New trends in rock slope stability analyses

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Abstract The paper discusses alternative approaches to stability analyses including the traditional deterministic principle, the so-called partial factor principle and the probabilistic approach which assesses the probability of failure rather than the factor of safety. To illustrate the different principles and approaches, stability analyses of road cuts near Trondheim, central Norway, are used as examples. It is concluded that although the traditional deterministic approach has the advantage of being well established and easy to understand, to conform with new standards and guidelines it is likely to be replaced by the partial factor principle. To obtain the best possible basis for evaluation it is useful to include a probabilistic analysis.

Résumé L'article présente de nouvelles approches pour les analyses de stabilité comprenant l'analyse déterministe traditionelle, la méthode dite des coefficients partiels et l'approche probabiliste qui évalue une probabilité de rupture plutôt qu'un facteur de sécurité. Afin d'illustrer les différentes méthodes et approches, des analyses de stabilité relatives à des déblais routiers près de Trondheim, au centre de la Norvège, sont présentées à titre d'exemples. Bien que l'analyse déterministe traditionnelle ait l'avantage d'être bien admise et facile à comprendre, on conclut qu'elle sera probablement remplacée par la méthode des coefficients partiels afin de suivre les nouvelles règles et recommandations. Pour obtenir la meilleure base possible de diagnostic il est utile de considérer également une approche probabiliste.

Key words Rock slope stability · Deterministic analysis · Partial factors · Probabilistic analysis · Factor of safety · Input parameters

Mots clés Stabilité de pente rocheuse · Analyse déterministe · Facteurs partiels · Analyse probabiliste · Facteur de sécurité · Paramètres de calcul

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Introduction

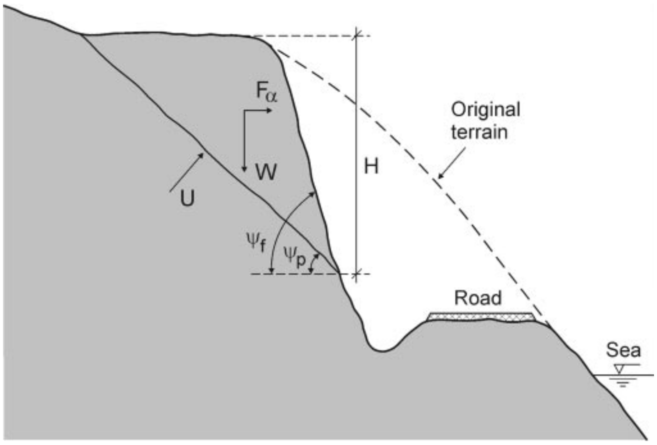
Traditionally, in the fields of geological engineering and rock mechanics, the deterministic principle of calculating the stabilizing and driving forces to arrive at a factor of safety has been the predominant method of rock slope stability analysis. In the last few years, however, alternative methods have become more widespread. To a great extent, this is due to the introduction of new standards and recommendations, such as Eurocode 7 (Comité Européen de Normalisation 1994) and NS 3480 "Geotechnical Planning" (Norwegian Council for Building Standardization 1988) which require all stability analyses to be carried out according to the so-called partial factor method, with partial factors for action and strength. In soil mechanics this principle was commonly adopted many years ago, while in geological engineering/rock mechanics the calculation principles have been changed only to a minor extent.

In addition, the probabilistic approach, with the calculation of the probability of failure instead of a factor of safety against failure, has become more common practice, as reflected by the many international conferences over the last few years focusing on this issue (Li and Lo 1993; Shackelford et al. 1996; Lee and Lee 1998).

The steps of the analysis that precede calculation, i.e. definition of potential stability problem and quantification of input parameters, are crucial for the final result. The main focus of this paper, however, is the advantages and disadvantages of the alternative methods, in particular the interpretation of calculation results.

Calculation example

To illustrate the different approaches and principles, a case representative of a common stability problem with road cuts around Trondheim, central Norway, has been used.



H = slope height = 22 m
ψ_f = slope angle = 77°
ψ_p = inclination of potential failure plane = 42°
γ_r = unit weight of rock mass (greenstone) = 30 kN/m^3
γ_w = unit weight of water = 10 kN/m^3
W = $(\gamma_r H^2/2)\cdot(1/\tan\psi_p - 1/\tan\psi_f)$
= 6388 kN/m = weight of potentially sliding rock
U = water pressure resultant (kN/m)
α = seismic acceleration in fraction of g (m/s^2)
F_α = mα = earthquake load (kN/m)

Fig. 1
Schematic sketch of typical stability problems with road cuts in the Trondheim area

The geometry and the acting forces of the road-cut example are shown in Fig. 1. The bedrock is a greenstone with uniaxial compressive strengths typically in the range 150–200 MPa. The potential stability problem is a result of the special topography along fjords and valley sides caused by undercutting of distinct exfoliation joints.

The water pressure during heavy rainfall is assumed to have a triangular distribution, as suggested by Hoek and Bray (1991), representing water entering freely at the top of the slope and being fully drained at the toe after having reached a maximum pressure corresponding to the hydrostatic at a height equal to 50% of the slope height. Thus, for the worst case situation, the maximum water pressure is assumed to be

$$u_{max} = \gamma_w \cdot H/2$$

and the resultant water pressure

$$U_{max} = \gamma_w \cdot H^2/4 \sin\psi_p$$

Due to the inhomogeneous and discontinuous character of rock masses, this idealized triangular distribution seldom corresponds perfectly with the real situation during heavy rainfall. However, in the absence of better alternatives, this is the configuration most commonly seen in literature. The author's own experience is that the triangular distribution often exaggerates the resultant pressure, because joints and cracks frequently provide a degree of drainage towards the slope face.

Due to irregularities, a distinct non-linear relationship normally exists between shear strength (τ) and the normal stress imposed on rock joints (σ_n). In this example the so-called active friction angle (φ_a) is used to define discontinuity shear strength:

$$\tau = \sigma_n \cdot \tan\varphi_a$$

As φ_a is not a constant but highly dependent on normal stress, it is crucial to adjust the friction parameters to the actual normal stress level. If this is not done, serious calculation and design errors may result (see Nilsen 1985). To quantify φ_a, the following empirical equation developed by Barton and Bandis (1990) is used here:

$$\tau = \sigma_n \cdot \tan[\mathrm{JRC} \cdot \log(\mathrm{JCS}/\sigma_n) + \varphi_b]$$

where JRC is joint roughness coefficient, JCS is joint compressive strength, and φ_b is basic friction angle. For the road-cut case, based on field observations and laboratory testing and taking into account the scale effects as described by Barton and Bandis, the following values of the respective parameters are: JRC = 9, JCS = 110 MPa, and $\varphi_b = 35.5°$.

Although central Norway is basically a low seismicity region, minor earthquakes are occasionally experienced and, from a safety point of view, it is recommended that seismicity is taken into consideration for critical structures (Dahle et al. 1992). Earthquake load is modelled here as an equivalent horizontal load, generally representing the most unfavorable condition for stability (pseudo-static principle). In the example, maximum possible seismic acceleration during the lifetime of the project is assumed to be

$$\alpha_{max} = 0.1\, g$$

This corresponds to a maximum earthquake load of

$$F_\alpha = 0.1\ W$$

Traditional factor of safety analysis

The basic principles of traditional, deterministic analysis are to calculate the stabilizing and driving forces and arrive at a factor of safety (F). For the situation in Fig. 1:

$$F = (W\cos\psi_p - U - F_\alpha \sin\psi_p)\tan\varphi_a/(W\sin\psi_p + F_\alpha\cos\psi_p)$$

As φ_a is not a constant but a function of σ_n, the latter has to be calculated in each individual case to define φ_a:

$$\sigma_n = (W\cos\psi_p - U - F_\alpha\sin\psi_p)/l$$

where:

$$l = H/\sin\psi_p = 32.9\ \mathrm{m} = \text{length of potential failure plane}$$

The first four columns of Table 1 give the results of the factor of safety calculation for the basic conditions in the example in Fig. 1. As shown, for a dry slope and no earthquake (best case) F is 1.88, while for a combination of maximum earthquake and maximum water pressure (worst case) the factor of safety reduces to 1.02. For maximum earthquake but no water pressure F is 1.59,

Table 1
Factor of safety calculation (figures for the probabilistic approach are mean values after truncation)

Approach	Deterministic				Probabilistic
Situation	Worst case	Best case	Earthquake/ no water	Water/no earthquake	
U (kN/m)	1809	0	0	1809	508
α (fraction of g)	0.1	0	0.1	0	0.03
F_α (kN/m)	639	0	639	0	179
σ_n (kN/m^2)	76	144	131	89	125
σ_n BestFit (kN/m^2)					124
φ_a (deg)	62.7	59.4	60.2	61.3	61
F	1.02	1.88	1.59	1.26	1.68

while for maximum water pressure but no earthquake it is 1.26.

The partial factor method

According to the partial factor method, partial factors for actions and materials are to be applied instead of an overall safety factor. In principle, the calculation is carried out as follows:

$F_d = F_k \cdot \gamma_f$

$M_d = M_k / \gamma_m$

where F_d is dimensioning action and M_d is dimensioning strength; F_k is characteristic action and M_k is characteristic strength; γ_f is partial factor for action and γ_m is material factor. The design is considered to be satisfactory if:

$M_d > F_d$

In terms of slope stability, this means: if stabilizing forces are greater than driving forces.
Guidelines for defining partial factors are described by the Comité Européen de Normalisation (1994) and Norwegian Council for Building Standardization (1997). For the example:

W, U: $\gamma_f = 1.0$
F_α: $\gamma_f = 1.3$
tan φ_a: $\gamma_m = 1.2$

Table 2 presents the results of the stability analysis using the partial factor method for the same basic cases as for the factor of safety approach. As can be seen, according to this concept the stabilizing forces are greater than the driving forces for all situations except for the worst case.

Probabilistic analysis

For the road-cut example, the parameters related to geometry and the unit weights are unambiguously defined and in the analysis represent constants. Other input parameters, i.e. water pressure, seismic acceleration and friction angle, may, however, vary within wide limits. Thus, a probabilistic approach, with probability distributions assigned to those parameters, has obvious advantages.
The following probabilistic analyses are based on the computer programs BestFit and @RISK developed by the Palisade Corporation (1996, 1997) and inspired by analyses described by Hoek (1998). In the probabilistic analysis, the considerations and quantifications of the respective variables are as follows:

Water pressure (U)

The build-up of water pressure is assumed to be according to the triangular distribution described earlier. However, during the lifetime of the slope, it is unlikely that the water pressure will equate to the maximum value given by the equation in the figure. The most common situation will be a practically dry slope, i.e. $U \sim 0$. The most realistic probabilistic model of water pressure is believed to be a truncated exponential function, with truncation represented by

Table 2
Stabilizing and driving forces for the road-cut example based on the partial factor method

Situation	Worst case	Best case	Earthquake/ no water	Water/no earthquake
$F_\alpha \cdot \gamma_f$ (kN/m)	830	0	830	0
σ_n (kN/m^2)	72	144	127	89
φ_a (deg)	62.2	59.4	59.9	61.3
Stabilizing forces (kN/m)	3763	6683	6021	4469
Driving forces (kN/m)	4890	4272	4890	4272
Stabilizing forces/driving forces	0.77	1.56	1.23	1.05

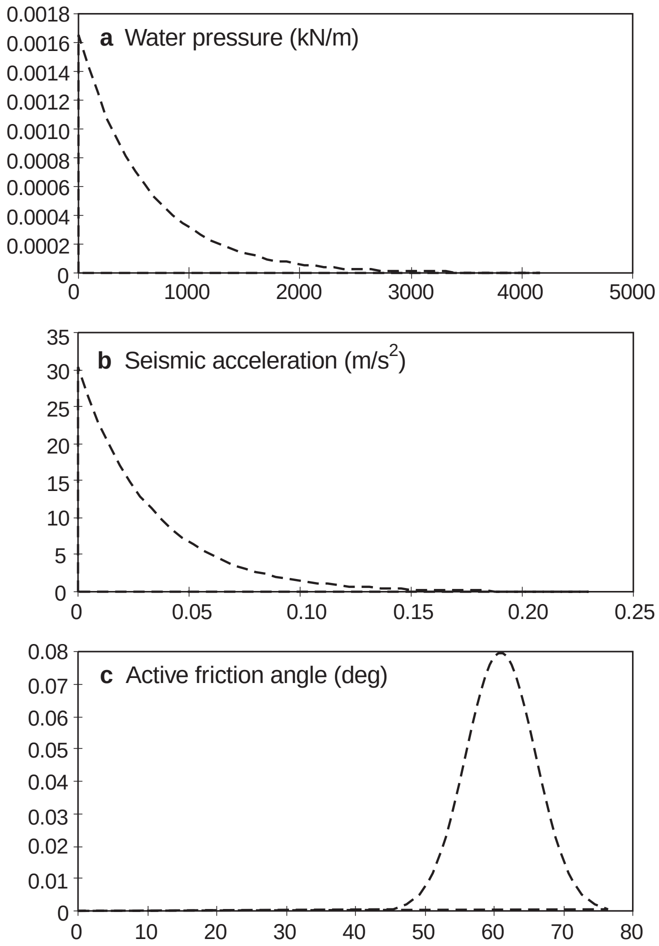


Fig. 2
Assumed probability distributions of **a** water pressure (U), **b** seismic acceleration (α_α) and **c** active friction angle (φ_a)

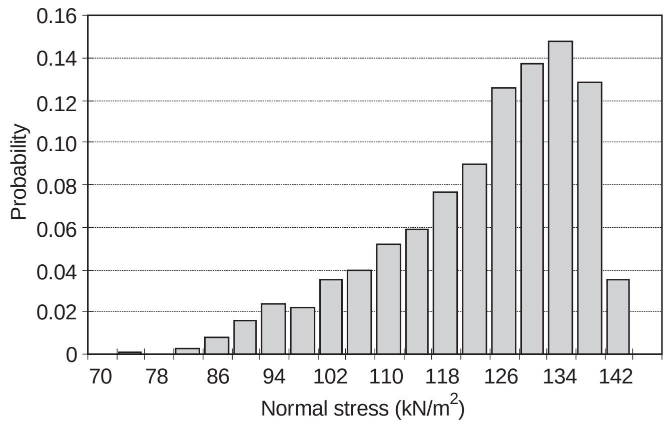


Fig. 3
Probability distribution of σ_n as calculated by @RISK based on 2000 iterations

the U_{max} value defined earlier and the mean value (probably somewhat conservatively) defined as $U_{max}/3$ (see Fig. 2a).

Seismic acceleration (α)

Most realistically, earthquake activity is modelled by an exponential distribution indicating that large earthquakes are very rare while small ones are common (Hoek 1998). In the present example, the maximum seismic acceleration during the lifetime of the slope is assumed to be $\alpha_{max} = 0.1\,g$ and the mean value $\alpha_{max}/3$. The resulting exponential distribution of α is shown in Fig. 2b; in the calculation the distribution is truncated according to $\alpha_{max} = 0.1\,g$.

Active friction angle (φ_a)

The input parameter of active friction angle is generally associated with a considerable amount of uncertainty and, as discussed earlier, it is also a function of the normal stress. For the present example, based on the rock mass data and calculated normal stresses (Table 1), a mean φ_a value of 61° is assumed and a truncated normal distribution of φ_a with a standard deviation of 5° within the actual normal stress range (between the "worst case" and "best case" in Table 1; see also Fig. 2c). Truncation is taken to be to the likely highest and lowest realistic φ_a values – 76 and 46° respectively.

Generally, most probabilistic analyses are based on mutually independent input parameters. In this case, however, two of the input parameters, φ_a and σ_n, are distinctly interrelated and, as a consequence, a two-step calculation procedure has to be followed. Step 1 of this procedure is the calculation and definition of the σ_n distribution. The calculation is carried out by the program @RISK (Palisade Corporation 1996). The result, based on Latin Hypercube sampling (a technique giving comparable results to the Monte Carlo technique but with fewer samples) and 2000 iterations, is shown in Fig. 3. According to BestFit, it can be described as a beta distribution with parameters $\alpha_1 = 3.12/\alpha_2 = 1.0$. Step 2 is an @RISK calculation of the factor of safety (F). Here, φ_a and σ_n are treated as interdependent variables with a dependency coefficient of −0.9 (representing a negative correlation during sampling, i.e. high values of φ_a are selected for low values of σ_n and vice versa). The result, based on Latin Hypercube sampling and 2000 iterations, is shown in Fig. 4.

According to the probabilistic approach, as illustrated by Fig. 4, the probability of failure of the actual slope is:

$$P(\text{failure}) = P(F < 1.0) = 0.015$$

This indicates that during the lifetime of the slope and for the assumed combinations of water pressure, seismic acceleration and friction, the probability of failure is 1.5%. Alternatively, the result may be interpreted as indicating that for the specified conditions, two out of a hundred slopes could be expected to fail.

As can be seen from Fig. 4, the probability distribution closely resembles a normal distribution. The mean value of F is 1.75 and the standard deviation is 0.43 (for a standard distribution indicating that 68% of the 2000 calculated F-values are between 1.32 and 2.18).

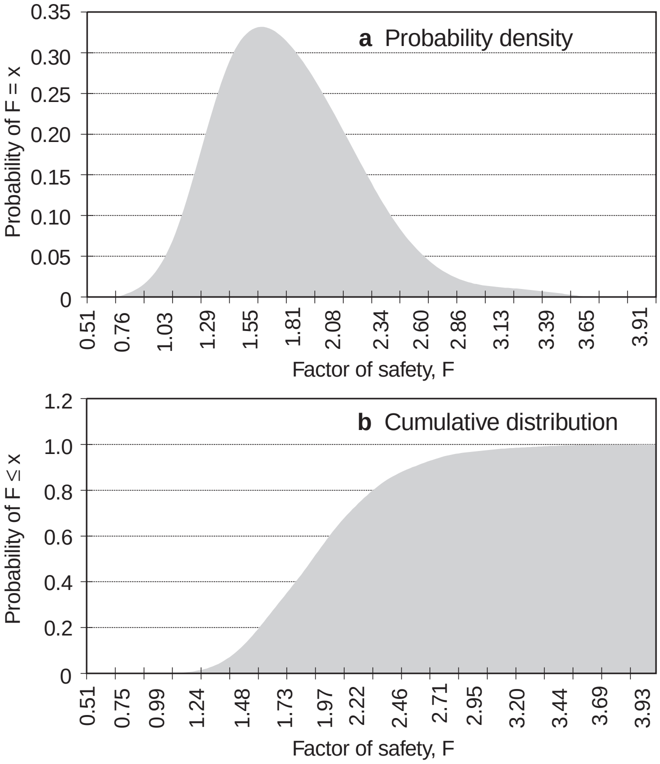


Fig. 4
a Probability density function and **b** cumulative distribution function of the factor of safety (*F*) for the road-cut example

Interpretation of results

The various calculation methods all give very distinct results. However, there is a considerable degree of uncertainty connected with the interpretation of the calculated values, i.e. what is the consequence of the calculated value in terms of stability and what value is required to be on the safe side?

Deterministic approach

According to its definition, a safety factor of $F > 1.0$ means stabilizing forces are greater than sliding forces and hence the slope should be stable. As there is always some degree of uncertainty connected to the input parameters, however, this may not necessarily be the case. To take the uncertainty into account and in order to allow for the different stability requirements of different types of structures, the following criteria for stability are often used: short-term stability (e.g. temporary slopes in an open pit mine), $F \geq 1.3$; long-term stability (e.g. permanent mine slopes or road cuts), $F \geq 1.5$

The factor of safety concept is easy to understand and has the advantage of having been standard procedure for a long time. In comparison, the partial factor method may be argued to give better control of the calculation by assigning partial factors to actions and materials. The available standards provide very concise descriptions on how to quantify the factors and what is to be considered as stable. However, the standards, such as Eurocode 7 and NS 3480, are intended mainly for the design of buildings and civil engineering works and make no clear distinction between different categories of structures or short- and long-term stability.

As a result of the basic differences, the partial factor method normally tends to give a more conservative design than when the approach uses the deterministic factor of safety. As shown by the example calculations of a road cut given in this paper, however, this is not always the case. The calculated stability according to the guidelines described here is satisfactory for two of the four cases when based on the safety factor approach and for three of the four cases when the partial factor method is used (see Tables 1 and 2).

It is important to be aware that, due to the uncertainty of the input parameters, even $F \gg 1.0$ (the calculated stabilizing forces much greater than the destabilizing forces) does not necessarily mean that the probability of sliding is equal to zero. Hence, if not fully understood, the deterministic approach may sometimes give a false impression of safety.

The final conclusion, i.e. the decision whether the calculated values represent a satisfactory level of safety or not, is not simple to make. In the case discussed in this paper, as earthquake activity is rare and simultaneous heavy rainfall and maximum earthquake conditions are very unlikely, it is realistic to assume the stability is satisfactory.

Probabilistic approach

To interpret the result of the probabilistic approach, i.e. to determine what probability of failure can be accepted, is often difficult. Guidelines do, however, exist. For example, the Norwegian national guidelines for buildings and civil engineering works in potential slide areas are shown in Table 3. According to the guidelines, a structure is considered to be safe when the probability of slide is lower than the respective limit given in the table.

For the road-cut example, with a $P(F < 1.0) = 0.015$ and an assumed structure lifetime of 50 years, the annual probability of slide would be 3×10^{-4}. According to the criteria in Table 3, this would satisfy the stability requirement of a structure in safety class 2, including roads of the category discussed here.

Table 3
Criteria regarding localisation of structures in potential slide areas according to Statens Bygningstekniske Etat (1995)

Safety class	Consequence of slide	Max. annual probability of slide
1	Minor	10^{-2}
2	Medium	10^{-3}
3	Major	$< 10^{-4}$

Conclusions

As illustrated by the calculation examples in this paper, rock slope stability analysis and particularly interpretation of the results is difficult even for simple slope geometry. In addition, different calculation principles give considerably different results. To conform with new standards and recommendations such as Eurocode 7, traditional deterministic analysis based on the calculation of one factor of safety is to be replaced by the partial factor method, involving partial factors on actions and material strength. The principle of considering individual factors obviously has certain advantages. However, the available standards and recommendations concerning interpretation of the results are at present largely limited to buildings and civil engineering projects. Therefore, in fields such as mining and dam foundation, for instance, it is likely that for some time to come the traditional factor of safety method will continue to be the most commonly used.

Due to the uncertainty of input parameters, including their variability, the probabilistic approach for stability analysis has obvious advantages. In addition, safety criteria are sometimes based on given probability limits. In important rock slope stability analyses, the option to include the probabilistic approach as a supplement to more routine deterministic analyses should always be considered.

It is hoped to further investigate the applicability of probabilistic methods and particularly the interpretation of calculation results during a research project to be initiated at the Norwegian University of Science and Technology.

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