Towards High Order Numerical Simulation of Aeolian Tones

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Strictly stable high order finite difference operators have been applied to the compressible Navier-Stokes equations in perturbation form for low Mach number computational aeroacoustics. Aeolian tones generated by vortex shedding from a circular cylinder have been simulated.

1 Introduction

High order finite difference methods have been developed, which are constructed to be strictly stable for linear hyperbolic and parabolic problems [1][2]. These methods satisfy the summation by parts (SBP) property, which is analogous to integration by parts and essential to obtain energy estimates. We have been using and extending the strictly stable high order difference approximations for the linearized and nonlinear Euler equations in Computational Aeroacoustics (CAA) [3] [4] [5]. We have been employing the perturbation form of the nonlinear Euler equations, which directly yields the linearized Euler equations, if the nonlinear terms are neglected. Cancellation errors are minimized for low Mach number CAA, if the perturbation form instead of the standard form of the conservation laws is discretized. Recently, we have extended the numerical approach to the nonlinear Navier-Stokes equations in perturbation form to simulate aeolian tones generated by vortex shedding from a circular cylinder at $Re_\infty = 150$, $M_\infty = 0.1$ and 0.2. The perturbation form of the Navier-Stokes equations is presented in section 2. In section 3, the high order difference method based on SBP operators is outlined. Results for aeolian tone simulations are presented in section 4 followed by conclusions in section 5.

2 Navier-Stokes equations in perturbation form

The perturbation formulation is used to minimize cancellation errors when discretizing the Navier-Stokes equations for compressible low Mach number flow. The 2D compressible Navier-Stokes equations in perturbation form read [6]

$$\mathbf{U}'_t + \mathbf{F}'_x + \mathbf{G}'_y = \mathbf{F}^{v\prime}{}_x + \mathbf{G}^{v\prime}{}_y \,, \tag{1}$$

where

$$\mathbf{U}' = \begin{pmatrix} \rho' \\ (\rho u)' \\ (\rho v)' \\ (\rho E)' \end{pmatrix}, \ \mathbf{F}' = \begin{pmatrix} (\rho u)' \\ (\rho u)'u' + p' \\ (\rho v)'u' \\ (\rho_\infty H_\infty + (\rho H)')u' \end{pmatrix}, \ \mathbf{G}' = \begin{pmatrix} (\rho v)' \\ (\rho u)'v' \\ (\rho v)'v' + p' \\ (\rho_\infty H_\infty + (\rho H)')v' \end{pmatrix}$$

with the perturbation variables (superscript $'$) with respect to the freestream values (subscript $_\infty$) $\rho' = \rho - \rho_\infty, (\rho\mathbf{u})' = \rho\mathbf{u}, (\rho E)' = \rho E - (\rho E)_\infty \,, (\rho H)' = (\rho E)' + p' \,, \mathbf{u}' = \frac{(\rho\mathbf{u})'}{\rho_\infty + \rho'}, p' = (\gamma - 1)[(\rho E)' - \frac{1}{2}((\rho\mathbf{u})' \cdot \mathbf{u}')] \,.$ t is time, and x and y are the Cartesian coordinates, respectively. ρ denotes the density, u and v the x- and y-direction velocities, E the specific total energy, and p the pressure. ρ_∞, $(\rho E)_\infty$ and $(\rho H)_\infty$ denote the uniform freestream quantities of density, total energy density and total enthalpy density, respectively. The viscous flux vectors $\mathbf{F}^{v\prime}$ and $\mathbf{G}^{v\prime}$ are the same as for the standard conservative form, except for using the temperature perturbation instead of temperature for the heat flux terms. General geometries are treated by a coordinate transformation $x = x(\xi, \eta)$, $y = y(\xi, \eta)$.

3 High order finite difference method

The finite difference methods considered here employ high order central approximations in the interior and special boundary stencils to satisfy the summation by parts (SBP) property leading to discrete energy estimates. The SBP operator with sixth order accuracy in the interior and third order accuracy near the boundaries by B. Strand [2] is used to approximate the spatial first derivatives in the transformed 2D Navier-Stokes equations. Second derivatives are computed by applying the SBP

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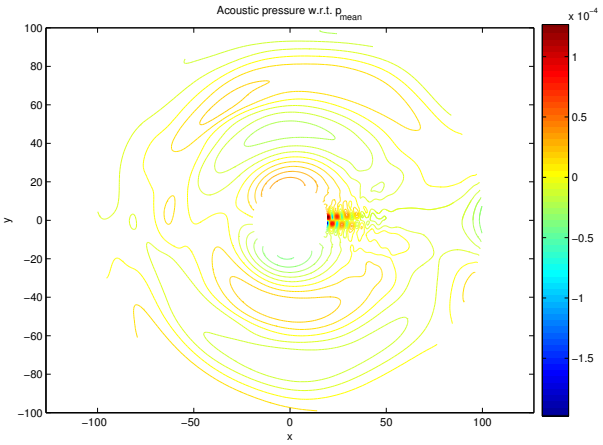

Fig. 1 Instantaneous fluctuation pressure for $M_\infty = 0.1$, $Re_\infty = 150$.

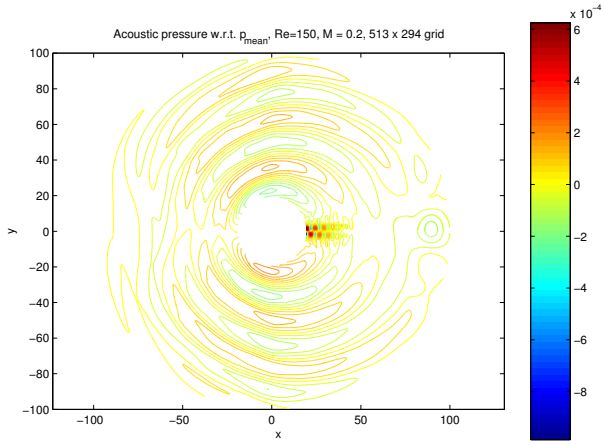

Fig. 2 Instantaneous fluctuation pressure for $M_\infty = 0.2$, $Re_\infty = 150$.

operator for the first derivatives twice. High wave number oscillations are damped by a sixth order explicit filter. The explicit fourth order Runge-Kutta method is used for time integration. No-slip isothermal wall boundary conditions ($T_w = T_\infty$) and characteristic farfield boundary conditions (first approximation of Engquist-Majda nonreflecting boundary conditions) are imposed after each Runge-Kutta step. The wall pressure and the outgoing characteristic variables at the farfield are not extrapolated, but obtained from the Navier-Stokes equations solved at the boundary. For $M_\infty = 0.2$, a sponge layer in a radial strip ($83.5 \leq r \leq 100$) near the farfield boundary at $r = 100$ has been used. A 513×294 grid ($r \times \theta$) O-grid is employed.

4 Results of aeolian tone simulations

The freestream Mach numbers $M_\infty = 0.1$ and $M_\infty = 0.2$ are considered. For the Reynolds number $Re_\infty = \frac{\rho_\infty u_\infty d}{\mu_\infty} = 150$ based on the cylinder diameter d, a von Karman vortex street is observed. Each time a vortex is shed from the cylinder, a sound wave is emitted. The frequency f of the vortex shedding and the periodic variation of lift and drag have been found in excellent agreement with a reference solution [7]. For the sound field, nonreflecting farfield boundary conditions proved to be decisive. The instantaneous pressure fluctuations $\tilde{p}' = p'(x, y, t) - \bar{p}'(x, y)$, where $\bar{p}'(x, y)$ is the mean pressure perturbation, in Fig.s 1 and 2 are qualitatively and quantitatively in good agreement with [7]. Note that only the region between a radius of about $r = 20$ and the farfield boundary at $r = 100$ is plotted. Since the Strouhal number is essentially equal for $M_\infty = 0.1$ and $M_\infty = 0.2$, namely $St = \frac{fd}{u_\infty} = 0.183$ for $Re_\infty = 150$, and because the sound waves are generated by vortex shedding, the wave length of the acoustic waves seen in Fig. 2 for $M_\infty = 0.2$ is essentially halved compared to Fig. 1 for $M_\infty = 0.1$. The amplitudes of the instantaneous pressure fluctuations $\tilde{p}'$ scale with $M_\infty^{2.5} r^{-0.5}$ as found in [7]. After this verification of the high order difference method, the direct numerical simulation and large eddy simulation of aeolian tones at higher Reynolds numbers can be attacked in the future.

5 Conclusions

The 2D compressible Navier-Stokes equations in perturbation form are solved by a strictly stable fourth order difference method to simulate the generation of aeolian tones by uniform flow over a circular cylinder at $Re_\infty = 150$, $M_\infty = 0.1$ and 0.2. The Strouhal number, drag and lift coefficients and the instantaneous pressure fluctuations with respect to the mean pressure are in good agreement with a reference solution.

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