can see that the measured pulse in the focal point of the two test media is somewhat different. The pulse is smoothed by propagation through the dome. The effect can partially be explained by the frequency-depended absorption in the dome material, which attenuates high frequencies more than low frequencies. The difference is also caused by phase aberrations over each element. These phase aberrations are zero in the focal point of the homogeneous medium, but they are not zero at the "focal point" (z=53 mm) of the dome in water medium. The phase aberrations are small, and it is therefore only the highest frequencies that are cancelled out by destructive interference.

To study the consequence of not taking care of the frequency-depended absorption, the two pulses were used as input to the same simulation. Fig. 82 shows that the difference in the pulse form gives a neglectable small change in the simulated beam profile. It seems thus that the modifications of the pulse form can be neglected for the dome in water medium.

The frequency-depended absorption in the dome in water medium is relatively weak, because the dome is thin and the absorption in water is neglectable. Can we neglect the frequency-depended absorption if the medium consists of human tissue,
The effect of frequency depended absorption

![Graph showing the effect of frequency depended absorption.](image)

Figure 83: The two pulses are attenuated by propagation 70 mm through human tissue. The '-' pulse is attenuated as in Ultrasim, i.e. the whole pulse is attenuated with the absorption of its center frequency. In the '-.-' pulse, it is taken care of the different absorption of each frequency component. The absorption parameters \( \alpha = 15 \) and \( \beta = 1 \) are used. The absorption is described in the formula: \( I = I_0 e^{-\alpha f^{\beta} v} \), where \( r \) is the propagation distance, \( f \) is the frequency and \( I \) is the intensity.

Normalized beam profiles for the two pulses

![Graph showing normalized beam profiles for the two pulses.](image)

Figure 84: The figure shows two simulations of the off-axis beam profile at depth 70 mm. The medium is homogeneous. The two signals shown in the previous figure are used as input to the simulation program.
too? In human tissue, the absorption parameters are typically $\alpha = 15$ and $\beta = 1$ [1].

Fig. 83 shows how frequency-depended absorption changes a pulse by propagating through 70 mm human tissue. This is done by using the Fourier transform on the emitted signal, multiplying the spectrum with the frequency depended absorption and then using the inverse Fourier transform. The frequency depended-absorption shifts the pulse to a pulse with lower center frequency. The two pulses shown in fig. 83 are used as input to Ultrasim. Fig. 84 shows that the shift to lower frequencies cause a broader off-axis beam profile, and the sidelobes increase approximately 1 dB. The conclusion is that Ultrasim can be improved by also taking care of the frequency-depended absorption.

The effect of frequency-depended absorption will increase if we use a longer propagation distance in tissue or if we use a more broad-banded pulse than shown in fig. 83.

For full compensation of the frequency-depended absorption in a layered medium with dome, fat, muscle etc., the compensation must be done for each ray. This is time-consuming. Different rays will propagate roughly the same distance in the different layers. This means that the main effect of the frequency-depended absorption is almost equal for each ray. A reasonable approximation is thus to calculate how a pulse changes by propagation through a typical ray, and then using these changes for all the rays. This approximation will be implemented in the next version of Ultrasim.
6 Fat lens tuning (FLT) by annular array transducers

To do optimal phase aberration correction in medical ultrasound imaging a 2D array transducer is needed. It will take some years before such transducers are available. In this section it is shown that phase aberration correction also can be done with annular array transducers.

6.1 Models of a thin and an obese patient

Everyone who uses an ultrasound scanner knows that some patients give better ultrasound images than others. To understand why image quality depends on the patient, ultrasound and Magnetic Resonance (MR) images of two patients who gave ultrasound images with different quality were recorded. The MR-images are used as a gold standard to compare the two patients. The images are shown in fig. 21 and 22. The tissue of the two patients, including the probe fluid and the spherical cap, are modelled as layers with smooth boundaries.

The patient who gave ultrasound images with low quality had more fat than the one who gave images with high quality. The obese one had a 2-3 cm thick outer layer of fat. The fat-layer was so soft that it was formed by the dome. The boundary between the fat-layer and the dome become spherical, while the boundary to the muscle layer still was plane. The speed of sound in fat is lower than the speed of sound in the probe. This causes a lens effect that move the focus deeper into the body. The fat-layer of the thin person was bent towards both the dome and the layer of muscle and will therefore not work as a lens. To explore the effect of the observed inhomogeneities four simplified models were studied.

1. The homogeneous case; assumed by the focusing algorithm.
2. The probe in water case; the test tank.
3. A thin person; high image quality.
4. An obese person; low image quality.

The four cases are shown in fig. 85. It is used an example with an annular transducer with diameter=15 mm, N=4 rings, Radius Of Curvature =50 mm and a dome with inside radius=9.7 mm and outside radius=10.5 mm. The velocities of sound in the different media is probe fluid=1579 m/s, dome=1974 m/s, fat=1440 m/s and homogeneous tissue=1540 m/s.
6.2 Algorithm for electronic focusing

If we assume a homogeneous medium and use an annular equal-area transducer, it is possible \[1\] to focus to depth \(z\) by using the delays \([0, \Delta T, 2\Delta T, 3\Delta T]\) on elements \([1,2,3,4]\). Element 1 is the inner ring.

\[
\Delta T \approx \left( \frac{1}{z} - \frac{1}{ROC} \right) \frac{D^2}{8Nc}
\]

Expected velocity in tissue

Diameter of transducer

Radius Of Curvature

6.3 Simulations

To explore how the focusing algorithm will work on the different models, the developed simulation program is used. Snell’s law is used in the simulations and it is corrected for the 1/r spherical loss factor. It is not taken care of reflection- and absorption loss. We want to detect the effect of the phase aberrations, so it is an advantage to neglect the absorption factors. The different attenuation in the models would disturbed the study of the phase aberration’s effect on the beam-profiles. In the study presented here, the effect of the phase aberrations are isolated, so that the difference in the simulated beam profiles are caused by phase aberrations only.

In the first simulations we did dynamic focusing with eq. 66. The results are presented in fig. 86. In the homogeneous medium, the signals of the elements are in phase for all depths after the correction. The algorithm works well. This is not the case for the three other models, not even for the probe in water case. The simulations show especially large phase aberrations for the obese patient: If we use a transducer with center frequency 5 MHz (\(T=200\text{ns}\)), the smallest and largest ring will have a phase difference of 0.5\(\pi\) in the far field. Fig. 89 shows the corresponding beam profiles using a 5 periods cosine pulse with cosine shape. The on-axis curves are normalized to 1 at depth 50 mm for the case with no phase errors. We see that the focusing algorithm does not work well when the patient is obese.

Since the phase aberrations are different in the different cases, we need a method that can handle different media, and we will show that eq. 66 can handle this. We can tune three parameters; ROC, D and c. Tuning c gives the same effect as tuning D, so we have only two independent parameters; ROC and D.

Fig. 85 shows that the refraction in the dome reduces the effective transducer diameter. The parameter D used in the algorithm, is therefore tuned down to 0.9*D. A simulation example with the dome in water case is shown in fig. 87. The phase differences between the elements are approximately the same for all depths after the tuning of D. By adding constant delay on each element, optimal dynamic focusing through the dome and the water is obtained.
Figure 85: Different problems to analyse

Figure 86: Delays [ns] on each element as a function of depth after dynamic focusing with the uncorrected algorithm for electronic focusing.
Figure 87: Phase aberrations between elements for dome in water before/after tuning D and add. const. delay

Figure 88: Phase aberrations between elements for a thin and an obese patient before/after tuning ROC
Figure 89: Energy after focusing with the uncorrected algorithm for el. focusing. PW. f=5MHz. Two-way energy is calc. as tr. energy multiplied with rec. energy.

Figure 90: Energy after focusing with the tuned algorithm for electronic focusing. PW. f=5MHz. Two-way energy is calc. as tr. energy multiplied with rec. energy.
In fig. 88 the focusing algorithm optimized for probe in water is used for the thin and the obese patient. The focusing is not optimal here. By studying the thin and the obese patient, we found that the phase aberrations are minimized at depth = 47 mm for the thin patient, while the phase errors are minimized at depth = 59 mm for the obese patient (see fig. 11). If we use these values as ROC in the algorithm, fig. 88 shows that the algorithm works well for the focusing in both the thin and the obese patient.

Fig. 90 shows that the beam profiles are improved compared to the untuned focusing in fig 89. An interesting observation is that the on-axis focusing is better with the dome than without, for depths different from ROC. The main result is the improvement of the beam profile of the obese patient. The two-way side-lobe level is reduced from 21 dB to 34 dB in this example.

### 6.4 Correction by parameter tuning

It is found that it is possible to correct for phase aberrations with annular array transducers. In the analysed example with the obese patient, the two-way side-lobe level was reduced 13 dB. The focusing can be optimized by tuning two parameters in the algorithm for electronic focusing (eq. 66). The first parameter to be tuned is the diameter of the transducer. This can be done by simulations or by experiments. Tuning the diameter corrects for the fact that rays from the outer elements are refracted more through the dome than rays from the inner elements. The transducer seems therefore smaller than it is. This effect is independent of the patient.

The other parameter to tune is the Radius Of Curvature (ROC). If we observe a homogeneous tissue, the real focus will be at ROC, and it will be natural to use this value in the algorithm for electronic focusing. But, if we observe on an obese patient we get large focus displacement in depth. The displacement is caused by a spherical dome that makes a lens of the thick and soft outer fat-layer. If we still use ROC in the dynamic focusing algorithm, we will not focus optimally. But, if we replace the ROC parameter used in the algorithm with the depth of the focus of the transducer-medium system, the algorithm will work well. Since the effect depends on the patient, the correction has to be done for each examination. This can be done by introducing an operator controlled adjustment of the ROC parameter used in the algorithm. It may also be possible to make an automatic algorithm that is based on methods as max. correlation, max speckle brightness or the time reversal mirror method.

Improvements made possible by the fat lens tuning (FLT) method are shown experimentally in fig. 91. An annular transducer with four rings, D = 15 mm, ROC = 75 mm and f = 3.25 MHz is used in the experiments. The dome was the same as in the simulation example. The figure shows ultrasound images of balloons in water.

The image to the left shows the balloons in water taken with default focusing (D = 15 mm and ROC = 75 mm). The second image shows the image after changing the D and ROC parameter used in the algorithm for electronic focusing. The
Parameters are set to the values that the simulation program found to be optimal for a dome in water medium (D=13.8 mm and ROC=84 mm). It is possible to see significant improvements.

Simulations with an obese fat-layer and no tuning of the parameters (D=15 mm and ROC=75 mm) gave the same phase aberrations as simulations with a dome in water medium with parameters tuned to D=15 mm and ROC=61 mm. In the third image of balloons in water, the parameters were set to D=15 mm and ROC=61 mm to show the fat lens effect. The quality of the ultrasound image of the balloons is now significantly reduced. The FLT method is shown to correct for the lens effect caused by a bent outer layer of fat. The improvements made possible by the method are thus given by the different quality of the balloon-images in fig. 91.

Figure 91: a) Image of balloons in water taken with standard equipment, b) Image after phase aberration correction, c) Image with phase aberrations as caused by an outer bent fat-layer as seen in the MR-images of the obese patient. The gain is equal for the three images.
6.5 Dome geometry

Figure 92: Analyse of phase error using a plane dome

Fig. 92 shows that it is possible to avoid much of the patient-dependent phase error by using a plane dome. By tilting the transducer, there may be generated phase aberrations since we do not look perpendicular through the dome. The simulations show an angle movement of 0.8 degrees when tilting the main beam 20 degrees, but only a small defocusing of the beam profile. Unfortunately, the dome material is not stiff enough to make a plane dome for a mechanical transducer. Making a plane dome small enough to observe through the ribs for cardiac imaging is also a problem.
7 Delay and amplitude focusing (DAF)

It has been experimentally proven that phase aberration correction based on time delay focusing can correct phase aberrations generated close to the transducer ([67], [84]). It is also shown experimentally that the effect of time delay focusing is reduced when the phase aberrations are generated a few centimeters from the transducer. The time reversal mirror (TRM) method does not have this limitation [97]. To explore why, the focusing through a phase screen 50 mm from the transducer is explored by simulations. The simulations and some important conclusions are presented in this section.

The observations from the simulations are used to suggest a new and improved method for focusing through an inhomogeneous medium. The method is based on both delay and amplitude focusing (DAF). The improvement compared with the time delay focusing techniques is that the suggested method can correct distortions caused by phase aberrations introduced far from the transducer. The improvement compared with the TRM method is that the suggested method does not need a point reflector, but it can be based on reflections from diffuse scatterers which are common in the human body.

7.1 Simulation of phase and amplitude aberrations

A phase screen is a simplified model of a thin layer of inhomogeneous tissue. Delay aberrations will be introduced to waves that propagate through the phase screen. This delay aberrations are simulated by an AR1 process:

\[ x(k) = ax(k - 1) + w(k), \quad w(k) \sim N(0, \sigma^2) \]  \hspace{1cm} (67)

The delay, \( x(k) \), in position \( k \) is given as a weighted sum of the delay in position \( k - 1 \) and a random term \( w(k) \). The correlation structure of \( x(k) \) is described by the autocorrelation function \( \rho(m) \) [3]:

\[ \rho(m) = \frac{R(m)}{R(0)} = a^{|m|}, \quad R(m) = \frac{a^{|m|}}{1 - a^2\sigma^2} \]  \hspace{1cm} (68)

The correlation length is defined as the distance where \( \rho(m) > 0.5 \). To simulate a phase screen with specified correlation length, we set the parameter \( a \) to:

\[ a = \left( \frac{1}{2} \right)^{\frac{1}{|m|}}, \quad m = \frac{\text{corr. length}/2}{\text{discretization step}} \]  \hspace{1cm} (69)

The standard deviation (std\(_z\)) of the delay aberrations, \( x(k) \), is \( \sqrt{R(0)} \). The parameter \( \sigma \) is then given by:

\[ \sigma = \text{std}_z\sqrt{1 - a^2} \]  \hspace{1cm} (70)

To simulate realistic phase aberrations for the human abdominal wall, the parameters in the AR1 model are based on measurements from Hinkelman et.al. [39]. The
average correlation length and standard deviation for the measured delay aberrations were 7.9 mm and 43.0 ns. The corresponding values of the amplitude aberrations were 2.3 mm and 3.3 dB.

If we set \( a = 0.983 \), \( \sigma = 8.0\text{ns} \) and the discretization step to 0.1 mm, eq. 67 can be used to simulate phase aberrations typical for the human abdominal wall.

### 7.2 The effect of typical phase and amp. aberrations

It is simulated how aberrations typical for the human abdominal wall will influence the beam profile of a transducer. The simulations are done for a 128 elements 50 mm linear transducer that is electronically focussed to 100 mm. A three wave lengths long cosine shaped pulse with center frequency 3.5 MHz is used. Typical phase and amplitude aberrations (fig. 93) are introduced on the transducer. The beam profile calculated 100 mm away from the transducer is presented in fig. 94. The phase and amplitude aberrations make the mainlobe broader and increase the sidelobe level. If we have amplitude aberrations only, i.e. remove the phase aberrations, the beam profile is improved, but it is not as good as when the beam is undisturbed.

Simulations with uncorrelated aberrations are presented in fig. 95. The mainlobe is not disturbed and the level of the sidelobes is constant. The simulations indicate that the locally focusing and defocusing of the correlated phase aberrations disturb the beam profile more than uncorrelated phase aberrations.

Figure 93: Phase and amplitude aberrations typical for the human abdominal wall. The simulations are done with an AR1-model.
Figure 94: Distortions of the beam profile due to aberrations typical for the human abdominal wall.

Figure 95: Distortions of the beam profile due to aberrations with size typical for the human abdominal wall, but with correlation length=0. The distortions decrease when the correlation length of the aberrations decreases.
7.3 Propagation of a rough wavefront

Phase aberrations caused by a rough boundary between for example fat and muscle can be modeled by a phase screen. A thin layer of muscle with islands of fat also causes phase aberrations that can be modeled by a phase screen. When a plane wave propagates through a phase screen, phase aberrations are introduced to the wave front. The change of the aberrated wave front by further propagation in a homogeneous medium is simulated. The results are presented.

To simulate how an aberrated wave changes through propagation, the integral equation in eq. 56 is used. It is assumed a homogeneous and non-absorbing medium. The wave front is represented by points, where each point generates a spherical wave with amplitude and phase as the wave-front. The cosine factor in eq. 56 is neglected and the calculations can therefore be viewed as Huygen’s principle or the Rayleigh integral [18]. Continuous waves with frequency 1, 3.5 and 5 MHz are used in the simulations.

The phase aberrations of the simulated wave front are presented for increasing propagation distances in fig. 96. The standard deviation and the correlation length of the phase aberrations are not significantly changed by propagation, i.e. the phase aberrations are not smoothed by propagation. Phase aberration correction of the received wave can therefore improve the focusing even when the phase aberrations are generated several centimeters from the transducer.

The change of the wave-front can be studied in fig. 98. The figure shows the same simulations as in fig. 96, but the phase screen is now subtracted from the wave-front. We see that the rough wave front is changed by propagation. Fig. 99 shows that the change is larger for low than high frequencies.

A phase screen close to the transducer can be corrected by emitting a pulse with phase aberrations opposite as the phase screen. The effect of such corrections will decrease with the distance between the transducer and the phase screen. The correction effect will also decrease with decreasing frequency.

The amplitude distortions of the wavefront are shown in fig. 100. The size and correlation length of the amplitude aberrations increase by propagating away from the phase screen. Fig. 101 shows that the amplitude aberrations also increase by using higher frequencies.

The amplitude aberrations caused by waves propagating more than 20 mm (f=3.5 MHz) have the same standard deviation as measured for the human abdominal wall. The amplitude aberrations are caused by local focusing and defocusing of the wavefront. This effect can probably explain the amplitude aberrations measured in [39].
The wavefront from \(-50\) mm to \(50\) mm

Figure 96: Time delay aberrations of the wavefront of a plane wave that has propagated through a phase screen at \(z=0\). The figure shows the phase front at increased distance to the phase screen, \(f=3.5\) MHz, CW.

Figure 97: Standard deviation and correlation length of the time delay aberrations to the rough wave front as a function of propagation distance. The three graphs show simulations done with continuous waves with frequency 1, 3.5 and \(f=5\) MHz.
- human abdominal wall aberrations, — no aberrations

<table>
<thead>
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<th>z</th>
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<th>corr</th>
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<tr>
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<td>0 ns</td>
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</tr>
<tr>
<td>2 mm</td>
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<td>50 mm</td>
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<td>100 mm</td>
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<td>200 mm</td>
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The wavefront from -50 mm to 50 mm

Figure 98: The difference between the time delay aberrations of the wavefront to a plane wave that propagated through a phase screen at z=0 and the phase screen. The figure shows the phase differences at increased distance to the phase screen, f=3.5 MHz, CW.

<table>
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<tr>
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<td>20</td>
<td>30</td>
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<tr>
<td>Distance to phase screen [mm]</td>
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<table>
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<th>3.5 MHz</th>
<th>5 MHz</th>
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Figure 99: Standard deviation and correlation length of the difference between the time delay aberrations to the rough wave front and the phase screen. The three graphs show simulations done with continuous waves with frequencies 1, 3.5 and f=5 MHz.
The wavefront from \(-50\) mm to \(50\) mm

Figure 100: Amplitude aberrations of the wavefront of a plane wave that has propagated through a phase screen at \(z=0\). The figure shows the amplitude when the distance to the phase screen is increased, \(f=3.5\) MHz, CW.

![Figure 100: Amplitude aberrations of the wavefront of a plane wave that has propagated through a phase screen at \(z=0\).](image)

Figure 101: Standard deviation and correlation length of the amplitude aberrations of the rough wave front as a function of propagation distance. The three graphs show simulations done with continuous waves with frequency 1, 3.5 and 5 MHz.

![Figure 101: Standard deviation and correlation length of the amplitude aberrations of the rough wave front as a function of propagation distance.](image)
7.4 Focusing of an emitted wave

7.4.1 The medium model

The medium shown in fig. 102 is used to evaluate methods for phase aberration correction. The medium is homogeneous except for a phase screen 50 mm from the transducer.

A phase screen can be used to model the phase aberrations caused by propagation through a rough boundary between fat and muscle. It can eventually be used to model the phase aberrations caused by propagation through a thin layer of muscle with islands of fat. The phase screen introduces phase aberrations, but no amplitude aberrations. Fig. 103 shows the phase screen that will be used in the simulations. The size and the correlation length are typical for phase aberrations caused by propagating through the human abdominal wall [39].

![Figure 102: The medium model.](image)

To simplify the simulations and the presentation of them, a point reflector is located far from the phase screen. The reflected wave will thus be approximately plane at the phase screen. The effect of a point scatterer closer to the phase screen and the effect of diffuse scatterer will be discussed in sec. 7.6.

A 100 mm long transducer consisting of 500 elements is used in the simulations. The simulations are done for continuous waves and for pulsed waves. The frequency of the signal is 3.5 MHz.
7.4.2 The DAF method

Delay and amplitude focusing (DAF) is a new method to focus through inhomogeneous tissue. The purpose is to emit a wave that does not have phase and amplitude aberrations after it has propagated through the aberrating area.

The usual methods use delay compensation as a tool to optimize the focusing [30] [60]. The electronic delays of the emitted wave are set equal to the time delay distortions in the previously received wave, but with opposite signs.

The time reversal mirror (TRM) method offers a better focusing. The TRM method [29] uses both the delay- the amplitude- and the pulse shape of the previously received wave to do optimal focusing. The divergent pressure field reflected from the acoustic point source is received and stored in shift registers at the transducer. One signal for each transducer element is stored. The signals are reversed in time and then retransmitted. This converts the acoustic field into a convergent wave focusing on the point source.

Experiments show that the TRM method does better corrections than the time delay focusing techniques when the aberrating area is a few centimeters from the transducer [97]. This shows that there must be some important information in the amplitude and the pulse form of the received wave.

The simulations presented in sec. 7.3 show that a rough wave front generates amplitude aberrations through propagation. This observation can explain the disappointing effect of time delay focusing through a phase aberrator a few centimeters from the transducer. The delay focusing emits a rough wave front to compensate for the phase aberrator. This phase front generates amplitude aberrations by propagating to the phase screen. When the wave has propagated through the phase screen, the wave front may be without phase aberrations, but the amplitude aberrations are large. The simulations presented in sec. 7.2 show that amplitude aberrations reduce the focusing.

The amplitude aberrations are caused by local focusing and defocusing of the wave front. It is possible to cancel out this amplitude aberrations by using apodization on the emitted wave. The amplitude of signals that are locally focused must be
decreased, while the amplitude of signals that is locally defocused must be increased. By setting the delay and the apodization correct, the amplitude fluctuations set by the apodization will be smoothed out by propagation to the phase screen. The phase aberrations will be canceled by propagating through the phase screen.

The idea behind the DAF method is to manipulate the delay and the amplitude of the signal emitted from each element. The goal is to emit a wave that does not have phase and amplitude aberrations after it has propagated through the aberrating area. The DAF method uses the following delay and amplitude compensation: The electronic delays of the emitted wave are set equal to the time delay distortions in the previously received wave, but with opposite sign. The apodization is set equal to the amplitude aberrations of the received wave.

The DAF method and the TRM method is equal if we use continuous waves, but different if we use pulsed waves. The delay focusing, the DAF method and the TRM method will be compared in the next sections.

7.4.3 Adaptive focusing of continuous waves

Fig. 104 shows the phase and the amplitude aberrations of a reflected wave that has propagated from the point reflector, through the phase screen and 50 mm further to the transducer. The phase aberrations at the transducer have \( \text{std} = 33\text{ns} \) and \( \text{corr.length} = 7.9\text{mm} \), while the amplitude aberrations of the received wavefront had \( \text{std/mean} = 0.33 \) and \( \text{corr.length} = 1.7\text{mm} \). The effect of similar aberrations are demonstrated in fig. 94. We see that corrections are necessary.

Figure 104: The figure shows the wave front of a plane wave that has crossed the 100 mm long phase screen and propagated 50 mm to the transducer. CW.
Figure 105: The figure shows the properties of a wave that has propagated from the transducer, through a 50 mm homogeneous medium, and finally through the phase screen. The electronic delays of the emitted wave were set equal to the time delay distortions in the previously received wave, but with opposite sign. Apodization was not used. CW.

Figure 106: This figure shows the same as the previous figure, but the apodization of the emitted wave was here set equal to the amplitude aberrations in the received wave. CW.
The wave observed on the transducer will be used to do modifications of the next emitted pulse. We want the emitted wavefront to be without phase and amplitude aberrations after it has crossed through the phase screen. The most simple idea is to use electronic delays with delays opposite of the phase aberrations of the phase screen. This will work well only in the near-field, because an aberrated wavefront changes its shape through propagation.

Another possibility is to use the opposite of the phase aberrations of the received signal as electronic focusing of the next emitted wave. This is the usual way of delay focusing. Fig. 105 shows the wavefront after propagating from the transducer through a 50 mm homogeneous medium, and after crossing the phase screen. A perfect correction will give no phase or amplitude aberrations in this wavefront. Unfortunately, the standard deviation of the phase aberrations is still as high as 13.5 ns and the $\text{std/mean}$ for the amplitude aberrations is as high as 0.21. This is an improvement compared with no corrections, but the focusing is not optimal.

A third possibility is to use the amplitude information in the received signal as well. The delay correction is the same as for delay focusing, but additionally we use the amplitude in the received signal as an apodization of the emitted signal. The wavefront observed directly after crossing the phase screen is shown in fig. 106. The standard deviation of the phase aberrations is now reduced to 5.8 ns and the $\text{std/mean}$ of the amplitude aberrations is reduced to 0.11. This is a significant improvement compared with the delay focusing technique. This method can be viewed as the time reversal mirror method [29]. The TRM method corrects usually for delay-, amplitude- and pulse form aberrations, but in the case with continuous waves there are no pulse form aberrations. This is because a sum of continuous waves of the same frequency, but with different phase and amplitude, is a continuous wave with the same frequency. The TRM method therefore simplifies to amplitude and delay focusing (DAF) in this example.

The conclusion from this section is that the time delay focusing improves the focusing even when the phase screen is several centimeters from the transducer. The simulations also show that the delay focusing can be improved significantly by extending it to delay and amplitude focusing.
7.4.4 Adaptive focusing of pulsed waves

In the previous section, the simulations show that delay focusing emits a wave that improves the focusing through the phase screen. It is also shown that the focusing can be improved by using the amplitude and pulse form aberrations in the reflected wave. The question is whether it is the amplitude aberrations or the pulse form aberrations that contain the necessary information to do the improvements. To explore this, the simulation experiment from the previous section is repeated with a pulsed signal.

Fig. 108 shows an image of a plane wave that has propagated through the phase screen and 50 mm further to the transducer. If the phase screen was removed, the signal received by the transducer would look as shown in fig. 107. The difference between these two images, shows the effect of the phase screen. We see that the phase and amplitude aberrations are great when the phase screen is included in the medium. The distortion of the pulse form looks small. The phase and the amplitude of the two received signals are shown in fig. 109 and fig. 110.

Two methods are used to focus the emitted wave. The effects of the delay focusing method are shown in fig. 111 and fig. 113. The electronic delays of the emitted wave were set equal to the time delay distortions in the previously received wave (fig. 110), but with opposite signs. Apodization was not used. Fig. 111 shows an image of the wave that has propagated from the transducer, through the 50 mm homogeneous medium, but not yet through the phase screen. Fig. 113 shows the phase and amplitude aberrations after the wave has propagated through the phase screen. Some phase and amplitude aberrations are still left. The standard deviation of the phase is 13.9 ns, while the std/mean for the amplitude is 0.18.

The effects of delay and amplitude focusing (DAF) are shown in fig. 112 and fig. 114. The electronic delays of the emitted wave were set as for the delay focusing, while the apodization were set to the amplitude aberrations in the previously received wave (fig. 110). The standard deviation of the phase is reduced to 8.6 ns, while the std/mean for the amplitude is reduced to 0.08. This shows that the DAF method improves the focusing compared with the delay focusing.

The focusing can probably be even better by the TRM method, but do we need better focusing? Note also that the aberrations do not disappear completely with the TRM method. Look for example at the CW-case in the previous section (fig. 106). Some small aberrations (std=5.8 ns) are still left after the TRM-correction. These aberrations are caused by the finite size and the finite number of elements (500) of the transducer. The great improvement by including amplitude focusing, and the small aberrations that are left by neglecting the pulse form shows that amplitude focusing is more important than pulse form focusing.

The main conclusion is that the DAF method is easier and almost as effective as the TRM method.
A plane wave after 50 mm propagation.

Figure 107: The figure shows a 100 mm long plane wave after it has propagated 50 mm in a homogeneous medium.

A plane wave 50 mm after crossing the phase screen.

Figure 108: The figure shows a plane wave that has propagated normally through the 100 mm long phase screen and 50 mm further to the transducer.
Properties of a plane wave after 50 mm propagation.

Figure 109: The figure shows the properties of a 100 mm long plane wave that has propagated 50 mm in a homogeneous medium. PW.

Properties of a plane wave 50 mm after crossing the phase screen.

Figure 110: The figure shows the properties of a plane wave that has propagated through the 100 mm long phase screen and 50 mm further to the transducer. PW.
Figure 111: The figure shows a wave that has propagated from the transducer, through a 50 mm homogeneous medium, but not yet through the phase screen. The electronic delays of the emitted wave were set equal to the time delay distortions in the previously received wave, but with opposite signs. Apodization was not used.

Figure 112: This figure shows the same as the previous figure, but the apodization of the emitted wave was set equal to the amplitude aberrations in the received wave.
Figure 113: The figure shows the properties of a wave that has propagated from the transducer, through a 50 mm homogeneous medium, and finally through the phase screen. The electronic delays of the emitted wave were set equal to the time delay distortions in the previously received wave, but with opposite signs. Apodization was not used. PW.

Figure 114: This figure shows the same as the previous figure, but the apodization of the emitted wave was set equal to the amplitude aberrations in the received wave. PW.
7.5 Focusing of a received wave

So far, only the focusing of the emitted wave is explored. The focusing of the received wave is somewhat different.

The simulations show that the aberrated wave front is not smoothed by propagation. Delay compensation can therefore be used to improve the focusing of the received wave. The time delay focusing adds delays to each element until the phase aberrations in the received wave are compensated for.

The defocusing caused by the amplitude aberrations of the received wave cannot be corrected by delay compensation. The amplitude aberrations can be corrected by back-propagation [51]. The back-propagation is done until a waveform similarity factor is maximized. The back-propagation to the distance of maximum waveform similarity is followed by time-shift compensation. If the signal is a pulse, the waveform similarity is maximized when the wave is back-propagated to the phase screen. The suggested similarity function [51] measures the shape-similarity of the signal and is not influenced by the amplitude. The correction method is shown to improve the focusing of the received wave.

The simulations in the previous section indicate that the pulse form similarity criterion is a weak one, because the pulse form is relatively constant. The simulations show that an equal amplitude criterion is probably a better optimization criterion.

The simulation presented in fig. 106 can be interpreted as a back-propagation from the transducer to the phase screen. The amplitude aberrations are minimized after 50 mm propagation, i.e. when the wave is back-propagated to the phase screen. Further propagation in a homogeneous medium will increase the amplitude aberrations. This simulation shows thus that uniform amplitude is a good criterion for deciding where to stop the back-propagation.

7.6 Diffuse scatterers

The presented simulations show that the TRM-, the DAF-, and the back-propagation methods focus well on a single point scatterer. Is it possible to use these methods to focus on an area with diffuse scatterers too?

If the diffuse scatterers are located far from the phase screen, the reflected wave will be long and irregular, but the phase and the amplitude of the wave front will still be uniform at the phase screen. The phase and the amplitude aberrations to the wave received at the transducer will therefore be caused by the phase aberrations from the phase screen only. The DAF-focusing, based on the phase and the amplitude aberrations observed at the transducer, will therefore work well on diffuse scatterers located far from the phase screen.

The TRM method gets into trouble because it retransmits the whole of the reflected signal. The signal reflected from diffuse scatterers is very long, and the radial resolution by using the TRM method will therefore be poor.

The DAF method gets into some problems when the diffuse scatterers are close to the phase screen. The signals reflected from a single point scatterer and observed
at the different transducer positions will get identical shapes. Phase and amplitude fluctuations to the signals received over the transducer are then easy to estimate. Diffuse scatterers will reflect signals that looks different at different transducer positions. The van Cittert Zernike theorem [52] shows that the waves that hit the transducer far from each other not are correlated at all. How can we find the phase aberrations between two signals that not are correlated at all?

Let us look at two neighbor rays from an area with diffuse scatterers. If the rays are close to each other, their signals will be highly correlated. It will thus be possible to estimate amplitude and phase aberrations between signals in neighboring elements. To get the the phase and amplitude aberrations over the whole transducer the aberrations must be estimated from element to element, and the final estimation error will accumulate up. This effect can be reduced by using a 2D array with many small elements. There will then be high correlation between several received signals, and the phase and amplitude aberrations can be estimated by an average of several phase and amplitude aberration estimates. The error in each estimation step will decrease and the estimation of the total phase and amplitude aberrations will be improved. A simulation example with a 5 × 80 array transducer showed that this estimation technique estimated the phase aberrations over the whole transducer well, even through the signals received far from each other were not correlated at all [58]. The estimation of the amplitude aberrations are similar to the estimation of the phase aberrations, and I will therefore assume that the estimation technique can be used to estimate the amplitude aberrations also.

Note that the correlation between signals received at two neighboring elements increases if the focusing of the emitted wave is improved [54]. The estimation error from element to element will thus decrease and the total estimation error will decrease. The DAF method combined with this estimation technique will therefore be well suited for iteration. Note also that it is best to use signals reflected from the focal point of the transducer to estimate the aberrations, because they have the highest correlation.

An important question is to decide whether the estimated phase and amplitude aberrations are caused by the phase screen only. The wave reflected from diffuse scatterers may have phase and /or amplitude aberrations before it crosses through the phase screen. These amplitude and phase aberrations will be included in the phase and amplitude aberrations estimated on the transducer, and the effect of the DAF method will be reduced. This happens if we use a short time window to estimate the aberrations. The effect can be reduced by using a longer time window.

Let us look at two neighbor signals at the phase screen. The phase of the first part of signal 1 is maybe behind the first part of signal 2, while the last part of signal 1 is maybe before the last part of signal 2. The cross correlation algorithm will not find any phase aberrations between the two signals. The small amplitude variations of the two signals will also average out if we use a time window with some length. On the other hand, the effect of the phase screen will be the same over the whole time window, such that the cross correlation between the two signals at the
transducer will be given by the phase aberrations from the phase screen only. The difference between the average amplitude of the two signals will give the effect of the local focusing/defocusing from the phase screen.

Since we know that the amplitude aberrations are caused indirectly by the local focusing and defocusing of the phase screen, the phase and amplitude aberrations can be compared to evaluate the estimates. At a position where the received wave is locally focused, we will for example expect that the amplitude is high.

The estimated phase and amplitude aberrations may also be used in a back-propagation algorithm to improve the focusing of the received wave. So far, back-propagation has been used with a point reflector [51], because it has not been possible to stop the back-propagation of diffuse signals. The original data must be back propagated, but the estimated phase and amplitude aberrations may be "back-propagated" first to decide how far we should back-propagate the wave-field.

Absorption may be a problem, but since signals of neighbor rays have approximately the same absorption, the problem is expected to be small. Eventually consequences of absorption will be low frequent as for instance those caused by the dome to an annular transducer (fig. 10). In such cases, the absorption effect can be removed by filtering the estimated amplitude aberrations with a high pass filter.

To use the DAF method, you will need a 2D array and access to the phase and the amplitude of the signal received at each element, or time to shoot enough beams to estimate the phase and amplitude with an optimization criterion. You must also have the possibility to set the delay and the apodization of each transducer element independently under the excitation of the transducer.

The conclusion of this section is that the DAF method can be used to focus on an area with diffuse scatterers, and will therefore be an improvement of the TRM method. The DAF method is also an improvement to the delay focusing, because it focuses better through a phase screen some centimeters from the transducer.
8 Discussion & Conclusion

The thesis starts with observing the fact that the quality of ultrasound images depends on the patient. Two main questions arise: How can we explain patient-dependent image quality, and how can we improve images of poor quality? Possible answers to these two questions are explored through experiments and simulations.

Human tissue has a layered structure, and MR-images of different patients show that the layered structure changes from patient to patient (fig: 21). To explore the focusing through different layered media, a simulation program is developed. The theory behind the simulation program is deduced in sec. 3, the simulation possibilities are described in sec. 4 and the program is verified by experiments in sec. 5.3. The basic modeling of the wave propagation in human tissue, the improved ray-tracing method developed to solve the differential equations, and the algorithms used in the implementation are discussed in sec. 8.1.1- 8.1.3.

Possible explanations for the patient depended image quality are discussed in sec. 8.2. Experiments and the simulation program are used to evaluate the different hypotheses. The conclusion is that phase aberrations caused by propagation through the outer layers of human tissue reduce the quality of medical ultrasound images. One has found that the phase aberrations are due to an outer layer of fat which is so soft that the outer boundary of the fat is formed by the probe, while the shape of the inner boundary is not affected by the probe. Even larger phase aberrations are caused by propagation through a layer of fat-containing muscle. Both tissue types are typical for an obese patient, but not so for a thin patient.

A review of articles on corrections of phase aberrations, published before and during the work on the thesis, is presented in sec. 2. The work done by the writer and the works done by other scientists are discussed in sec. 8.4. The author's contribution is a method to compensate for phase aberrations caused by a spherical dome and a thick outer fat-layer. The author has also combined the best of published correction methods and suggested a way to correct phase aberrations generated at a distance from the transducer using waves back-scattered by diffuse scatterers. This is an important improvement compared to other phase aberration correction techniques.

8.1 Theory

The discussion of the theory deduced in the thesis, is divided into four sections. First, the model used for describing the wave-propagation in human tissue is discussed. Then the method used for solving the wave equation is discussed. Next, the algorithms used in the simulation program are discussed and then, finally, simulation results are compared with measurements.
8.1.1 The model

The wave equation used in the thesis is based on three equations: the law of conservation of mass (eq. 1), Newton's second law (eq. 2) and an equation for the relationship between mass density and pressure (eq. 3). An absorption term is included in the last equation.

The law of conservation of mass is an exact equation, while the equation for Newton's second law neglects viscosity. These two equations are well known in acoustical text books and do not need any further discussion.

The third equation is an equation by Stokes which is known to work well for fluids. The sound absorption is modeled to be proportional to the time derivative of the mass density. The proportional factor, $\mu_0$, is constant for water. For a constant $\mu_0$, the absorption of the wave will be proportional to the square of the frequency. This frequency dependence fits with measurements in water.

The oscillation of the molecules in the medium is large for long wave-lengths and small for short wave-lengths. There is little interaction between water molecules. The proportionality factor in water is therefore modeled to be independent of the wave-length. In human tissue there is more interaction between molecules. The resistance in tissue is therefore assumed to increase with the relative displacement of the cells. The larger the compression or decompression is, the larger the resistance to the movements is. At increased resistance the wave will give off more heat, i.e. the absorption will be larger. For this reason it is assumed that in tissue $\mu_0 = \text{const}/\omega$.

This assumption may not be exact, but it describes the absorption in a way that makes it possible to solve the wave equation. The absorption of the wave, found by solving the wave equation, has the same frequency dependency as in human tissue.

The wave equation used in the thesis is deduced from these three equations. Three approximations are done. The two first are common. They neglect the gravity and assume that the particle velocity $\ll$ the speed of sound. As a third approximation, the author assumes that $\rho(r, t) \frac{\partial}{\partial t} u(r, t) \approx \frac{\partial}{\partial t} \rho(r, t) u(r, t)$. This approximation is valid (plane wave analysis) if $\frac{u}{c} \sqrt{1 + \left(\frac{u}{c} \lambda \right)^2} \ll 1$, i.e. if $u \ll c$ and the mass density $\rho$ do not vary too rapidly compared to a wave length $\lambda$. For solids and liquids $u \ll c$ [17]. The goal of the third approximation is a simple wave equation (eq. 12) that allows relatively rapid spatial fluctuations in the mass density.

8.1.2 The methods

The deduced wave-equation is solved for wave-propagation through a layered medium. The solution is an integral equation. It is deduced by combining the ray-tracing approximation with the Kirchhoff-Helmholtz’s integral theorem. No other researcher has been found who has deduced or used the presented integral formula. If the medium is homogeneous, the integral equation simplifies to the Rayleigh integral.

An interpretation of the integral formula is that each point on the transducer emits a spherical wave. It is only a conical part of the wave that hits the observation point. The conical part is denoted as a ray-tube. The ray-tube is refracted due to
Snell’s law at the boundaries. The distance along the ray-path, and the speed of sound in each layer are used to calculate the propagation time. If the wave inside the ray-tube is not scattered by discontinuities, no energy will disappear out of the ray-tube. The amplitude of the emitted signal will decrease with the broadening of the ray-tube. The wave is also attenuated due to absorption along the ray-path and due to reflection at the boundaries.

The main approximation used to deduce the suggested integral solution is

$$|\nabla A/A| << \frac{\omega}{c}$$  \hspace{1cm} (71)

A is the amplitude of a wave emitted from a point source at the observation point, ω is the angular frequency and c is the speed of sound in the medium. A is calculated along the ray-path. For a point source in a homogeneous and absorbing medium the model gives $A = -\frac{A_0}{4\pi r}e^{-\mu r}$, where r is the distance from the point source and μ is the absorption factor (eq. 32). The approximation is valid if $\left(\frac{1}{r} + \mu\right) << \frac{\omega}{c}$, i.e. if $\frac{\lambda}{2\pi r} << 1$ and $\frac{\mu c}{2\pi f} << 1$. The last condition can also be written as $\mu_0 k_0 \omega << 1$.

A realistic example is studied: The point source emits a pulse with a frequency $f = 3MHz$ in a medium with a speed of sound $c = 1540m/s$. This gives a wave length of 0.5 mm. The wave will be absorbed by propagation. In human soft tissue, the absorption is approximately 0.75dBcm$^{-1}MHz^{-1}$ ($\Rightarrow \mu = 26m^{-1}$). The values in this example give $\frac{\lambda}{2\pi r} = 0.008$ (r=1 cm) and $\frac{\mu c}{2\pi f} = 0.002$. Both of the values are considerably lower than 1, and the approximation is therefore good for distances between the observation point and the transducer of more than 1 cm. Since we are mostly interested in the acoustic field 1-16 cm from the transducer, the main approximation can be concluded to be valid for a homogeneous medium.

The ray approximation is traditionally used for plane waves that propagate through plane boundaries. It is also used in optics where high frequencies are used. It is not common to use ray approximation in medical ultrasound since diffraction is a dominant effect. If the wave propagates through a rough boundary, it will introduce phase aberrations. The Huygen’s principle is used to simulate how a rough wave front is being changed by propagation (sec. 7). Phase aberrations with properties as caused by propagation through the human abdominal wall and waves with frequencies from 1-7 MHz were used. Propagation changed the wave front, but when the wave propagated less than 1 cm the changes were small. Phase aberration in itself does not affect on the validity of the approximation in eq. 71, but the phase aberrations cause amplitude aberrations by propagation. For small propagation distances, the generated amplitude aberrations were minor. The ray approximation is thus valid. For larger propagation distances, the amplitude aberrations increase and the approximation in eq. 71 will not be valid. The conclusion is that the ray-approximation can be used right after the wave has crossed the boundary, but not when the wave has propagated a longer distance from the rough boundary. The limit depends on the size and the correlation length of the phase aberrations. Approximation is valid for a large area if the medium has smooth boundaries, but it can only be used when the wave is close to the boundary if the boundary is rough.
The simulation examples using Huygen's principle are presented in section 7 and can be used to explore the validity of the ray approximation. The propagation of a rough wave front is simulated. The introduced phase aberrations had similar properties to aberrations caused by waves propagating through the human abdominal wall. After 5 cm of propagation, large amplitude aberrations were generated. The amplitude aberrations fluctuated so rapidly that \( \frac{\Delta A}{A} \) is up to 700m\(^{-1}\). The frequency of the wave was 3.5 MHz. This gives \( \frac{\Delta v}{v} = 14280^{-1} \). The ray-approximation in eq. 71 will thus still be valid. This indicates that the ray-approximation is more applicable than expected.

The theory developed by the author also shows that an inhomogeneous layer of continuous inhomogeneities causes phase aberrations and refraction, but not scattering. The refraction of rays caused by soft tissues is small. The effect of a layer with continuous inhomogeneities can therefore be modeled as a phase screen that causes phase aberrations. This model is valid as long as the phase aberrations do not cause significant amplitude aberrations.

Normally, the finite element method (FEM) must be used if the medium consists of arbitrarily discontinuous inhomogeneities. Note that the FEM is not effective for simpler media. It needs a large computer memory, especially if the simulations are done in three dimensions.

### 8.1.3 The algorithms

It would be desirable that the input to the simulation program is the electrical input, as done in [80], where the Mason model for the transducer is used in combination with a field simulator. The transfer function from voltage and current excitation to pressure and velocity is given by the model, and the output is used as an input to the field simulator.

The simulation program does not use the electrical pulse as input. Instead, the pressure pulse measured at the focal point is used. At this point the summation of the integral equation is coherent. The pulse measured at the focal point is therefore, a measured approximation of the time derivative of the pressure at the transducer's surface. This means that the transducer in the simulation program emits a pulse that is a measured approximation of the time derivative of the pressure.

The simulation program uses a discrete version of the integral equation. The transducer surface is represented by a lot of points. Each point represents an area that emits a broad beam. The area is assumed to be so small that it emits a spherical wave. The wave does not necessarily have to be spherical, but the amplitudes of the waves in the ray direction and the direction normal to the transducer should be almost identical. At the far-field of the transducer, the angles between the ray-paths and the normal of the transducer are small. The approximation is therefore good in the far-field. The transducer must be divided into more points to simulate the near-field than to simulate the far-field.

An iterative algorithm is used to find the rays between the observation point and
all of the points on the transducer. The rays are drawn from the observation point and to the transducer. The starting direction of the first ray is a vector that goes from the observation point to the desired transducer point. The cross-point between the ray and the first boundary is found analytically or numerically by Newton’s method. The direction of the transmitted ray is given by Snell’s law. The ray will miss the desired transducer point if the ray is refracted. The next iteration step is to change the starting direction of the ray. The vector from the desired transducer point and normal to the first ray are used in the correction. The starting direction of the ray is turned in the opposite direction of this vector. This is done iteratively until the ray-path between the observation point and the wanted transducer point is found. A control routine that ensures that the algorithm converges is concluded.

The attenuation factor due to the broadening of a ray, $dB_i/dB_j$, is found by calculating two neighboring rays. The three rays have the same starting point, but different directions. The two neighbouring rays span out a ray tube. The area of the cross section of the ray tube, $dBi$, can then be calculated. $dB_i$ is the area of the cross section close to the start of the ray and $dB_j$ is the area of the cross section at the end of the ray. In a homogeneous medium, this will give the well-known spherical attenuation; $1/r^2$. If the ray is refracted, $dB_i/dB_j$, will give a more correct calculation of the "spherical attenuation".

The absorption factor depends on the frequency. So far only the center frequency is used to calculate absorption along a ray path. It is found that a typical pulse used in medical ultrasound imaging does not change dramatically by propagating in human tissue. The approximation will thus be valid. For more broad-banded pulses frequency depended absorption has to be included.

8.1.4 Accuracy of the simulation program

The simulations are compared with measurements in section 5.3. Wave fields in water are measured and simulated. A 3 MHz annular array transducer with two rings and no dome was used. For this homogeneous medium, simulations fit well with measurements. An example with a transducer with dome in water is also studied. The simulations and the measurements fit well for this layered case to.

8.2 Explanations of patient - depended image quality

Ultrasound images of a thin and an obese patient are shown in fig. 16. The image of the thin patient is of high quality, while the image of the obese patient is of low quality. To improve ultrasound images, it is important to understand why the obese patient gives ultrasound images of lower quality than the thin one. Several hypotheses are suggested and explored in the thesis.
8.2.1 Hypothesis 1; Thick outer fat-layer

The first hypothesis is that a thick outer layer of fat causes distortions. An ultrasound image of the heart of a pig in water is observed through a layer of fat with uniform thickness to explore the hypothesis. Fig. 17 shows the heart imaged through water and a layer of fat. The beam was highly attenuated when propagated through the fat-layer. It was thus necessary to increase the gain until the electronic noise became visible in the image. Significant acoustical noise caused by the layer of fat was not observed. In the image of the obese patient (fig. 16), the gain is high, but not as high as the electronic noise visible in the image. The distortions seen in the image of the obese patient, are therefore acoustical noise and not electronic noise. The conclusion is that a layer of fat with uniform thickness does not explain the low image quality of an obese patient.

The speed of sound in human fat is 1440 m/s, while the speed of sound in muscles is 1580 m/s at 37°C [8]. The simulation program shows that a plane boundary between human fat and muscles at 37°C causes phase aberrations with minor consequences (fig. 11). The simulations also show that the phase aberrations do not increase significantly when the thickness of the fat-layer is increased, as long as it has uniform thickness.

Although the fat-layer does not explain the low image quality of an obese patient, it causes some distortions. These distortions are explored in section 5.2.3, where it is measured how much a beam-profile is disturbed by propagation through different specimens of tissue. The main distortion is due to frequency - depended attenuation. In human tissue, waves with high frequencies are more attenuated than waves with low frequencies. The low frequency components of the wave will therefore be more dominant. The result is a broader beam, which reduces the resolution. The distortions caused by frequency - depended attenuation may be avoided by using a narrow banded pulse or by using a bandpass filter on the received signal.

8.2.2 Hypothesis 2; The ribs

A layer of fat with uniform thickness does not explain the poor image quality of an obese patient. Another hypothesis is that a thick layer of fat increases the distance between the transducer and the ribs. This can be a problem since we have to observe between the highly attenuating ribs. To explore this hypothesis, a thin person was imaged with and without an extra layer of fat. The two images are shown in fig. 18. Some acoustical and electronic noise is observed in the disturbed image, but it is still not as bad as for the obese person in fig. 16.

8.2.3 Hypothesis 3; Bent outer fat-layer

A thorough study of the patient - depended image quality is conducted by Magnetic Resonance (MR) images. Two persons are selected, one thin who gives ultrasound images of high quality and one obese who gives ultrasound images of low quality.
MR-images of these patients are used to make models of the outer tissue layers of the two patients. The ultrasound and MR-images of the two patients are shown in fig. 21 and 22.

It is possible to see the ultrasound probe to the right in the MR-images. We observe that the tissue is formed by the spherical dome. The thick outer fat-layer of the obese patient is so soft that the outer boundary is formed by the dome, while the inner boundary remains plane. The fat-layer of the thin patient is formed by the dome at both boundaries. Models of the two different outer fat-layers are shown in section 6, and the simulation program is used to explore the transducers' ability to focus through the different models. The outer fat-layer of the obese patient works as a lens and moves focus deeper into the body. There is only a minor focus displacement in the thin patient.

Calculations of the consequences of the focus displacement predicted for the obese patient are shown in sec. 6.3. At transmission, the focus displacement reduces the focus properties in the near-field, while it is improved in the far-field. At reception, where dynamic focusing is used, the quality of the focusing is reduced at all depths. The total effect on the image can be studied in fig. 23, where images of balloons in water are shown. The defocusing of the image to the right is as large as by the bent fat-layer observed in the obese patient.

The conclusion is that a layer of fat that is being bent by a spherical dome at the outer boundary but not at the inner boundary, generates phase aberrations that disturb the focusing significantly. The effect will appear in obese patients, but not in thin patients. The distortions are caused by the spherical dome of the annular transducer. A plane array transducer will not cause similar phase aberrations. Over the past few years, plane array transducers have become a lot more common than annular transducers, but annular transducers are still in use. A method to improve images taken with annular array transducers is therefore important. Note that the plane array transducers used today are one-dimensional, and can therefore only perform electronic focusing in one direction. In the other direction it is common to use a physical lens to do the focusing. A lens like this can give the same distortions as a spherical dome. A method for making corrections with an annular array transducer is presented in sec. 6, and discussed in sec. 8.4.1.

8.2.4 Hypothesis 4; Thick fat-layer around the heart

The MR-images of the thin and the obese patient shown in fig. 21 and 22, show a difference not only in the outer fat-layer, but also in the fat-layer around the heart. The obese patient has a thick layer of fat around the heart, while the thin patient has only a thin layer. Both simulations and experiments show that a plane layer of fat with uniform thickness causes only small distortions. The fat-layer around the heart is bent. This makes half the beam propagate in fat while the other half propagates through other types of tissue. The difference in speed of sound will cause large phase aberrations. An example of this effect is shown in fig. 24. The effect
causes large distortions. The effect is especially large in this example, since the vertical boundary between fat and meat are close to the transducer. Beams emitted in almost all directions will be disturbed. When the vertical boundary is as deep inside the body as shown in the MR-images, only a few beams are disturbed.

8.2.5 Hypothesis 5; Rough boundary/ high fat contents in the muscles

The last hypothesis is that the content of fat in the layers of muscle changes from patient to patient. It is reasonable to assume that an obese patient has a higher fat-content in the muscles than a thin patient. It is not easy to verify this hypothesis by MR-images. The resolution is too weak. An optical image of an obese person is shown in fig. 31 A lot of fat can be seen inside the muscle tissue.

To explore this hypothesis, focusing through two different specimens of fat and meat were measured. The difference between the specimens was the structure of the layer of meat. The first specimen was relatively homogeneous, while the second specimen contained several islands of fat (fig. 26). The islands had an irregular form. The size was approximately 0.1-2 mm. The speed of sound in the fat was measured to 1437 m/s and 1591 m/s in the meat. The - fat containing meat looked similar to the muscle layer of the obese person imaged in fig. 31. The experiments were made on bacon in water at 37°C. The speed of sound and the absorption in the fat and the meat fit well with similar values for human tissue.

The beam-profiles measured after the wave had propagated through the two specimens are shown in fig. 67 and 68. Distortions caused by the homogeneous meat were minor, while distortions caused by the fat - containing meat were large. The propagation through the latter caused a beam with decreased main-lobe and increased side-lobes. The two specimens were of equal thickness and were therefore assumed to cause the same attenuation. Distortions caused by frequency- depended attenuation are therefore assumed to be equal in the two experiments. The decreased main-lobe and the increased side-lobes are concluded to be caused by phase aberrations. The distortions in the ultrasound image caused by the two specimens are shown in fig. 29 and 30. The lateral resolution of the image taken through the fat - containing meat is low compared to the image taken through the homogeneous meat. The lateral resolution was decreased to a level where it was difficult to see the vertical boundary between the object and the water. The image taken through the fat - containing meat also shows a sky of reverberations below the specimen placed close to the transducer. This sky looks similar to the sky that can be seen in the ultrasound image of the obese person shown in fig. 16. The reverberations and the phase aberrations caused by fatty meat can explain the low quality of images of an obese person. It is necessary to do phase aberration correction to make good images of patients with high fat content in the muscles.
8.3 The phase screen model

The model developed in the thesis shows that a thin layer of spatially continuous inhomogeneities does not scatter the wave, but it introduces phase aberrations to the wavefront. As long as the phase aberrated wave causes small amplitude aberrations by propagating through the thin layer of inhomogeneous tissue, the effect of the layer may be modeled as a phase screen. To simulate the effect of the inhomogeneous layer, the delays in the phase screen will be added to the wavefronts that propagate through the phase screen.

The phase screen model is verified by simulations. The simulations presented in sec. 7 show how a rough wave-front is changed by propagation. The simulations are done on a homogeneous medium and is therefore not done by ray-tracing, but by Huygen’s principle. The roughness of the phase aberrations of the wave front is set to have similar properties as if it were caused by propagation through the human abdominal wall. The simulations show that the wave front changes through propagation, but the change is small if the waves propagate less than 5 mm. It is therefore possible to model the effect of a 5 mm thick layer of fatty meat as a phase screen. If the wave-front propagates further, the shape will change. The author's method can handle a thicker layer with inhomogeneities by modeling it as several phase screens.

8.4 Phase aberration correction

8.4.1 Parameter tuning

To focus on a specified point inside the human body, electronic delays are added to the signals at the different elements on the transducer. If these delays are set correctly, the signals emitted from the transducer will arrive at the specified point at the same time. This results in good focusing.

To calculate the electronic delays, it is assumed that the speed of sound is constant inside the human body. It is common to use a speed of sound equal 1540 m/s. The algorithm that calculates electronic delays for the transducer is based on the size and the shape of the transducer. The algorithm will work well if these values fit with reality.

If too low or too high a speed of sound is used in the algorithm for electronic focusing, the focusing properties of the transducer will decrease. The distortions depend on the size of the electronic delays. If there are large differences between the electronic delays specified for the different elements, the algorithm will be sensitive to errors in the speed of sound. Plane phased array transducers use large delays to do the focusing and are therefore sensitive to errors in the speed of sound. Spherical phased annular array transducers need less delays and are therefore less sensitive to such errors. However, all phased arrays get reduced focusing if the focusing algorithm uses the wrong speed of sound. It is therefore important that the speed of sound used in the focusing algorithm is tuned correctly.
Annular array transducers are tilted to emit beams in different directions. To do the tilting and still have good acoustical contact with the human tissue, the transducer is encapsulated in a probe, i.e. a spherical dome filled up with fluid. The acoustical properties of the fluid and the dome are chosen so that they equal the acoustical properties of human tissue, but the probe still causes some distortions. The distortions are explored in section 6.

The speed of sound in the dome is higher than the speed of sound in the dome-fluid and in human tissue. The propagation path between a point on the transducer and an observation point will therefore not be straight-lined. These ray-paths will be refracted due to Snell's law. The refraction causes a reduction of the effective size of the transducer. The electronic delays are not set optimally if the physical transducer diameter in the algorithm for electronic focusing is used. This causes large phase aberrations close to the transducer, and small distortion in the far-field of the transducer. The phase aberrations can be reduced by using the effective transducer diameter in the algorithm for electronic focusing. An example of the improvements are shown in section 6. The optimal transducer diameter used in the algorithm does not significantly depend on the patient. The main effect of such distortions can therefore be removed by tuning the transducer diameter until the focusing in a water medium is optimal. The tuning can be done through experiments or with the help of the simulation program developed by the author.

The radius of curvature (ROC) of a spherical transducer is also used as a parameter in the algorithm for electronic focusing. If the ROC of the transducer equals the depth where the signals emitted from the transducer arrive at the same time, the delays specified by the focusing algorithm will be optimal. This is the case for the thin patient, but not for the obese one (see sec. 8.2.3). The effective focal depth in the obese person is larger than the ROC of the transducer. The algorithm for electronic focusing will therefore specify delays that are not optimal for focusing in the obese person. If the ROC parameter in the algorithm equals the effective focal depth, the algorithm will specify delays that give improved focusing in the obese person.

The improvements done by tuning the ROC parameter are verified by the simulations shown in section 6. Improvements made possible by the method are shown experimentally in fig. 23. The illustration shows ultrasound images of balloons in water. The image to the left shows the balloons in water taken with default focusing. In the second image, the parameters are set to values that the simulation program found to be optimal for water. It is possible to see a significant improvement. In the third image, the parameters are set so that phase aberrations are generated as caused by a fat-lens as observed in the MR-image of the obese patient. The quality of the ultrasound image of the balloons are now significantly reduced. The improvements made possible by the parameter tuning are given by the different quality of the balloon-images in fig. 23.
8.4.2 Correction of rough phase aberrations

The tuning of the diameter and the radius of curvature used in the algorithm for electronic focusing can only compensate for some of the distortions in the image of the obese person.

The obese person has a muscle layer between the outer layer of fat and the hearth with high fat content. (fig. 31). This layer will cause phase aberrations because of the difference in speed of sound in fat and muscle. The experiments done by the author to explore the size of the distortions are discussed in section 8.2.5. The distortions caused by a 9 mm thick fat - containing muscle layer were so large that they can explain the low image quality of the obese patient.

The properties of the phase aberrations generated by propagating through the human abdominal wall and through the female breast are presented in the literature. The time delay aberrations caused by propagation through the human abdominal wall were measured to typically std. = 43 ns and has a correlation length = 7.9 mm [39]. The time delay aberrations caused by propagation through the female breast were measured to typically std. = 36 ns and has a correlation length = 2.1 mm [85]. The measured delay aberrations were rough compared with typical transducer sizes, and they were different from specimen to specimen. These delay aberrations reduce the focusing for frequencies around 3 MHz. These frequencies are common in medical ultrasound imaging. The distortions caused by delay aberrations will increase with increasing frequency of the emitted wave.

About 75 articles on phase aberration correction and related subjects have been published over the last 10 years. Articles published before 1995 are reviewed. The works done to show that time delay aberrations are introduced to a wave-front which propagate through human tissue are included in the review. The correction methods can be classified according to the tools that are used when doing corrections:

1. Both the correlation methods and the quality factor methods do corrections by regulating the delays on the elements. Equal delays on transmission and reception are used. The main idea is to manipulate the delays until the focusing is optimal, i.e. until it has compensated for the phase aberrations caused by propagation through human tissue. The methods will be denoted as time delay focusing. ([30], [44], [45], [54], [60]).

2. The time reversal mirror (TRM) [29] method regulate the delays, the apodization and the pulse form. The wave received from a point reflector is retransmitted backward to emit an optimal focused beam. Back-propagation [51] is a way to use the TRM-principle at reception.

Information in the previous back - scattered field is used to estimate the corrections. Experiments presented in literature show that the TRM-method works better than the time - delay focusing methods if the phase aberrations are generated at a distance to the transducer. The TRM-method requires a point reflector to focus on. This is a problem since the human body only consists of diffuse scatterers. The van Cittert
Zernike theorem shows that the waves back-scattered from diffuse scatterers and received by adjacent elements are correlated. The correlation is found to be high enough to estimate time delays necessary to do time delay focusing.

There has still not been presented a method that uses waves back-scattered by diffuse inhomogeneities to correct phase aberrations generated at a distance to the transducer. To make good ultrasound images of an obese patient it is necessary to develop such a method. The phase aberrations due to the fat containing muscles of an obese person are generated a few centimeters away from the transducer. There is no strong point scatterer in the human body. In spite of that, the delay and amplitude focusing (DAF) suggested by the author is expected to perform the desired corrections. This is discussed further in section 8.4.4.

The phase aberrations caused by propagation through human tissue are two-dimensional. It is therefore necessary to build two-dimensional phased array transducers to perform optimal phase aberration correction. The elements must be smaller than the correlation length of the phase aberrations. This requires some rather large transducer elements. Smaller elements are necessary to electronically focus a beam in an arbitrary direction. The distance between the center of neighboring elements must be less than half a wave length to avoid grating lobes. A dimensional array that satisfies this demand usually has 64-128 elements. One dimensional array can scan two-dimensional ultrasound images, but not three-dimensional images electronically. To do a 3-dimensional scan electronically and to do phase aberration correction, it is necessary to build two-dimensional arrays. A 64 x 64 or 128 x 128 array will not be available in the foreseeable future. To do two dimensional scans and phase aberration corrections it is necessary to build 1.5D arrays. These arrays have about 64-128 elements in the azimuth direction and about 5-10 elements in the elevation direction. A 1.5D array is plane in the azimuth direction while it is spherically focused in the elevation direction. The spherical focusing can be done by a lens. Prototypes on 1.5D arrays exist today. It is possible to do phase aberration correction with even fewer elements. This can for instance be done by dividing the rings of an annular transducer in pieces that are shorter than the correlation length of the phase aberrations. It will then be necessary to do mechanical scanning.

8.4.3 Focusing through a phase screen close to the transducer

Previous publications show that time delay focusing can correct well for phase aberrations generated close to the transducer. The time delay focusing methods reported in literature are discussed in this section. The main difference between these methods is that one has used different ways to find the optimal focusing delays.

The estimation of the delays are suggested found by two different principles: 1) The delays between signals received on neighboring elements are estimated directly by correlation methods. 2) The optimal focusing delays are found indirectly by changing the delays until a quality factor is maximized. One suggested quality factor is the intensity in a region of interest in the image [60]. It is also suggested to
normalize the intensity [54]. A normalized quality factor can be used to evaluate the focusing. Direct estimation of the phase aberrations can be done by cross correlation between signals received on adjacent elements [30]. An alternative method is to do time delay estimation via minimization of the sum of the absolute differences (SAD) between waves received on adjacent array elements [45]. It is also possible to use 'averaged' phase information. That is made available by the quadrature detection of the signals received on each element [44]. All the methods are reported to estimate the phase aberrations well.

The estimation of the focusing delays can alternatively be divided into one-way and two-way estimation methods:

1. The one-way methods compare the signals received on adjacent elements to estimate the phase aberrations of the received wave. The estimation of the phase aberrations can be done by one excitation of the transducer. The correction delays are set equal to the estimated phase aberrations, but with opposite signs. The correction delays are used both when the wave is emitted and when it is received. All of the suggested time - delay focusing methods can be based on one-way estimation.

2. The two-way method works adaptively. The electronic delays for the emitted- and for the received wave are modified simultaneously. A new wave is emitted for each modification of the delays. The delays are modified until the quality factor is maximized. The maximum speckle brightness (max intensity) method is deduced for two-way estimation. It is shown theoretically and experimentally to work on waves back scattered from diffuse scatterer (speckle).

Even when arrays that can measure two - dimensional phase aberrations are built, it will not be trivial to do corrections in real time. The one-way estimation methods need access to the signals received on each element to estimate the time delays between them. A large memory is needed to save the signals, and it is required time to estimate the delays. It may be difficult to do the estimation in real time. The main problem is the necessary complexity to do the corrections.

The two-way method (max. speckle brightness) does not need access to the signals received on each element and is therefore easier to implement. The maximum speckle brightness method corrects the elements one by one. The delay on one element is modified until the intensity of the total received signal is maximized. Then a new element is corrected. The corrections are done until the delays on all elements are corrected. The drawback with this method is the necessity to emit several waves to correct one beam. Since each emitted pulse needs time to propagate through human tissue, the algorithm will not work in real time unless the number of excitations can be reduced dramatically.

It may be possible to make the two-way estimation more effective than correcting the elements one by one. Such an algorithm is not yet presented. One idea may be to estimate the phase aberrations as a n-the order polynomial. The constant
term and the 1 order term must be set to zero. There is thus n-2 parameters left to estimate, which is considerably fewer than the number of elements on the transducer. A further minimizing of number of excitations can be found by using statistical methods. An advantage with this method is that the corrections can be done little by little. The doctor will therefore see the image be improved during the examination.

The one-way estimation methods need for the signals received in adjacent elements to be correlated. There is high correlation between signals back-scattered by a point reflector, but what about signals backscattered from diffuse scatterers? Distinct point scatterers are not available in human tissue, while diffuse scatterers are common. Inside an apparently homogeneous layer of fat or in the liver there are inhomogeneities much smaller than the wave length. They work as diffuse scatterers and cause speckles in the ultrasound image. The van Cittert Zernike theorem shows that the transducers can be built in such a way that there is high enough correlation between signals received in neighboring elements to do phase aberration correction even when the wave is back-scattered by diffuse scatterers. The correlation decreases with the distance between the elements. The consequence is that the phase aberrations must be estimated from element to element. The estimation error will thus accumulate. Simulations show that in spite of the accumulated estimation error, the final error is considerably smaller than the size of typical phase aberrations [58]. It is therefore possible to estimate the phase aberrations which are necessary to do time delay focusing even when the wave is back-scattered by diffuse scatterer.

8.4.4 Focusing through a phase screen a few centimeters away from the transducer

If the phase aberrations are generated close to the transducer, the phase aberrations will be equal for most beam directions. The phase aberrations estimated for one beam direction can thus be used to correct several beams. If the phase aberrations are generated a few centimeters away from the transducer, the phase aberration correction has to be individual for each beam direction. The time it takes to do corrections increases with the number of beams that has to be corrected individually. It will therefore take longer time to correct for phase aberrations generated several centimeters away from the transducer than to correct for phase aberrations generated closer to the transducer.

In this section, human tissue is modeled as a phase screen in a homogeneous medium. The question is where to place it. Large phase aberrations are introduced to waves propagating through the fat content muscle layer of an obese person (sec. 8.2.5). There is a thick layer of fat between the transducer and the fat-containing muscle layer of an obese person. To model the tissue of an obese person it will therefore be natural to place the phase-screen at a distance from the transducer.

Simulations done to explore the distortion caused by this model are presented in section 7. The phase screen is placed 5 cm away from the transducer. A point
reflector is placed so far away from the transducer that the back-scattered wave will be plane when it propagates through the phase-screen. It is not necessary to assume a plane back-scattered wave, but it makes the presentation easier to read. The phase screen used in the simulations has statistical properties typical for a plane wave that has propagated through the human abdominal wall.

Let us first look at the back-scattered wave. Directly after the plane wave has propagated through the phase screen, phase aberrations are introduced to the wavefront. Amplitude aberrations are not introduced. Simulations show how the rough wavefront changes by further propagation (sec. 7). It is observed that the phase front is not smoothened by propagating 5 cm, but the shape is changed. It is also demonstrated that phase aberrations generate amplitude aberrations that increase by propagation and when the frequency is increased.

The simulation examples show that a plane wave that propagates through a phase screen and 5 cm further before it reaches the transducer has phase and amplitude aberrations large enough to disturb the focusing significantly. The largest defocusing was caused by phase aberrations, but the amplitude aberrations also increased the side-lobe level. The phase-screen caused phase aberrations typical for the human abdominal wall and waves with a frequency of 3.5 MHz were used, which is common in medical ultrasound imaging.

The main question is how to do optimal focusing through the phase-screen. The tools used for manipulating the wave field emitted from the transducer are the form of the pulse, the electronic delays and the apodization. The goal is to emit a wave-front that is spherically focused and that has uniform amplitude after it has propagated through the phase-screen. To find the optimal excitation of the transducer, information from an earlier back-scattered wave is used. The question is what to emit to recreate a wave-front that is spherically focused and that has uniform amplitude after it has propagated through the phase-screen. The wavefront back-scattered by a point reflector located in the desired focal point has the same shape that we want the next emitted wave-field to have. If we can emit a wave that is a back-propagation of this reflected wave the beam will be optimally focused.

The time delay focusing was tested first. The next emitted wave used extra delays. The delays were set equal to the time fluctuations in the previous received wave-front, but with opposite signs. Apodization was not used. This wave propagated 5 cm to the phase-screen. The delays from the phase screen were added after the propagation. The correction is optimal if the wave-front becomes a plane wave with uniform amplitude. The simulated wave is more plane than the phase screen, but it still has phase aberrations with standard deviation 14 ns. The amplitude of the wave-front is far from uniform. The std/mean to the amplitude was 0.18. The amplitude aberrations are due to the emission of a wave with rough wave-front. The correction delays added to the emitted wave cause local focusing and defocusing of the wave-front. The parts that were focused generated high amplitudes, while the local defocusing reduced the amplitude. The amplitude aberrations increased with increased propagation distance. This effect explains why the time delay focusing
works best for aberrators placed close to the transducer.

In the thesis, it is suggested that apodization be used actively during the corrections. This is to compensate for the amplitude aberrations caused by the phase aberrations. Elements that are delayed to cause a local focusing must be emitted with low amplitudes. The elements that are locally defocused must be emitted with high amplitudes. By setting the apodization correctly, the amplitude of the emitted wave will be uniform when it has propagated to the phase-screen.

Simulations of delay and amplitude correction (DAF) are presented in 7. The time delay correction is set to the opposite of the phase aberrations in the received wave, while the apodization is set to the amplitude aberrations in the received wave. The simulations show that this method improves the focusing through the phase-screen compared to time-delay focusing only. After the emitted wave front has propagated through the phase-screen, it is more plane and has a more uniform amplitude. The standard deviation of the phase aberrations were reduced to 9 ns, while the std/mean to the amplitude was reduced to 0.08. These distortions are so small that they will not disturb the focusing significantly. Good corrections with delay and amplitude focusing (DAF), and the fact that the pulse form was almost undisturbed by propagation, show that the amplitude information is more important than the pulse form.

To estimate the delay and amplitude used in DAF, it is required that the waves received in adjacent elements are correlated. If the wave is back-scattered by a point reflector the correlation will be high. The method will thus work well for waves back-scattered by a point reflector at the point we want to focus on. The van Cittert Zernike theorem shows that the waves back-scattered from diffuse scatterer and received by adjacent elements also are correlated. The correlation is found to be high enough to estimate the time delays necessary to perform time delay focusing. The correlation is therefore also high enough to estimate the amplitude aberrations. To get high enough correlation, it is required that the distance between adjacent elements is much shorter than the size of the transducer aperture used to emit the wave. The correlation also depends on how well the emitted beam is focused. If the transducer is well focused, the correlation between signals received in adjacent elements will be high. If the emitted beam is broadened by for instance phase aberrations, the correlation will decrease. However, this can be compensated for by building a transducer where the distance between adjacent elements is even smaller compared with the total size of the transducer.

How small must the elements be to use the waves back-scattered from diffuse scatterer to estimate phase and amplitude aberrations? The distance between adjacent elements must be so much smaller than the transducer aperture that the correlation between signals received in adjacent elements are highly correlated. If the correlation is low, errors in the estimated time delay between neighboring signals will be high. Using diffuse scatterer where the correlation is low, the totally estimated phase and amplitude aberrations must be estimated element by element. The error of the estimated aberrations, will thus accumulate. In spite of the ac-
cumulation of the estimation errors, simulations are done that show that a $5 \times 80$ elements array is sufficient to estimate the phase aberrations [58].

To explore the DAF, simulations are done without taking care of absorption. If the medium has a highly inhomogeneous absorption, it will cause amplitude aberrations. These amplitude aberrations should be corrected in the opposite way as suggested by the DAF method. It is assumed that the amplitude aberrations between neighboring waves mainly are caused by diffraction of a rough wave front. Amplitude aberrations caused by absorption are assumed to be small. It is in other words assumed that the absorption of neighboring waves is almost the same. If not, even the TRM method will fail.

The suggested delay and amplitude focusing (DAF) can be viewed as a way of combining the best of previously published methods, *i.e.* a method that combines the best of time delay focusing and the time reversal mirror. The improvement compared with the time delay focusing is that the DAF method correct for phase aberrations generated several centimeters away from the transducer. The DAF method improves the TRM method since it can be used on waves back-scattered by diffuse scatterer. The DAF method is therefore expected to be the best way to do phase aberration correction in medical ultrasound imaging.
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Articles:


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