Small is Beautiful
How to make wave energy economic
A Study of Economies as if People Mattered

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([London 1973])
Once upon a time
our cat committ'd a crime: (?)
In the basement of our house
it caught a tiny lovely mouse.

Then suddenly it saw a rat,
a lot of food for hungry cat!

Then came the time for mouse release
when cat's desire was the rat to seize.
Oh, where is the wanted food?
It's time to be in poorer mood!

**MORAL:** My dear cat! The coming night
stick to the mouse and hold it tight.
Average capture
High or Low "Efficiency"?

Envisaged example:

Wave energy resource \( J_{\text{average}} = 2.5 \text{ kW/m} \)
\[ \Rightarrow 1000 \text{ MW} = 1 \text{ GW incident on a 40km coast line} \]

<table>
<thead>
<tr>
<th>&quot;Efficiency&quot;</th>
<th>Plant capacity</th>
<th>kWh cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>1000 MW</td>
<td>( \infty )</td>
</tr>
<tr>
<td>50%</td>
<td>500 MW</td>
<td>0.25 ACU</td>
</tr>
<tr>
<td>10%</td>
<td>100 MW</td>
<td>0.15 ACU</td>
</tr>
<tr>
<td>2%</td>
<td>20 MW</td>
<td>0.08 ACU</td>
</tr>
</tbody>
</table>

In this example you would go for a low "efficiency" if the market price were 0.10 ACU.

What would you do if the market price was 0.20 ACU and if more than 40 km coast line was available?

"ACU" = arbitrary currency unit
\[
\frac{P}{C} = \frac{\text{Power}}{\text{Cost}} = ?
\]
\[
C = a_1 + a_2 \text{Surface} + a_3 \text{Volume} + a_4 \text{P}_{\text{max}} + \ldots
\]

Difficult!

Simpler: \[ \frac{P}{V} = ? \] (or \[ \frac{P}{S} \])
Floating body in heave due to incident sinusoidal wave of amplitude $A$ and angular frequency $\omega$.

Net force $= \text{excitation force} - \text{radiation force} - \text{friction force}$

$$\hat{F} = \hat{F}_e - Z_r \hat{u} - R_f \hat{u}$$

Radiation impedance $Z_r = R_r + i\omega m_r$

$R_r = R_r(\omega)$ = radiation resistance (= added damping)

$m_r = m_r(\omega)$ = added mass

Net converted power (time average):

$$P = P_e - P_r - P_f$$

"Excitation" power:

$$P_e = \frac{1}{2} |\hat{F}_e| \cdot |\hat{u}| \cos(\gamma)$$

$$[\gamma = \text{phase angle between } \hat{u} \text{ and } \hat{F}_e]$$

$$\hat{F}_e \propto A$$

Radiated power:

$$P_r = \frac{1}{2} R_r |\hat{u}|^2$$

Lost power:

$$P_f = \frac{1}{2} R_f |\hat{u}|^2$$

$P_e \propto |\hat{u}|$  $P_e \propto |A|$

$P_r$ and $P_f \propto |\hat{u}|^2$ (independent of $A$)
\[ P_m = \frac{F_x^2}{8(R_r + R_f)} \]  
\[ U_m = \frac{F_r}{2(R_r + R_f)} \]  
\[ F_x = |\hat{F}_x| \cos(\gamma) \]

If \( \gamma = 0 \) and \( R_f = 0 \)  
\[ P_m = \int d_{r,\text{max}} \]
Maximum capture width \( d_{r,\text{max}} = \lambda / 2\pi \) for a point absorber,  
50% for an array or symmetric terminator  
Approaching 100% for a sufficiently non-symmetric term.
$S_w = \text{water plane area of floating body}$

source-like mode (e.g. heave)

dipole-like mode (e.g. surge)

$\lambda = \frac{2\pi}{k}$

$\hat{F}_e$ vs $k\alpha = (\omega^2/g)\alpha$
UPPER BOUND TO POWER-TO-VOLUME RATIO:

\[ P = \frac{1}{2} |F_e| \cdot |\hat{u}| \cos(\phi) - \frac{1}{2} (R_r + R_s) |\hat{u}|^2 \]
\[ < \frac{1}{2} |F_e| |\hat{u}| \cos \gamma < \frac{1}{2} |F_e| |\hat{u}| \]
\[ |\hat{u}| < \omega \delta_{max} \]
\[ |F_e| < \frac{3}{2} g S_w |A| \]

\[ P < \frac{1}{2} |F_e| |\hat{u}| = \frac{1}{2} \delta_{max} g S_w g S_{max} \omega |A| < \frac{3g}{4} V \cdot \omega |A| \]

\[ \frac{P}{V} < \frac{3g \omega}{4} |A| = \frac{\pi \rho g |A|}{2T} = \frac{\pi \rho g H}{4T} \]

\[ \rho = 10^3 \text{ kg/m}^3 \quad g = 9.8 \text{ m/s}^2 \]

Example: Wave height \( H = 2 |A| = 2m \)
Wave period \( T = 10 \text{ s} \)
\[ \frac{P}{V} < 1.6 \text{ kW/m}^3 \]
HEAVE-LIMITED SPHERE IN IRREGULAR WAVE

Optimum phase

Optimum heave amplitude if $S_{max}$

Radiation resistance $R_r$ (Hulme 1982, Falnes 1984)
Loss ("friction") resistance $R_f = 10^4 \text{ Ns/m}$ (assumed)

Given a typical winter wave in the Norwegian Sea
$H_{m0} = 3.2 \text{ m}$ $T_{m0} = 7.0 \text{ s}$ $T_{m-10} = 8.5 \text{ s}$ from a 2048 s time series (Barstow, 1993)

Numerical results (Eidsmoen 1993):

$2 \text{ kW/m}^3$
$1 \text{ kW/m}^2$

0.5

$\frac{P}{4\pi a^2}$
$P / V$

0.5

$P$

How big should one sphere be?

$\text{small}$

$\text{a = ?}$
100 MW wave-power plant
(50-100 km long array offshore)

200 units each 0.5 MW or 1000 units each 0.1 MW

(Mass production of units)

R&D required

Demonstration of full-scale WE converters

for which markets exist!

**Stages**

1. Develop 10 kW unit.
   After successful testing:

2. Develop 20 kW unit.
   After successful testing:

3. Develop 40 kW unit.
   After successful testing:

4. Develop 80 kW unit.
   After successful testing:

5. Perhaps develop 160 kW unit.
Before hunting the rat we must learn to catch the mouse