NGSSC CFD Course Part 2
Compressible Fluid Flow

21 August 2003

V. Methods for Euler and Navier-Stokes Equations (B. Müller)

Testcase Inviscid Flow over a Parabolic Arc Airfoil

Objective

To check the Euler solver for subsonic, transonic and supersonic inviscid flow over a parabolic arc airfoil by means of the analytical solution of the Prandtl-Glauert equation and by means of other numerical solutions.

Grid

61×21 grid, equidistant in x, $x_{min} = -1$, 20 points ahead of leading edge $x = 0$, 20 points behind trailing edge $x = 1$, $x_{max} = 2$, $y_{max} = 1$. Lower boundary given by $y_0(x) = max(0, h(x))$, where $h(x) = 2\tau(x - x^2),\ x \in [0,1]$, describes the upper side of the parabolic arc airfoil with thickness $\tau = 0.1$. Points in y-direction clustered near $y = y_0$ by

$$y_j \ = \ y_0 \ + \ (y_{max} - y_0)\left(1 + \beta\,\frac{1-\beta_j}{1+\beta_j}\right) \tag{1}$$

where $\beta_j = \left(\frac{\beta+1}{\beta-1}\right)^{(j-1)/(j_{max}-1)}$, $j = 1, 2, ..., j_{max}$, and $\beta = 1.1$.

Flow Cases

$M_\infty = 0.5$, 0.85 and 2 .
External flow over a symmetric airfoil at zero angle of attack; inviscid, steady, 2D.

Initial Conditions

$\mathbf{U} = \mathbf{U}_\infty$.

Boundary Conditions

Symmetry ($x < 0$, $y = 0$ and $x > 1$, $y = 0$): $\frac{\partial \rho}{\partial y} = \frac{\partial u}{\partial y} = \frac{\partial p}{\partial y} = v = 0$.
Wall ($x \in [0,1]$, $y = 2\tau(x - x^2)$): $\mathbf{u}\cdot\mathbf{n} = 0 = \frac{\partial p}{\partial n} = 0$.
Inflow ($x = -1$):
Outflow ($x = 2$):
Farfield ($y = 1$):

Time Integration

Local time stepping, $CFL = |RK| = 1.5$, $n_{max} = 2000$ for $M_\infty = 0.5$, $n_{max} = 1000$ for $M_\infty = 0.85$, $n_{max} = 500$ for $M_\infty = 2$.

Exercises

1. Which boundary conditions are used at inflow, outflow, and farfield? How are they implemented?

2. Suppose you introduce two layers of ghost cells next to each boundary. Which values of the flow variables would you prescribe in the ghost cells to satisfy the boundary conditions? (optional)

3. (a) Generate a grid with the code `bump.f`.

   (b) Compute the present flow case by running the code `navier_fvm.f`.

   (c) Visualize the results in MATLAB. You may use

   `plot1d.m` to plot pressure coefficient c_p in cells $J = 1$, Mach number M in cells $J = 1$, temperature T in cells $J = 1$, convergence history of residuals,

   `plot2d.m` to plot Mach number contours,

   `plot2dc.m` to plot density, velocity (u and v) and pressure contours,

   `plot2ds.m` to plot density, velocity (u and v) and pressure surface plots.

4. Compute the drag coefficient

$$c_D = 2\frac{1}{L}\int_0^L c_p \frac{dh}{dx}\,dx,$$

   with the nondimensional length $L = 1$. Note that the factor 2 accounts for the pressure force on the lower side of the symmetric airfoil at zero angle of attack. $c_p = \frac{p_w - p_\infty}{\frac{1}{2}\rho_\infty u_\infty^2}$ is the wall pressure coefficient.
   Compare with the result $c_D = 0$, if $M_\infty < 1$ and $c_D = \frac{16}{3}\frac{\tau^2}{\sqrt{M_\infty^2 - 1}}$, if $M_\infty > 1$ of the Prandtl-Glauert theory.

5. Plot $\frac{u - u_\infty}{u_\infty}$ in the cells $J = 1$ as a function of $\frac{x}{L}$, and compare with the Prandtl-Glauert theory

$$\frac{u(x,0) - u_\infty}{u_\infty} = \frac{4}{\pi}\frac{\tau}{\sqrt{1 - M_\infty^2}}\left(1 - \left(\frac{1}{2} - x\right) ln\left|\frac{1-x}{x}\right|\right)\,,\ 0 \neq x \neq 1 \quad if\ M_\infty < 1\,,$$

$$\frac{u(x,0) - u_\infty}{u_\infty} = \left\{\begin{array}{cl} -2\frac{\tau}{\sqrt{M_\infty^2 - 1}}(1 - 2x) & :\ x \in [0,1] \\ 0 & :\ x \notin [0,1] \end{array}\right\}\,, \quad if\ M_\infty > 1\,.$$

6. Plot $\frac{v}{u_\infty}$ in the cells $J = 1$ as a function of $\frac{x}{L}$, and compare with the Prandtl-Glauert theory

$$\frac{v(x,0)}{u_\infty} = \left\{\begin{array}{cl} \frac{dh}{dx} = 2\tau(1 - 2x) & :\ x \in [0,1] \\ 0 & :\ x \notin [0,1] \end{array}\right\}\,.$$

7. Characterize the subsonic, transonic and supersonic flow cases. Describe the numerical difficulties in computing each of them. Try to find the limits of the code by running it at a very low and a very large Mach number. Does the code give results, and are they reliable?

8. Vary the grid, and check the grid dependence of the results. Give recommendations on the grid (distance of boundaries from airfoil, clustering, number of grid points) to obtain grid converged results.