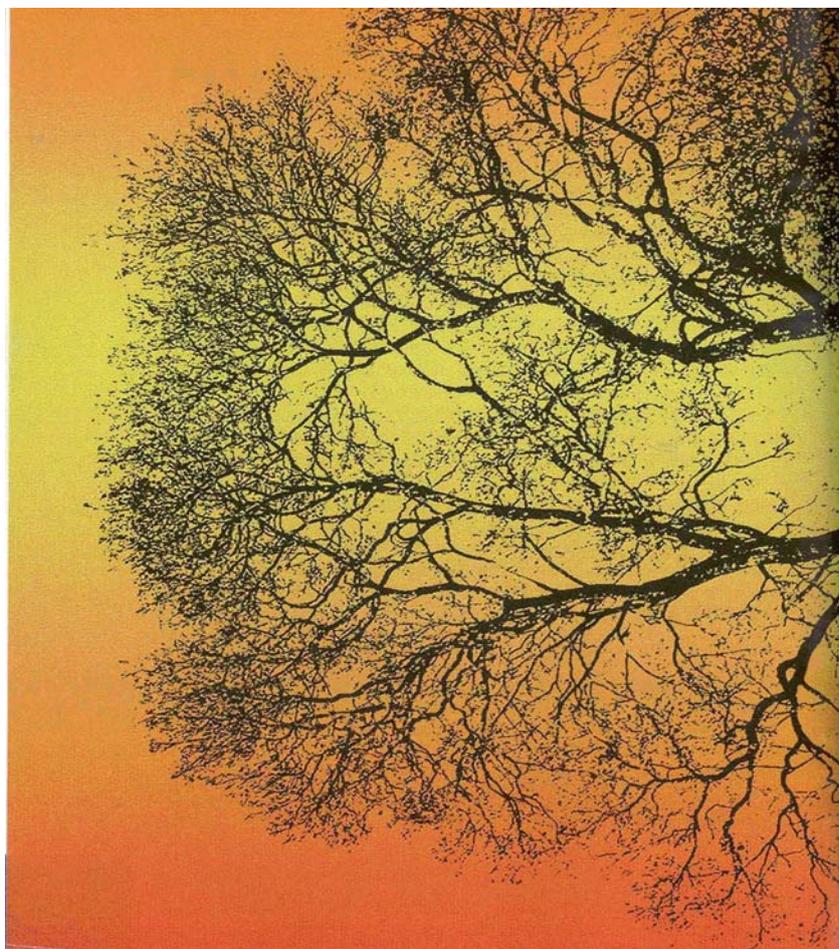


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Complexity - Systems and Interplays in Nature



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and

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Ideas thus made up of several simple ones put together, I call complex; such as beauty, gratitude, a man, an army, the universe; John Locke, English Philosopher (1632-1704)

Towards the end of the nineteenth century, before Modern Physics as we know it was founded, it was a widespread belief that Physics had come close to its end. The basic principles are known, it was said, and all that remains is exploring their applications. A similar attitude was common in the twentieth century after the unravelling of the fundamental forces. When one knows the elementary particles and the forces that act between them, one knows in principle everything ... Since everything eventually is built from such particles, one only needs a description of their behaviour in order to understand everything that in turn is built from them. Again, one might say that the Physics project is closed, in this case by the laws of the elementary particles.

Physics has, in order to understand the basics, focused on taking things apart, figuring out how the pieces work -- so focused that we have forgotten to put the pieces back together again! By this insight the study of complexity begins. Brand new and fundamental problem

definitions arise from the attempt to make a bridge between the behaviour of the smallest particles and the world as perceived in everyday life.

In an article named "Simple Lessons from Complexity" in the magazine *Science* (volume 284, 2nd April 1999), which includes a special section on complexity, the physicists Nigel Goldenfeld and Leo P. Kadanoff make the following statement:

"The ideas that form the foundation of our worldview are also very simple indeed: The world is lawful, and the same basic laws hold everywhere. Everything is simple, neat, and expressible in terms of everyday mathematics, either partial differential or ordinary differential equations. Everything is simple and neat—except, of course, the world. Every place we look—outside the physics classroom—we see a world of amazing complexity. The world contains many examples of complex “ecologies” at all levels: huge mountain ranges, the delicate ridge on the surface of a sand dune, the salt spray coming off a wave, the interdependencies of financial markets, and the true ecologies formed by living things. Each situation is highly organized and distinctive, with biological systems forming a limiting case of exceptional complexity. So why, if the laws are so simple, is the world so complicated?"

Think about ice ferns on a window a frosty winter day. We know that the molecules that water is made from are quite simple. They behave like small bricks that attract each other. Nevertheless, they form ice ferns, like those on the picture, when they are deposited on a cold substrate. The ice ferns are a direct result of the molecules' tendency to be deposited next to each other, that is to say their relative attraction. What appears to be a simple step by step process is still the origin of complicated shapes. The ice ferns on the picture are from a car window and look almost like living plants.



Figure 1. Ice fern.

The point is that we could not have guessed these forms only by looking at a single molecule. A vast number of molecules are needed to form an ice fern, typically on the order of billions. This is an example showing that a simple rule, repeated many times over, gives a complicated but identifiable result, where ***the whole is more than the sum of its constituents.***

That the whole is more than the sum of its constituents is not news. If you look at the Egyptian pyramids or L'église de Notre Dame simply as a pile of ten million stone bricks, then you miss all that has to do with the usage of the buildings and their architectonic styles. It is the manner in which the bricks have been put together that decides the appearance of L'église de Notre Dame viewed from a distance.

The cathedral was built after exact

drawings, but the ice ferns build themselves up step by step as the water molecules deposit on the cold glass surface.

This is complexity. There is no architect behind the ice ferns. The secret behind the ice ferns' complicated form must be hidden in the interplay between the individual water molecules. The question is how. Nature is loaded with similar examples that simple rules on a small scale give complex results on a level visible to the human eye. These include cumulus clouds on a summer day, an oak tree's thousand branches, waves on an agitated sea, or the stripes of the tiger.

Where to Start the Description?

Our understanding of a complex system depends on at which level we start to describe it. We may look at our ice ferns in an electron microscope, which shows single molecules, and on this scale they appear regular and simple. Looked at by eye from a close range, complexity emerges, but at a longer distance, say 20 meters, only a white spot is seen, which again is rather simple.

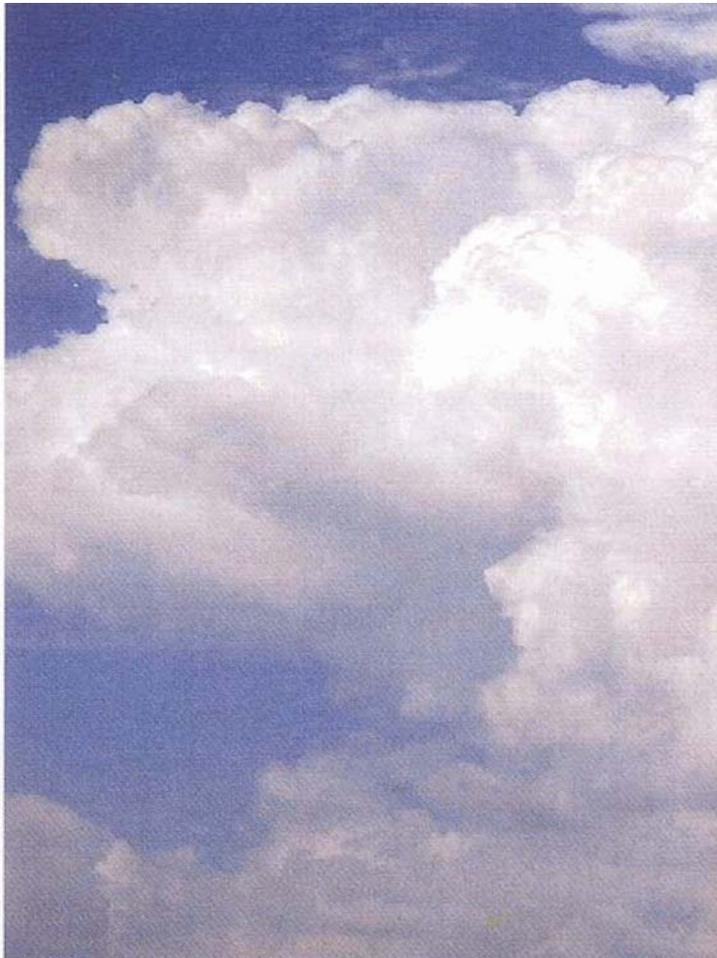


Figure 2. Cumulus clouds

Complex systems may also be considered as units, which can be pieced together to larger systems. Many trees make a forest; many clouds make a cloud cover, and so on. In the case of the forest, a tree plays the role of a component -- a green dot or a green particle in a forest that is complicated at some level. Maybe it grows along the coastline shaped by straights and bays, or maybe there is room for rivers winding through the landscape. From a far distance the forest itself can play the role as "particle", which together with all the other forests on the earth constitutes a substantial part of the earth's biosphere. In this manner we can go on making an entire hierarchy of interconnected systems assembled from interacting "particles".

These systems may be complex or simple. For example, think about the more than 50 000

people streaming out of Wembley Stadium after an important football match. The flow of people is comparable with syrup that flows out of a bottle; a rather simple phenomenon. The human beings that make up the stream are, on the other hand, immensely complicated. The choice of level of description is vital.

Imagine starting far out in our solar system and moving towards a human being on earth: This hierarchical list of complex systems can be made:

- Earth's orbit around the sun
- Earth's biosphere
- 50 000 people streaming out of Wembley
- A human being
- The human brain
- Blood (a complex fluid of blood cells in water)
- The blood cell
- DNA-Molecule
- Atoms
- Elementary particles

Every individual item on this list is a complex system from some level of description -- or a simple system on a lower level of description. The extreme points, the biggest and the smallest on such lists, tend to be rather simple. At the right level of description, simplicity may be found also in the middle of such a size hierarchy, exemplified by the human flow out of Wembley.

Complexity, Interaction, and Universality

Is it possible to give a simple definition of complexity? As for many collective terms in science, this is not an easy task. Collective terms change both with use and with new insight, and attempts to give exhaustive definitions will always be controversial.

Yet, complexity is interaction, and much research is devoted to understanding the relation between activity on a small scale and large scale behaviour. It is all about putting things together and attempting to understand how and why the whole assembly appears as it does.

In order to achieve this, these problems need to be condensed into their most essential basic elements. This is the case regardless of whether one attempt a characterisation of a complicated item by a single number or the explanation of something is sought by performing a simplified experiment or by running a computer simulation. We will return to this.

The objective is to find results of universal validity. When Newton worked on his theory of gravity, it was known that many planets' orbit had the same universal shape: they were ellipses. Newton's law of gravity (its strength goes as one divided by the square of the distance to the object causing it) explains why this shape is universally valid, and with that it applies to planets in all other solar systems as well.

In a similar way there exist amongst complex systems universal or general shapes and characteristics. When a description turns out to be of general validity in the sense that it is valid for a whole class of different systems, it is called *universal*, and the class in question is called a *universality class*. Often the theory itself which would explain such universal properties, is lacking, and is analogous to Newton's law of gravity. Nevertheless, the detection of a universality class is the first important step towards understanding the underlying laws.

The quest for the essential characteristics is not only a scientific activity. In the lithography in Figure 3 Picasso reduces an ox to the essential ox. His "first state" contains all details. An ox is a complicated system, not easily describable. All oxen are different, as are human beings.

Every human being has its own face, and every ox has its own face. How simplified may an ox be represented and still retain some "oxiness"? Picasso's answer to this question is given as his "eleventh state", the lower right panel in the Figure.

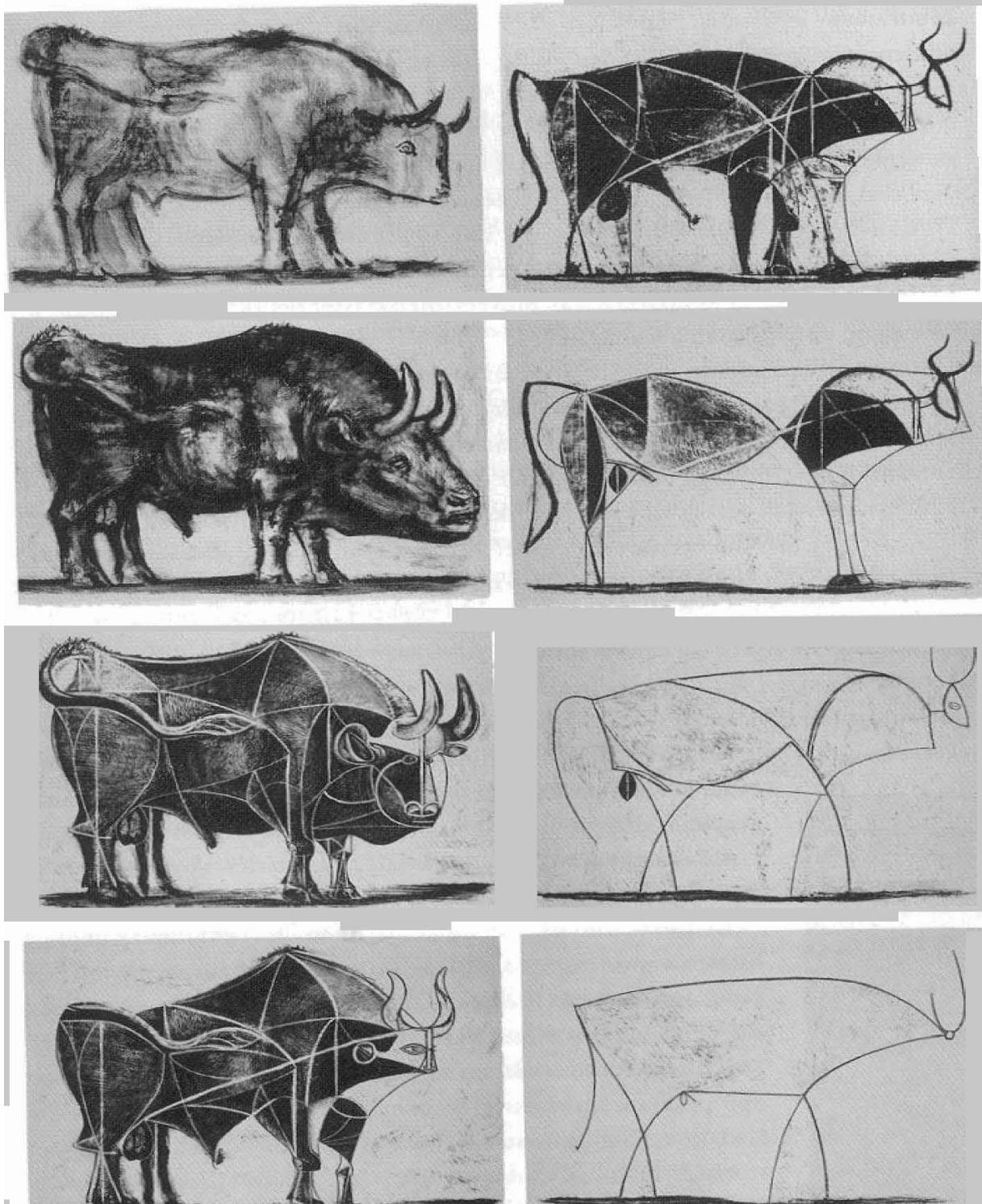


Figure 3. Picasso: Huit Etats du Taureau 1945-46.

Complex Fractures and Universality

An important example where universality is observed is the formation of fractures. Looking at a crushed ice cube or a piece of stone broken off from another, one sees a complicated surface, a landscape of crests and valleys, which has little in common with the shape of the

simple atoms building the material. Yet, these landscapes, or fracture surfaces, let themselves to be characterized by a single number in a way that allows one to compare fractures in different materials. The surprise is that this number is the same for widely different materials.

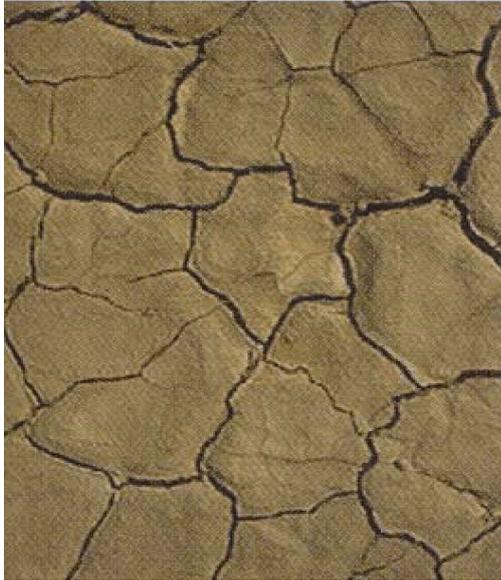


Figure 4. Fractures.

on their ideas about how cracks develop step by step.

In experiments this was first shown (in a study published in 1992) by the Norwegian physicists Knut Jørgen Måløy (now professor at the University of Oslo), Alex Hansen (now professor at the Norwegian University of Science and Technology), and Einar Hinrichsen (now research leader at SINTEF). For a number of years they had studied fundamental properties of the processes creating fractures. The question they asked themselves was: What happens when a material is exposed to stress? When will it break? Where will a crack first appear, and where will it end? And, maybe more important, how can one characterize the fractures? They did experiments, trying materials as different as wood, steel, bakelite, and ceramics, and they did computer simulations based

The formation of fractures is again an example expressing that what is simple on an atomic level gives complicated shapes on our length scale. The single atoms all interact in the same way. By applying force to the material, starting to deform it, microscopic cracks appear. This initiates the fracturing process. The *micro cracks* cause the forces within the material to be redistributed leading to more micro cracks. The micro cracks are correlated with each other, and evolve into larger connected cracks, which eventually make a complete fracture surface through the material. It is the shape of this kind of surface that was studied by Måløy and Hansen, and they found that they all could be described by the same number, only provided the materials is brittle. The fact that the choice of material does not play a role, made it possible to identify a universality class of different materials.

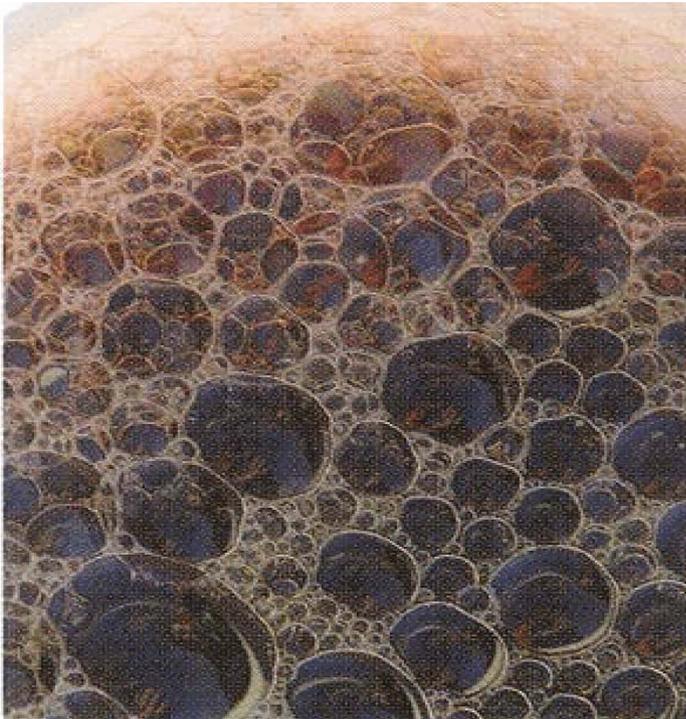
A universality class otherwise need not contain the same kind of systems. Apparently very different systems with completely different levels of description may have the same universal properties. Models that are used in this branch of Physics have shown that things may be grouped in surprising ways. Could it be that stock market crashes have the same underlying cause as the extinction of the dinosaurs? Do cracks in glass and granite have the same characteristics? Does the flow behind a tanker and above a volcanic outburst have profound similarities?

Description of Complex Systems

A main challenge in the study of complex systems is to find a description by numbers (preferably not too many numbers). Only when measurements can be compared with theoretical calculations, and vice versa, are we able to judge whether an idea is right or wrong. In order to do that, we need numbers.

A famous class of observations, which let themselves describe by a single number, are fractals. When something is fractal it consists of parts of many different lengths, and we can

put a number on how many of the smaller pieces there are in comparison to how many there are of the larger pieces. This number is called the fractal dimension.



For instance, in a cup of coffee, see Figure 5, there exist large, medium and small bubbles. If the relative amounts of these bubbles may be described by a number, then the bubbles may have a fractal dimension.

However, not all complex structures have a fractal dimension, and much work remains before we are able to capture all complex patterns and shapes in simple descriptions.

Do you see what the image to the right in Figure 6 shows? It shows a cut through red cabbage. The pattern is complex yet identifiable to the observer, but it is not clear that one may put a number on it. At least, so far that is not done. What about the

Figure 5. Bubbles in coffee.

left image in Figure 6, which we have borrowed from the article "Fractals" by M. Daoud and H. van Damme, in *Soft Matter Physics*, Springer Verlag 1999. Do you see what it is? It is a cut through a crumpled piece of paper, and in this case Daoud and van Damme showed that the fractal number 2.5 suffices to describe the structure. Maybe the red cabbage belongs to the same universality class, maybe not. Measurements are required in order to answer that question.



Figure 6. Left: Cut through crumpled paper. Right: Cut through red cabbage.

Computer Simulations and Complexity

Complexity has become a subject within modern Physics; not only due to the fact that so many interesting questions arise, but also due to our ability to address them. Modern

computers make exactly the tool needed in order to repeat simple rules very many times. The lower tree in Figure 7 is made by a computer program that has no more than ten lines of code. Together they say: "Draw a Y, and then draw another two Y's initiating on top of the first one." These Y's become smaller and smaller for every step, their directions are a little different, and the top level Y's are coloured orange and red. When this rule is repeated twelve times, the tree in the Figure is obtained. Compare it with the real tree in Figure 7, and you will see that the two trees share the same basic shape.

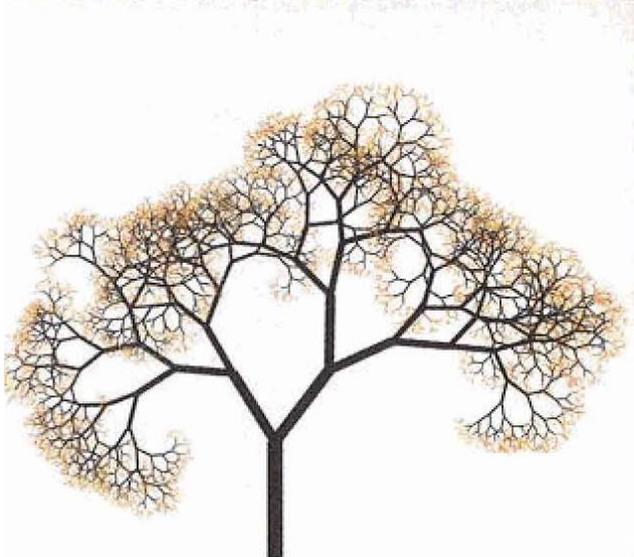
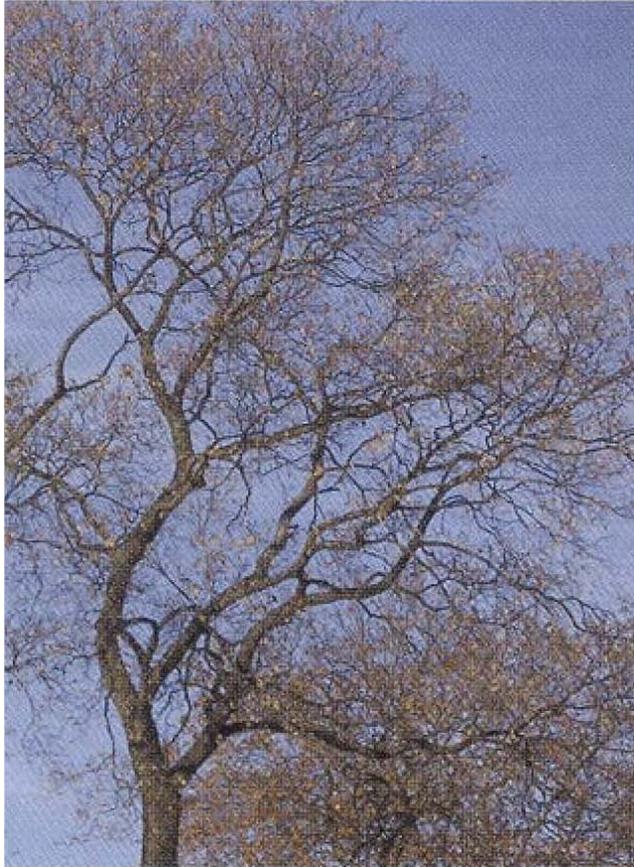


Figure 7. Over: Real tree. Under: Fractal tree.

Crystal growth (as for the creation of the ice ferns) may for instance be modelled without any details on the atomic level.

This is called to make a model, that is to say a simplification that still contains the essential features of the object of interest. In a similar manner we may make a model for ice crystals: We may let the water molecules be represented by particles with no other properties than having a certain spatial extension and jumping around until they meet and stick together. And maybe, they will have readjusted their position a little bit after being deposited (they have to adjust to their neighbours, but then crystals come out of the process). This way to do research in Physics is called simulation and is only possible with the help of computers. True, you could probably have drawn the tree in the lower panel of Figure 7 by hand, but that is also the exception that proves the rule. Often you would have to carry on for a life-time to complete what a computer does in minutes.

The point is that computer simulations may be used to isolate essential mechanisms or properties on the microscopic level, which are needed in order to create the macroscopic patterns observed. Such simulations can be carried out by testing how much simplification the microscopic description can undergo before the macroscopic results are no longer in accordance with reality. Hence, models and computers become a central tool to find universality classes, and it is astonishing how much may be removed from the micro description and still not change the macroscopic behaviour.

Networks

Network modelling is another common basis for computer simulations. Imagine an anthill, where the ants coordinate for collection of food and building materials. The ant society can be described as a simple network of different functions that all are connected.

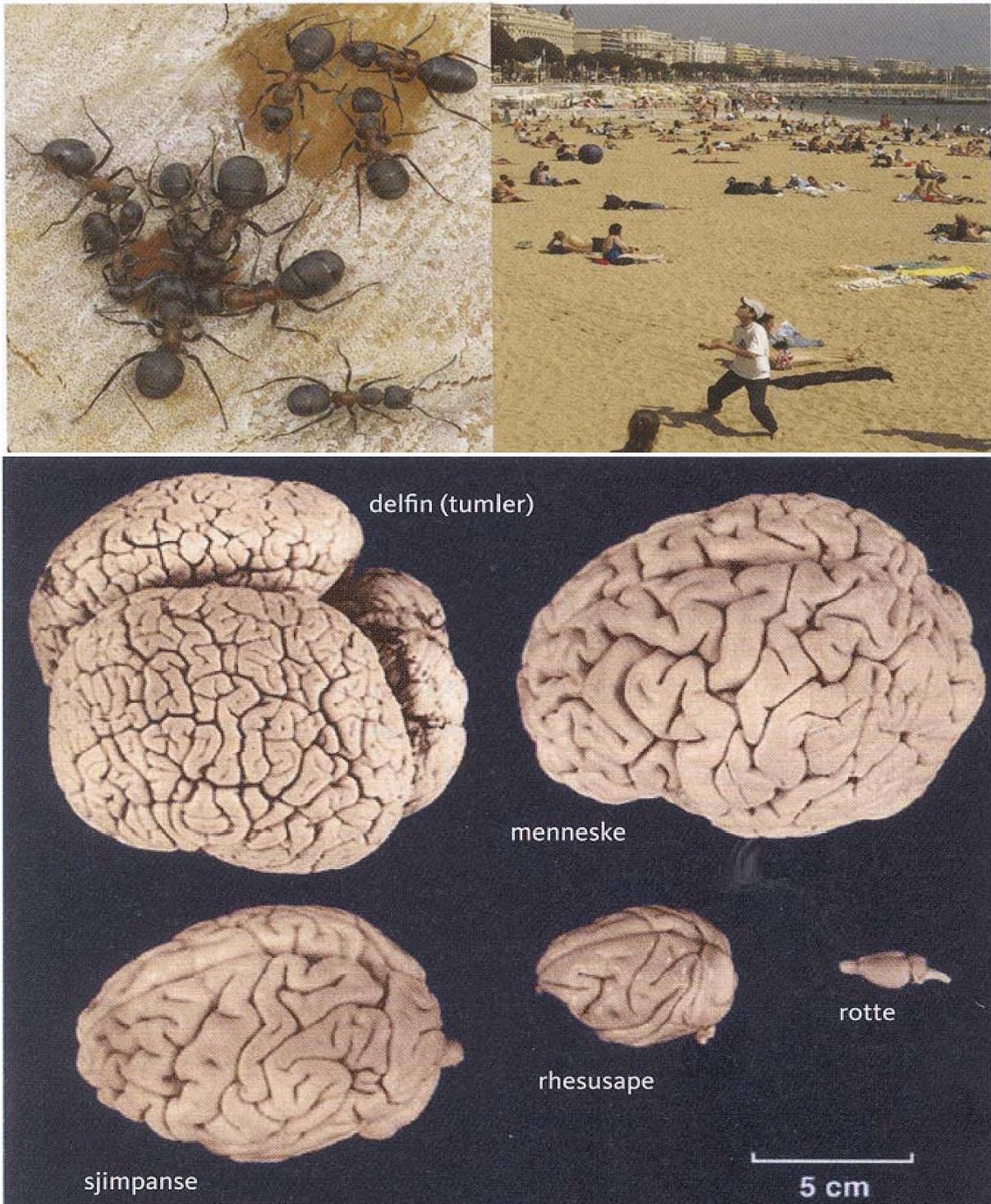


Figure 8. Over: From the proper perspective an ant colony and a crowd of humans share many things in common. Under: Different brains.

The same type of modelling can be used for various aspects of the human society. Human activities on a beach or traffic congestion during rush-hour are examples of this. The research field called econophysics has become a part of modern Physics and describes stock trading by

means of e.g. network modelling. One of the purposes is to understand the variations of the stock prices. In this case the level of description is the stock traders. They talk with each other and with other people and keep themselves informed about stock purchase and sale. The stock traders form a network with their surroundings, which can result in wealth or ruin, and which can have collective properties that we can calculate on a computer.

The human brain is possibly the most pronounced and advanced example of a complex system. The image in Figure 8 shows a set of different brains. We know that every single one of the depicted brains is a physical network. But how many of a brain's processes are "softwarelike" and how much is pure Physics and Chemistry? Such questions about the brain are one of the greatest challenges that face biological Physics today.

Models and Experiments

Ketchup appears rather simple, but it consists of so many different ingredients that an accurate description is almost impossible. Clay found in Nature is similar, that is to say that it contains many different ingredients, which may have impact on its properties.

How to isolate something's essence from all that which is irrelevant? The answer is synthetic clean materials, which consist of one a single type of particles that interact with each-other in a known way. By focusing on model materials, which are the simplest possible, we can begin to understand them. Experiments need to be simple enough for us to understand what is going on, but not so simple that the effects of interest are lost. The trick is to simplify matter as much as possible without losing the essence, just like Picasso's ox.

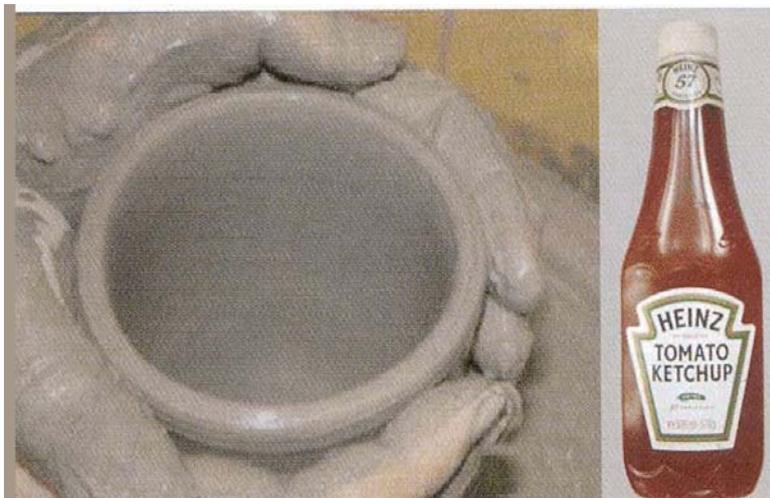


Figure 9. Clay and ketchup.

In other words we learn something about ketchup, caviar, avalanche in clay materials, paint and so on by studying simple model systems, which in themselves do not need to have any practical application. A model material of this kind is synthetic clay without any other constituents than the simplest kinds of clay particles and water. Other materials can be tiny plastic spheres suspended in water or liquid crystals, which are used in displays of calculators, mobile phones, and flat PC-screens. Such model systems, we can to a large extent understand since we have good control over them and at the same time we can compare with simulations.

Complex Fluids

A single substance like ice is a solid and hard material. Ice ferns are hard and keep their shape as long as it is cold. Only when the temperature rises above the freezing point, the water

Above we wrote about models that are used as input in simulations. Here we think about physical models made of materials, on which we can perform experiments. By that, a need for new theories is created, and thereafter a need for ways to test these theories. In real research simulations and physical experiments walk hand in hand seeking to understand complexity.

becomes liquid. But some substances are neither liquid nor solid, but both at once! These materials fall outside the standard classification and are often termed complex fluids. Examples of complex fluids are toothpaste, ketchup, ice cream, jelly, and other foods, foam, paint, cosmetics, clay, and flowing sand.

The complexity of complex fluids often has its roots on a nanometre scale, meaning that the relevant description level is molecules or nano-particles, and they have complicated and sometimes surprising properties on a macroscopic level. Consider toothpaste: If you open the tube with toothpaste and hold it upside down, nothing happens. In order to get the toothpaste out, you will have to squeeze the tube, and the toothpaste flows out like a thick liquid. But it does not flow like a regular fluid; all the deformation is concentrated within a thin layer next to the tube walls. It looks like a plug that flows out! The toothpaste on the toothbrush keeps its shape and behaves like a solid material.

Another example of a complex fluid is flowing sand. You can walk on sand, but at the same time it can flow inside an hour-glass. Sand is solid or liquid very dependant on how it is treated. Sand grains are about a millimetre large, so the description starts at the millimetre scale. Sand grains are simple, but their interaction makes complicated patterns. In the computer simulation of a sand pack shown in Figure 10, the forces between the grains are visualised by corresponding thicknesses of the red connection lines. Altogether they form a complicated and complex network where some of the grains carry much of the load, whereas other grains are almost completely screened. This phenomenon is the same as one may observe in old bridges and church ceilings: arches of stone or other hard materials can carry a tremendous load.

The materials studied to understand complex fluids and systems may be natural, synthetic, or

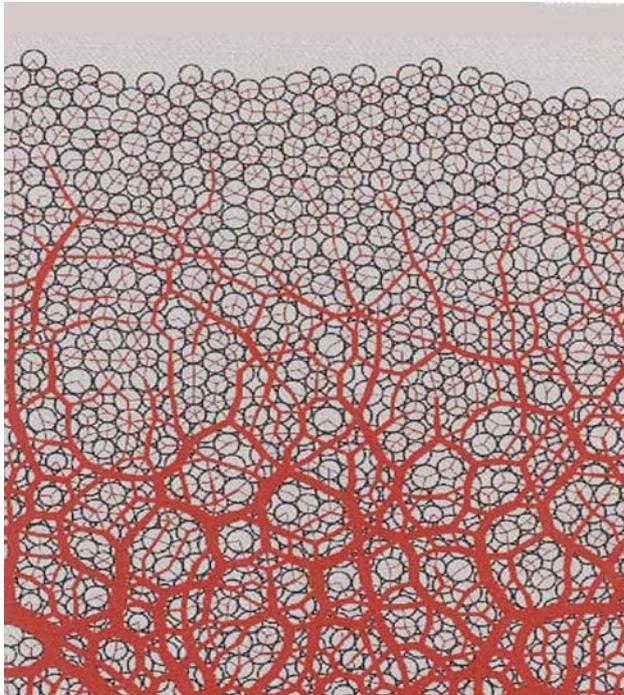


Figure 10. Simulation by Jan Ludvig Vinningsland of sand grain packing.

biological. Some materials are even called *smart*, because they change their properties in response to their surroundings. Some smart materials change properties when exposed to an electric or magnetic field. For instance the material may change from solid to liquid or cease to be transparent, or change its shape. Such smart materials may in the future form the basis of smart clothes, smart houses, smart cars, smart skis, smart muscle replacements, smart medicine, etc.

The well-known Norwegian physicist and Nobel laureate Lars Onsager was a key person in developing the understanding of universality and complex fluids. Onsager first found a theoretical foundation for liquid crystals. He published this in an article in 1949. Liquid crystals are otherwise a prominent example of nano-

technology in present everyday use. It is no exception that 50 years passes from the time Physics establishes the foundation of a technology until technological applications eventually are produced in large volumes for the consumers, as it has been the case of liquid crystals.

Do it Yourself

Osborne Reynolds (1842-1912) was an English scientist and technologist. He did the following experiment: He attached a balloon around the lower opening of glass tube held vertically, and filled the balloon with water that also filled the lower part of the glass tube. He squeezed the balloon and observed the rise of the water level in the glass tube, just like one should expect. But then, he filled the balloon with sand before attaching it to the glass tube. Again the balloon and the lowermost part of the glass tube were filled with water. When he squeezed the balloon, the water level in tube sank.

This might not be what you expected? The explanation is that when you squeeze sand, the sand grains have to move in order to pass each-other, and then small spaces open up between the sand grains, which are filled up with water, and thus the water level sinks in the glass tube. It is the same thing that happens when walking on a wet sandy beach; a dry zone appears around your feet for every step you make.

Another experiment that you may try at home is corn starch in water: Make a mixture of 50% corn starch and 50% water in a bowl. Upon gentle tilting the mixture flows smoothly, but if stirred with a spoon the mixture stiffens abruptly. This phenomenon is just the opposite of the one described above, the corn starch mixture thickens when exposed to large enough and fast enough external forces, as opposed to ketchup or toothpaste which becomes thinner. The explanation is that instead of tearing particles apart like in the ketchup case, the corn starch particles are initially separated but are forced into contact and eventually they build a network that can resist the exerted force. The corn starch mixture is called shear thickening and ketchup is called shear thinning.

Further Reading (Updated from original version):

- Ball, Philip: *Made to Measure: New Materials for the 21st Century*, Princeton, New Jersey: Princeton University Press 1997
- Ball, Philip: *The Self-made Tapestry: Pattern formation in nature*, New York: Oxford University Press 1999
- Ball, Philip: *Critical mass; How one thing leads to another*, Arrow books 2005
- Ball, Philip: *Flow; Nature's Patterns: A Tapestry in Three Parts*, Oxford university press 2009
- Barabasi, A-L.: *Linked; How everything is connected to everything else and what it means for business, science and everyday life*, Plume 2003
- Bentley, P.J.: *Digital Btology. The creation of life inside computers and how it will affect us*, London Headline Books Publishing 2001
- Benyus, J.M.: *Biomimicry: Innovation inspired by nature*, New York: W. Morrow and Company Inc. 1997
- Carroll, S.C.: *Endless forms most beautiful: The new science of evo devo*, W.W. Norton & Company, N.Y. 2005
- Coveney, P. and Highfield, R.: *Frontiers of Complexity: The search for order in a chaotic world*, New York: Fawcett Columbine 1995
- Fisch, M.R.: *Liquid crystals, laptops and life*, World Scientific Publishing Co. Ltd. 2004
- Flekkøy, E.G.: *Sandslott og flodbølger; Den fantastiske fysikken omkring oss* (in Norwegian), Cappelen Damm 2006
- Fossum, J.O.: *Physical phenomena in clays*: <http://folk.ntnu.no/fossumj/P2Akademiet/english.htm>: A recorded radio talk about Materials Science given at The National Norwegian Broadcasting NRK P2 Akademiet 2000
- Johnson, N.: *Two's company; Three is complexity*, Oneworld Oxford, 2007
- Lewin, R.: *Complexity: Life at the edge of chaos*, The University of Chicago Press 1999
- Pelesko, J.A., *Self Assembly: The science of things that put themselves together*, Chapman & Hall/CRC 2007
- Sardar, Z. and Abrams, L.: *Introducing Chaos*, Cambridge UK: Icon Books Ltd 1999
- Ward, M.: *Universality: The underlying theory behind life, the universe and everything*, Macmillan 2001
- Wilson, M., Karannagara, K., Smith, G., Simmons, M., and Raguse, B.: *Nanotechnology, Basic Science and Emerging Technologies*, Sidney University of New South Wales Press Ltd. 2002