NGSSC CFD Course Part 2
Compressible Fluid Flow

21 August 2003

V. Methods for Euler and Navier-Stokes Equations (B. Müller)

Testcase Boundary Layer on a Flat Plate

Objective

To check the Navier-Stokes solver for a laminar boundary layer on a flat plate by means of the Blasius similarity solution.

Grid

46×21 grid, equidistant in x, 15 points ahead of leading edge $x = 0$, $x_{max} = 1$, $y_{max} = 1$, $\beta = 1.01$. Points in y-direction clustered near $y = 0$ by

$$y_j \;=\; y_{max} \left(1 + \beta \, \frac{1 - \beta_j}{1 + \beta_j} \right) \tag{1}$$

where $\beta_j = \left(\frac{\beta+1}{\beta-1} \right)^{(j-1)/(j_{max}-1)}$, $j = 1, 2, ..., j_{max}$.

Flow Case

$M_\infty = 0.5$, $Re_\infty = 2000$, $Pr = 0.72 = constant$.
External flow over flat plate, laminar, steady, 2D.

Initial Conditions

$\mathbf{U} = \mathbf{U}_\infty$.

Boundary Conditions

Symmetry ($x < 0$, $y = 0$): $\frac{\partial \rho}{\partial y} = \frac{\partial u}{\partial y} = \frac{\partial p}{\partial y} = v = 0$.
Adiabatic wall ($x \geq 0$, $y = 0$): $u = v = \frac{\partial T}{\partial y} = \frac{\partial p}{\partial y} = 0$.
Inflow ($x = -0.5$):
Outflow ($x = 1$):
Farfield ($y = 1$):

Time Integration

Local time stepping, $CFL = |RK| = 1$, $n_{max} = 1500$.

Exercises

1. Which boundary conditions are used at inflow, outflow, and farfield? How are they implemented?

2. Suppose you introduce two layers of ghost cells next to each boundary. Which values of the flow variables would you prescribe in the ghost cells to satisfy the boundary conditions? (optional)

3. (a) Generate a grid with the code `bump.f`.
   (b) Compute the present flow case by running the code `navier_fvm.f`.
   (c) Visualize the results in MATLAB. You may use

   `plot1d.m` to plot pressure coefficient c_p in cells $J = 1$, skin friction coefficient c_f, temperature T in cells $J = 1$, convergence history of residuals,

   `plot2d.m` to plot Mach number contours,

   `plot2dc.m` to plot density, velocity (u and v) and pressure contours,

   `plot2ds.m` to plot density, velocity (u and v) and pressure surface plots.

4. Compute the drag coefficient
$$c_D = \frac{1}{L}\int_0^L c_f\,dx,$$
   with the nondimensional length $L = 1$.
   Compare with the result $c_D = \frac{1.33}{\sqrt{Re_\infty}}$ of the boundary layer theory.

5. Plot $\frac{u}{u_\infty}$ as a function of $\eta = \frac{y}{L}\sqrt{Re_\infty \frac{L}{x}}$ for a few x-stations on the flat plate.

6. Plot $\frac{v}{u_\infty}\sqrt{Re_\infty \frac{x}{L}}$ as a function of $\eta = \frac{y}{L}\sqrt{Re_\infty \frac{L}{x}}$ for a few x-stations on the flat plate.

7. Compare the computed velocity components with the Blasius similarity solution for an incompressible boundary layer on a flat plate. You may plot the Blasius solution with the MATLAB file `blasius_plot.m`. In the two plots, the Blasius velocity components are scaled as in 5. and 6., respectively.

8. Vary the grid, and check the grid dependence of the results. Give recommendations on the grid (distance of boundaries from flat plate, clustering, number of grid points) to obtain grid converged results.