



Norwegian University of  
Science and Technology

Department of Mathematical Sciences

## Midterm examination paper for **MA1301 Number Theory**

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**Examination date:** 9. October, 2019

**Examination time (from–to):** 18:15–19:45

**Permitted examination support material:** D: No printed or hand-written support material is allowed. A specific basic calculator is allowed.

**Other information:**

**Write clearly and orderly.** Illegible solutions and poorly organized work may receive little, if any, credit. Please organize your solutions in the same order that the problems are listed.

**Justify your work.** Unless otherwise stated, explain how you arrive at your solution with a proof or calculation.

**Scoring.** This exam will be scored out of 100 points.

**Language:** English

**Number of pages:** 1

**Number of pages enclosed:** 0

**Checked by:**

Informasjon om trykking av midtsemesterprøve	
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Date

Signature



**Problem 1:** Provide accurate statements for the following theorems and conjectures.

1. The twin prime conjecture
2. The division algorithm

**Problem 2:**

1. Use the Euclidean algorithm to calculate  $\text{GCD}(182, 95)$ .
2. Find all integer solutions to the equation  $95x \equiv 6 \pmod{182}$ .

**Problem 3:** Find all integer solutions to the system of congruences below.

$$\begin{aligned}x &\equiv 3 \pmod{4} \\2x &\equiv 1 \pmod{5} \\4x &\equiv 1 \pmod{9}\end{aligned}$$

What is the smallest positive solution?

**Problem 4:** Prove that  $11^{3k+1} \equiv 4 \pmod{7}$  for all  $k \in \mathbb{N}$ .

**Problem 5:** Let  $n$  be a natural number and suppose that the integers  $a, b, c$  and  $d$  satisfy  $a \equiv b \pmod{n}$  and  $c \equiv d \pmod{n}$ . Prove that  $a + c \equiv b + d \pmod{n}$ .