NGSSC CFD Course Part 2
Compressible Fluid Flow

18 August 2003

II. Governing Equations (B. Müller)

Exercise 1

Objective

To get familiar with the Navier-Stokes equations and train nondimensionalization.

a) The 2D compressible Navier-Stokes-Equations read in differential conservative form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{1}$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + p)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} = \frac{\partial(2\mu\frac{\partial u}{\partial x} - \frac{2}{3}\mu(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}))}{\partial x} + \frac{\partial(\mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}))}{\partial y} \tag{2}$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2 + p)}{\partial y} = \frac{\partial(\mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}))}{\partial x} + \frac{\partial(2\mu\frac{\partial v}{\partial y} - \frac{2}{3}\mu(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}))}{\partial y} \tag{3}$$

$$\begin{aligned}\frac{\partial(\rho E)}{\partial t} + \frac{\partial(\rho uH)}{\partial x} + \frac{\partial(\rho vH)}{\partial y} \quad = \quad & \frac{\partial[u(2\mu\frac{\partial u}{\partial x} - \frac{2}{3}\mu(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})) + v(\mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})) + k\frac{\partial T}{\partial x}]}{\partial x} + \\ & \frac{\partial[u(\mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})) + v(2\mu\frac{\partial v}{\partial y} - \frac{2}{3}\mu(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})) + k\frac{\partial T}{\partial y}]}{\partial y}\end{aligned} \tag{4}$$

Nondimensionalize the equations either for external flow over an airfoil with chord length L using uniform flow reference conditions or for internal flow over a turbine blade with chord length L using stagnation conditions as reference conditions.

Hint:
Insert $x = Lx^*$, $u = u_\infty u^*$, $\rho = \rho_\infty \rho^*$, etc., cf. II., 1.7, pp. 20-21, and simplify.

b) Nondimensionalize the equations of state for perfect gas

$$p = \rho RT \tag{5}$$

and

$$e = c_v T \tag{6}$$

with the same reference conditions as in a).

Hint:
$c^2 = \gamma RT$, where c is the speed of sound and $R = c_p - c_v$ the specific gas constant.