Stabilization of Gas-Lift Wells by Feedback Control

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Summary

Stabilization of casing-heading instability from gas-lift wells is important since it increases production and reduces operational problems. This thesis is a contribution within feedback control stabilization of gas-lift wells with casing-heading dynamics.

The main advantage of stabilizing the gas-lift well by feedback control, compared to traditional methods, is the limited production loss associated with feedback control stabilization. Two types of control structures are investigated in this thesis. The first type is control structures utilizing direct measurement of the process variable to be stabilized. The second type is control structures utilizing observers for estimation of the desired process variable.

Six different control structures for stabilization of the single gas-lift well have been investigated. All six control structures were able to stabilize the gas-lift well. These control structures have been investigated by simulation studies and thereafter verified by experiments in a laboratory scale gas-lift well. A simplified model of the single gas-lift well has been applied for analysis of the open-loop and closed-loop system. Two of these control structures are novel structures. The first control structure controls the pressure drop across a restriction upstream the production choke by manipulating the opening of the production choke. The second controls an estimate of the downhole pressure of the well by manipulating the opening of the production choke. The results from stabilizing the gas-lift well, both in simulation studies and in laboratory experiments, showed an increased production rate from the well.

The author has also presented a control structure for gas distribution in a dual gas-lift well. A simplified model of the dual gas-lift well has been developed and applied for simulations and analysis of the system. Simulation studies and laboratory experiments have been performed to verify the control structure. Both in the simulation studies and in the laboratory experiments an increased production rate from the dual gas-lift well was obtained.

Preface

This thesis is based on research carried out in the period July 2000 through January 2006, primarily at the Norwegian University of Science and Technology, partly at Shell International Exploration and Production, Rijswijk, the Netherlands and partly at Delft University of Technology, the Netherlands. Funding was provided by the Norwegian Research Council, ABB AS, and Norsk Hydro ASA; through the Petronics Program. The title of the research program was: "*Optimized production and automatic control of oil wells and pipelines*". The main objective of this research program was to develop innovative technological solutions for automatic control of oil wells and optimization of oil production through cooperation between different specialized fields. Knowledge within system theory and control in combination with multiphase flow technology and production technology were the key fields of the research program.

This thesis consists of an introductory part, followed by a collection of papers that have been published, are pending publication in scientific journals or have been presented at international conferences. In the introductory part, the background and motivation for the research is presented, along with a summary of the main findings reported in the papers.

Acknowledgements

First of all I would like to thank my family for their support during these years. Without their support this work would not have been possible. Thanks to Otto Eikrem, Anne-Karin Bergmann Eikrem, Erlend Eikrem, Pål Are Eikrem and Renate Nyborg.

I want to thank my supervisor Bjarne A. Foss for his support and patience during these years. He has pointed out the right direction of this work, and arranged for this PhD work to become successful. I will especially thank my co-supervisor Ole Morten Aamo, and also Lars Imsland for their support during these years.

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I want to thank my current employer Statoil ASA, and especially Jan Richard Sagli and John-Morten Godhavn.

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Gisle Otto Eikrem
Trondheim, January 2006

Contents

Chapter 1

Introduction

1.1 Background

Production of oil and gas from offshore installations is associated with high investments and high production costs. To maximize the profit from an oil-producing installation, the production rate of oil must usually be kept as high as possible. The pressure difference between the producing reservoir and the receiving end puts an absolute upper bound on what can be obtained in terms of production rate. The properties of the production equipment may impose additional restrictions on the achievable rate. One limitation is the oil, gas, and water separation and handling capacity of the processing equipment at the receiving end. Another limitation is the ability of the well and transportation pipeline to deliver a stable multiphase flow rate. A method for dealing with instability in wells and pipelines is to introduce friction to the system until the flow becomes stable. This thesis focuses on the development of automatic control strategies for stabilization of wells without reducing production rate, and is therefore a contribution to the overall objective of maximizing profit.

In the early phase of their production lifetime, most oil wells flow naturally and are referred to as flowing wells. This means that the pressure in the well is sufficient to overcome the total pressure loss due to frictional and gravitational forces along the flow path to the separator. As the reservoir is depleted, the pressure in the well decreases so that in the late phase of the production lifetime of the well, it ceases to flow naturally. To be able to produce from a well that does not flow naturally, or to increase production from a well that flows poorly, artificial lift needs to be introduced. Different methods for artificial lift are available, and they can be divided into two main categories: gas-lift, which is the topic of this work, and pumping. Information regarding the pumping technology can be found in Lea *et al.* (2003). The main idea in gas-lift technology is that by compressing gas at the surface and injecting it as deep as possible into the well, the density of the fluid decreases, thereby reducing the hydrostatic pressure loss along the flow path. Due to the reduced pressure drop in the well, the pressure in the bottom of the well becomes sufficiently low to continue production. Gas-lift is a widely applied technology. Shell International reports that 25% of their total oil production comes from gas-lift wells. Statoil ASA reports that ten Statoil operated fields will require the use of gas-lift in the near future. Further information regarding artificial lift can be found in Takács (2005) and Lea *et al.* (2003).

This work is concerned with two different well completions with gas-lift, referred to as single gas-lift well and dual gas-lift well, respectively. The single gas-lift well, which is the most common of the two, has one production tubing within its casing volume, as sketched in Figure 1.1, while the dual gas-lift well has two production tubings within its casing volume, as sketched in Figure 1.4. Lift-gas is routed from the surface to the tubings through the annulus volume, coupling the dynamics of the flows in the annulus and tubings. This coupling and the nature of multiphase flow cause the kind of instabilities which are studied in this thesis.

1.2 Multiphase Flow in Wells

The term multiphase flow is in this context used to refer to a flow of gas and liquid. The flow of gas and liquid in wells and pipelines can take many shapes. In vertical pipes one of the following two-phase flow regimes will be present; bubbly flow, dispersed flow, annular flow or slug/churn flow. In horizontal pipes one of the following flow regimes will be present; stratified flow, annular flow, slug flow or dispersed flow. The type of flow regime present is, among other factors, determined by the flow rates of the two phases, the properties of the fluids and the pipe geometry. More information regarding the flow regimes in multiphase flow can be found in Fuchs (1997), Taitel (2001), Oliemans (2001) and Brennen (2005).

The slug flow regime is the regime which presents the most significant challenges. Slug flow is defined as periodically varying rates of gas and liquid through a cross section. Slug flow may occur both at constant inlet and outlet conditions and during transient operations.

Different types of periodic slug flow are listed below:

- Hydrodynamic slugging
- Terrain induced slugging
- Casing-heading slugging
- Pipeline/riser (riser-foot) slugging

Terrain induced slugging, casing-heading slugging and pipeline/riser slugging are usually characterized as severe slug flow. Hydrodynamic slug flow is usually characterized by high frequency oscillations with a small amplitude, compared to the severe slug flow. Hydrodynamic slugging alone is usually not regarded as a problem for the production of oil and gas. Terrain induced slugging is hydrodynamic slug flow enhanced by the terrain. Slug flow becomes a challenge for the oil and gas industry when it exceeds the capacity of the receiving facility.

The two most frequently severe slugging phenomena are casing-heading slugging and pipeline/riser slugging. These two phenomena have significant similarities with respect to the dynamics of the fluid flow. For both systems the instability is caused by the buildup of a gas volume within the system. Both systems can be stabilized by the introduction of friction to a level where the pressure drop given by friction overcomes the driving force due to gas expansion during slug blowout. The main difference between these two systems is the location of the gas volume. For the gas-lift well the gas volume is given by the annulus volume, while for the pipeline/riser system the gas volume is a part of the upstream pipe volume.

Transient slug flow may be excited by operational events:

- Startup or blowdown of wells or pipelines
- Rate or pressure change in wells or pipelines
- Pigging of pipelines

Hence, this type of transient slug flow is caused by changes to the production system. The slug behavior vanishes soon after the new operating point is reached. This type of transient slug flow can not be removed, but can be minimized by slow process changes. More information regarding control of these types of slug flow can be found in Storkaas (2005) and Storkaas and Godhavn (2005).

Two-phase fluid flow has a complex nature. Several types of models exist for two-phase flow in pipes. The main model types are the homogeneous equilibrium model, the separated flow model and the two-fluid model. More information regarding these flow models can be found in Johnson (1998).

1.3 Dynamics of Gas-Lift Oil Wells

1.3.1 Single Gas-Lift Well

A single gas-lift well is a well completion where the borehole contains one production string with gas lift, see Figure 1.1. It consists of an annulus volume into which lift gas is injected through the gas-lift choke. An injection valve is located at the bottom of the annulus volume, allowing the gas to flow into the tubing. The annulus volume is closed at the bottom by the use of packers. As the gas is injected into the tubing volume it flows back to the wellhead and through the production choke together with the well fluid. The injected gas reduces the density of the produced fluid from the well, thereby reducing the pressure at the bottom of the well, referred to as the downhole pressure (DHP). The lower the downhole pressure of the well is, the higher the rate of produced fluid from the reservoir will be. The pressure at the top of the well, upstream the production choke, is referred to as the tubing head pressure (THP), while the pressure at the top of the annulus volume is referred to as the casing head pressure (CHP).

Under certain operating conditions the injected lift gas from the annulus into the tubing does not flow with a constant rate. Instead, it interacts with the well flow in a way that causes severe oscillations in the production rate. Figure 1.2 shows a typical time series of the production rate from such a well, obtained from the OLGA 2000 multiphase flow simulator. The production rate oscillates with a period of about 2 hours, and is characterized by a peak in production followed by a considerable time period of very low production. The solid line in Figure 1.3 shows the average production from a single gas-lift well as a function of choke opening. For small choke openings, the production is stable and increases with increasing opening. However, when the choke opening reaches 0.2, the flow becomes unstable and production drops significantly as the choke opening is increased further. The mechanism by which the oscillations come about is referred to as casing-heading instability, and can be described as follows:

1. Gas from the casing starts to flow into the tubing. As gas enters the tubing the pressure in the tubing falls. This accelerates the inflow of gas.

2. The gas pushes the major part of the liquid out of the tubing.

3. Liquid in the tubing generates a blocking constraint downstream the injection orifice. Hence, the tubing gets filled with liquid and the annulus with gas.

4. When the pressure upstream the injection orifice is able to overcome the pressure on the downstream side, a new cycle starts.

Onset of the casing-heading instability requires the presence of a gas volume and a gravity dominated pressure drop from this gas volume to the separator. More information regarding the casing-heading instability can be found in Torre *et al.* (1987), Asheim (1987), and Xu and Golan (1989).

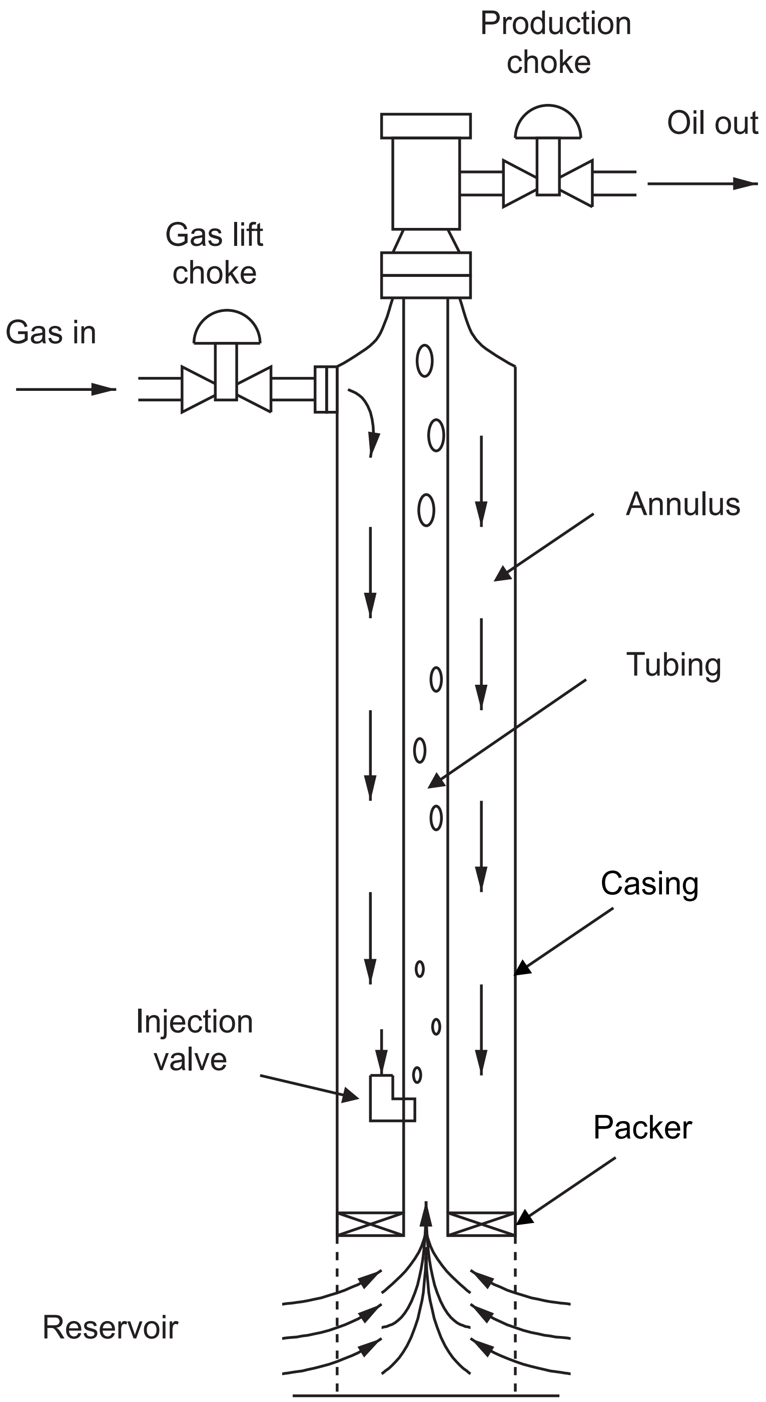

Figure 1.1: The single gas-lift well.

The oscillating flow occurring at large choke openings, due to the casing-heading instability, is highly undesirable because it reduces the total production and introduces significant disturbances to the downstream processing facility, which may give insufficient separation of gas, oil and water. The peak rates of gas and liquid may give liquid overflow in the separator, high pressure in the separator, or gas overload in the compressor train. In addition, the repeating impact from the oscillating fluid flow may cause fatigue upon the process equipment. Due to these problems, an unstable well can be choked back to prevent unstable behaviour, but this is a very expensive means of stabilization with respect to production loss. Another preferred method for stabilization is to increase the amount of lift-gas, if available. A significant "stability margin" is usually added by choking much more than strictly required, giving an even higher production loss. The dashed line in Figure 1.3 shows the potential production from a gas-lift well if the flow were stable at higher choke openings. The figure suggests that there is a large potential for increased profit in finding methods of stabilization that do not rely on excessive choking. This thesis considers automatic control as a solution to the problem.

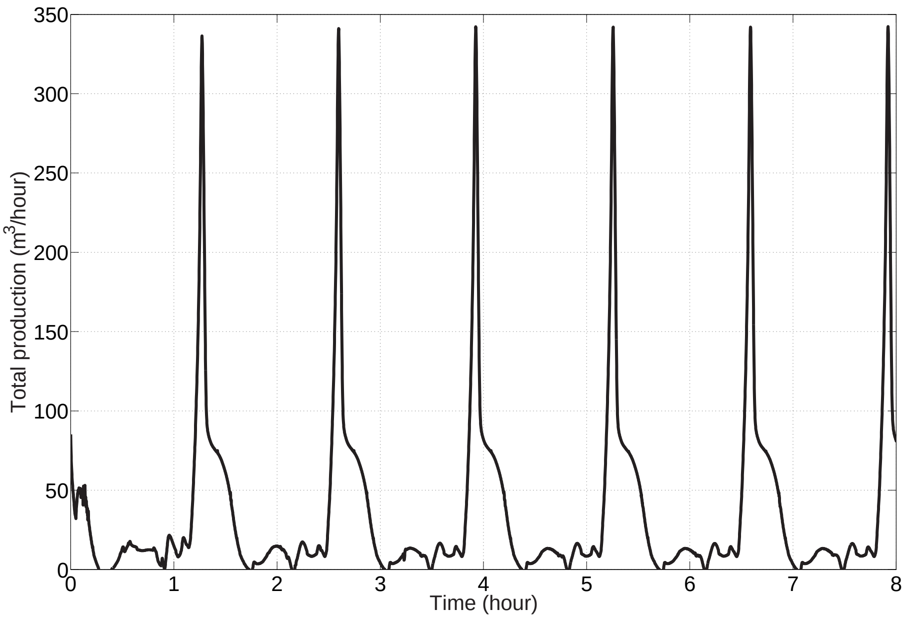

Figure 1.2: Typical production from an unstable gas-lift well.

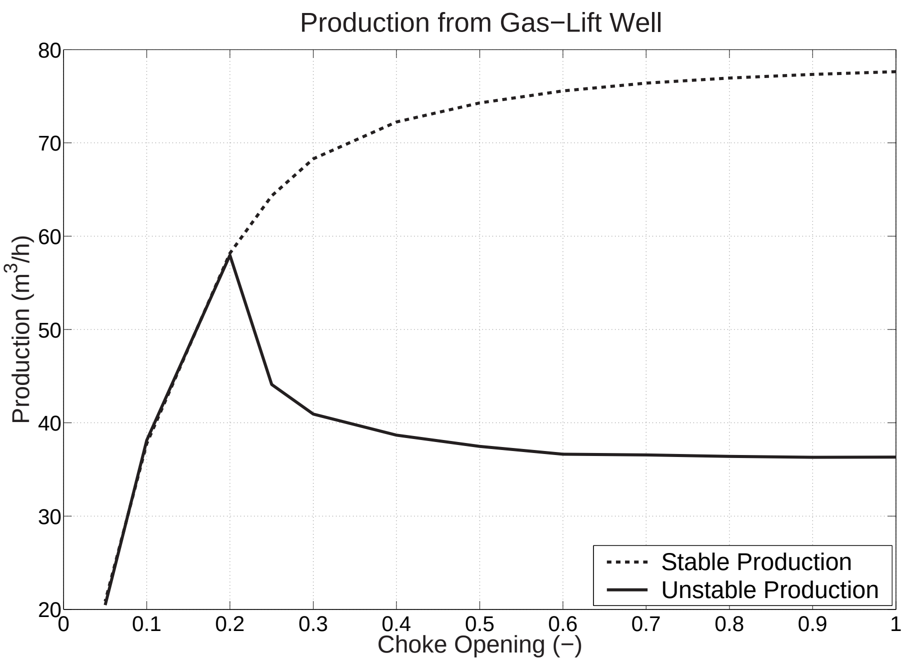

Figure 1.3: The production from an unstable gas-lift well compared to the theoretical available production.

1.3.2 Dual Gas-Lift Well

To reduce drilling and completion costs, several productive formations can be produced through a single borehole. This solution can be desirable if several formations are located vertically close above one another. Dual gas-lift is defined as the continuous production by gas lifting of two formations opened in the same well (Takács 2005). A drawing of a dual gas-lift well is given in Figure 1.4. When two formations are produced from one well, each zone should usually be produced independently from the other. Packers are applied to isolate the two formations, these packers prevent gas injection at points further down in the well. The two production strings share a common annulus, and the depth of the annulus volume is given by the depth of the packers above the upper zone. Both strings of the dual gas-lift well therefore have a common maximum depth for the gas injection points. Due to the common annulus, the two production strings must share a common gas injection source.

A challenge with dual gas-lift wells is to sustain a desired gas distribution between the two production strings. To obtain dual gas injection, suitable sizes of gas injection valves need to be selected. If the system is not properly designed, or the system is changed compared to the design case, which may very well happen over time, the situation may occur that the gas only flows into one of the strings. This is referred to as the gas distribution instability. Figure 1.5 shows typical downhole pressures for the two strings in a dual gas-lift well. Initially, they are equal and gas is evenly distributed. Due to the instability, gas starts re-distributing into string II. The result is a high pressure in string I, so that production stops, while the pressure in string II decreases leading to increased production. The corresponding total production decreases significantly as shown Figure 1.6. While the instability may be removed by replacing the gas injection valve, less expensive solutions are of great interest. This thesis considers automatic control as a possible solution to the problem.

To obtain stable production from a dual gas-lift well, the pressure drop along the path of the lift-gas needs to be friction dominated. For the dual gas-lift well completion the gas injected into the annulus has two possible paths back to the surface, the gas can flow through tubing I or tubing II. To maintain a stable gas flow through both strings, the pressure drop from the annulus volume to the separator, for both strings, is required to be friction dominated. Stable production from the dual gas-lift well can be obtained with gravity dominated pressure drop if feedback control is applied for stabilization.

1.4 Stabilization by Automatic Control

This section will give an overview of literature concerning the stabilization of gas-lift wells by feedback control. The literature overview will cover work published until current date.

Severe oscillating multiphase flow in wells and pipelines has been a known phenomenon for a long time. The problems related to this flow regime has

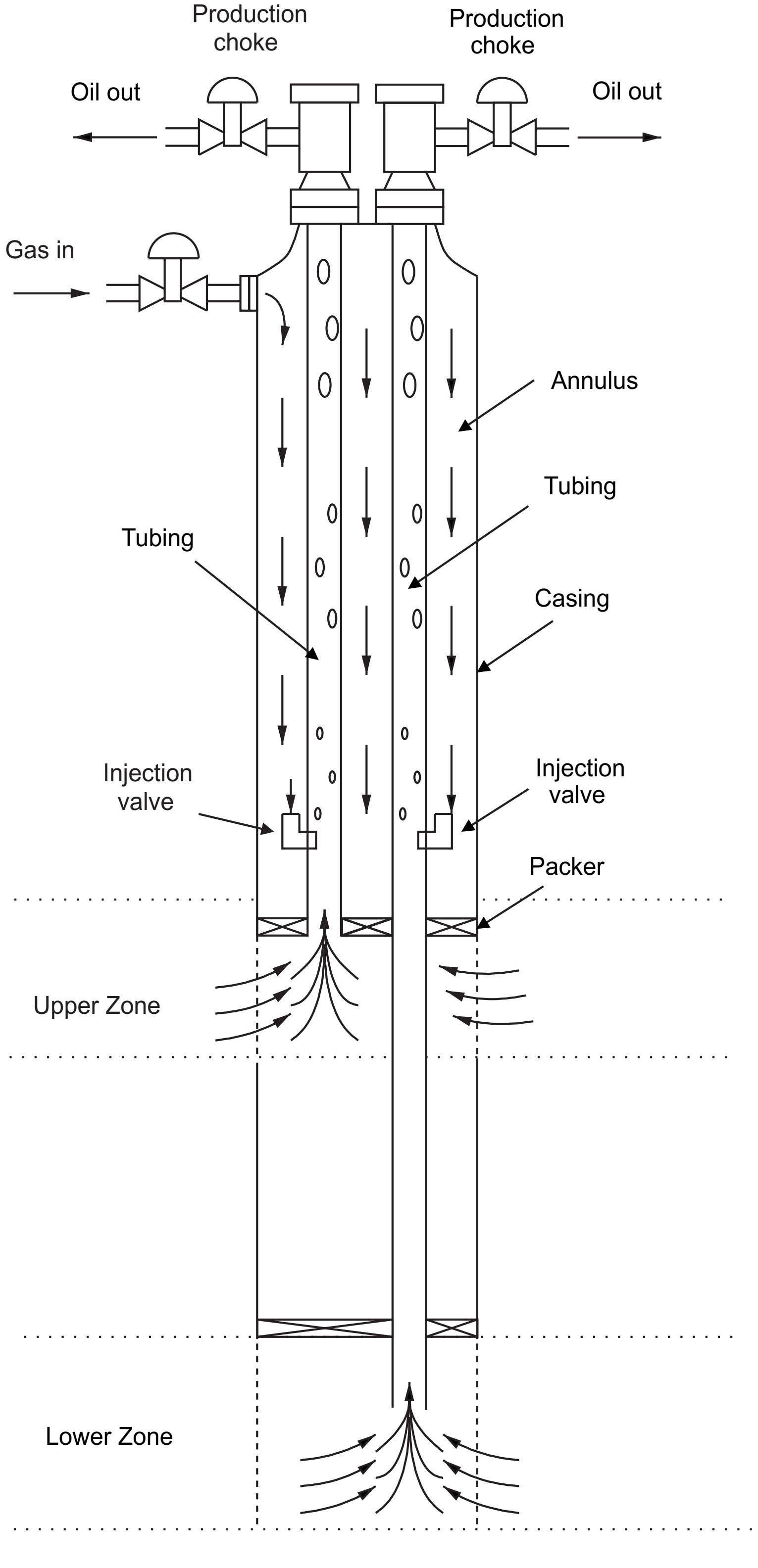

Figure 1.4: The dual gas-lift well.

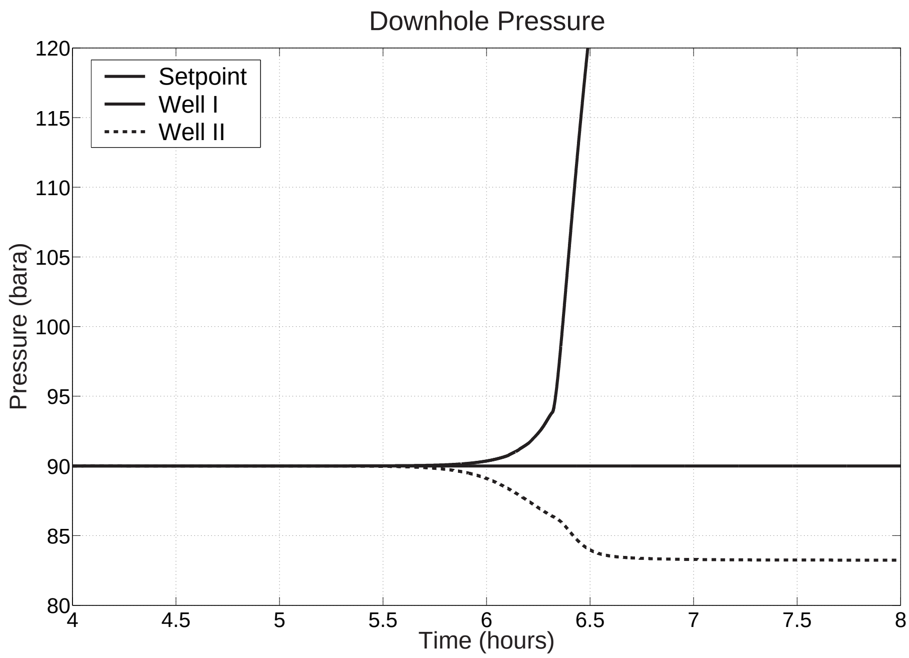

Figure 1.5: Unstable dual gas-lift well.

been handled in different ways. The traditional way of handling this unstable flow is to change the dynamics of the system by making changes to the system. In this context these methods are referred to as static stabilization methods. Further information about static stabilization methods for multiphase flow systems can be found in Tengesdal (2002). The drawbacks of the static stabilization methods are the high cost of the modifications and/or cost of lost production. These drawbacks are the motivation for seeking new solutions to this problem.

The solution presented to this problem is the use of feedback control. This solution changes the dynamics of the system, instead of making changes to the system itself.

The first publications addressing stabilization of gas-lift wells by feedback control are Blick and Boone (1986) and Blick and Nelson (1989). They present theoretical studies, which indicate that the pressure in the top of the well can be controlled by manipulating the opening of the production

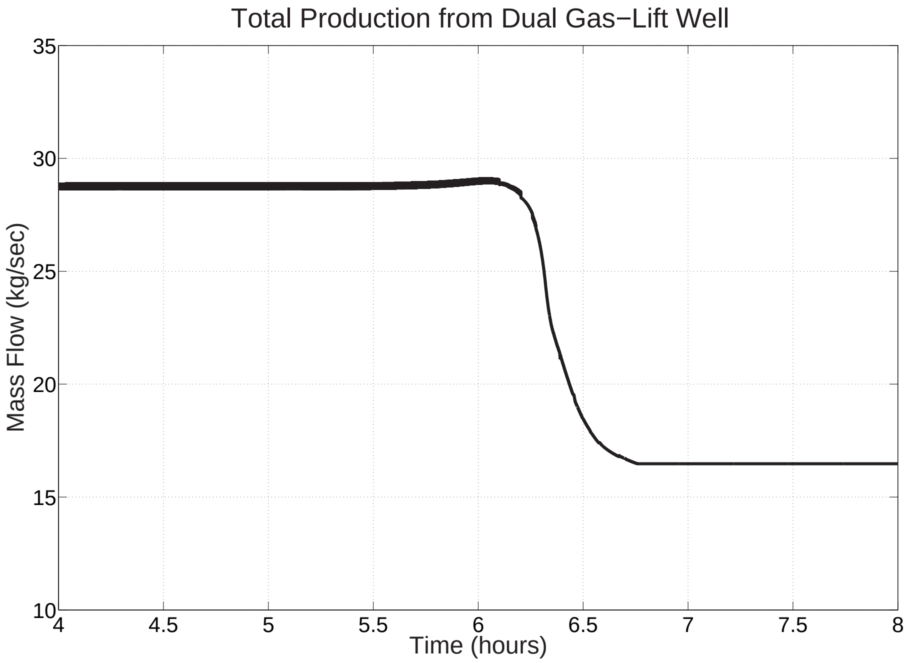

Figure 1.6: Non-optimal production from the dual gas-lift well.

choke.

A method for stabilization of gas-lift wells has been proposed by Der Kinderen and Koornneef (1997). This method applies the pressure in the top of the annulus as the controlled variable, and the opening of the production choke as the manipulated variable.

The stabilization methods of the gas-lift well have been further developed by ABB Research LTD. Several control structures supported by simulation results have been patented (Dalsmo *et al.* 2000). These control structures cover control of the downhole pressure by manipulating the opening of the production choke, control of the wellhead pressure by manipulating the opening of the gas injection choke, control of the downhole pressure by manipulating the opening of the gas injection choke and control of the casing pressure by manipulating the opening of the production choke. Based upon ABB's field patent application, an article describing the content of the patent was published (Jansen *et al.* 1999). A master thesis with focus upon development of a simplified model for the gas-lift well was performed the same year (Dvergsnes 1999).

In 2000, the Petronics research program was started, with the objective of continuing the work on stabilization of multiphase flow. A technical report from this research was published in 2001, describing the stabilization problem of the single gas-lift well, supported by simulation studies and model development of a simplified model (Hu *et al.* 2001). A simulation study of a system of single gas-lift wells was presented in Eikrem *et al.* (2002). This publication studied stabilization of the gas-lift wells by control of the downhole pressure and control of the casing-head pressure. The production choke was selected as the manipulated variable. At the same time an article describing stabilization of wells at the Brage field, by use of feedback control, was published (Dalsmo *et al.* 2002).

The next step in the development was to investigate the possibility of stabilizing the gas-lift well by control of the total mass in the well as reported in Imsland *et al.* (2003), Imsland (2002), Eikrem *et al.* (2004*a*) and Imsland *et al.* (2004). Another approach was to stabilize the gas-lift well by control of an estimate of the downhole pressure. The opening of the production choke was selected as the manipulate variable. This work was documented in Eikrem *et al.* (2004*a*), Aamo *et al.* (2004*a*), Aamo *et al.* (2004*b*) and Eikrem *et al.* (2004*b*).

A new control structure for stabilization of the gas-lift well was presented in Eikrem *et al.* (2005). The single gas-lift well was stabilized by control of the pressure drop across a restriction upstream the production choke. Again, the production choke was selected as the manipulated variable. A new control structure for gas distribution in the dual gas-lift well was studied in Eikrem *et al.* (2006). This control structure enables production from both tubings with a lower pressure drop in the system compared to traditional dual gas-lift well completions.

A simplified model of the gas-lift well using two states was presented by Sinegre *et al.* (2002). A detailed overview of the development within stabilization of multiphase flow in pipeline/riser systems can be found in Storkaas (2005) and Skofteland and Godhavn (2003).

The literature within dual gas-lift wells is very limited compared to the literature of the single gas-lift well. Stabilization of the gas distribution problem in dual gas-lift wells has to the authors knowledge not been reported in the literature. Dual gas-lift wells are being designed to prevent this type of gas distribution problem, see Takács (2005).

1.5 Summary of Papers

A complete list of the author's publications chronologically is given below. The thesis contains reprints of papers 1, 3, 6–9. Conference papers 2, 4, and 5 are covered by journal papers 6 and 7, and are therefore not included in the thesis.

1. "Stabilization of Gas Lifted Wells", G.O. Eikrem, B.A. Foss, L.S. Imsland, B. Hu and M. Golan, *Proceedings of the 15th IFAC World Congress on Automatic Control*, Barcelona, Spain, July 21–26, 2002. (Referred to as Paper I.)

2. "A state feedback controller for a class of positive systems: Application to gas lift stabilization", L.S. Imsland, B.A. Foss and G.O. Eikrem, *Proceedings of the 7th European Control Conference*, Cambridge, UK, September 1–4, 2003.

3. "Stabilization of Gas Lifted Wells Based on State Estimation", G.O. Eikrem, L.S. Imsland and B.A. Foss, *Proceedings of the 2003 International Symposium on Advanced Control of Chemical Processes*, Hong Kong, China, January 11–14, 2004. (Referred to as Paper III.)

4. "Observer design for gas lifted oil wells", O.M. Aamo, G.O. Eikrem, H.B. Siahaan and B.A. Foss, *Proceedings of the 2004 American Control Conference*, Boston, USA, June 30–July 2, 2004.

5. "Anti-Slug Control of Gas-Lift Wells - Experimental Results", G.O. Eikrem, O.M. Aamo, H.B. Siahaan and B.A. Foss, *Proceedings of the 6th IFAC Symposium on Nonlinear Control Systems*, Stuttgart, Germany, September 1–3, 2004.

6. "Observer design for multiphase flow in vertical pipes with gas-lift - theory and experiments", O.M. Aamo, G.O. Eikrem, H.B. Siahaan and B.A. Foss, *Journal of Process Control*, vol. 15, no. 3, pp. 247–257, 2004. (Referred to as Paper IV.)

7. "A state feedback controller for a class of nonlinear positive systems applied to stabilization of gas-lifted oil wells", L.S. Imsland, G.O. Eikrem and B.A. Foss, accepted for publication in *Control Engineering Practice*. (Referred to as Paper V.)

8. "Stabilization of Gas-Distribution Instability in Single-Point Dual Gas Lift Wells", G.O. Eikrem, O.M. Aamo and B.A. Foss, *SPE Production & Facilities*, vol. 21, no. 2, 2006. (Referred to as Paper VI.)

9. "On Instability in Gas-Lift Wells and Schemes for Stabilization by Automatic Control", G.O. Eikrem, O.M. Aamo and B.A. Foss, submitted to *SPE Production & Facilities*. (Referred to as Paper II.)

Next, summaries of each of the six papers included in the thesis are given, Paper I – VI. First, two papers (Paper I and II) describe control structures for single gas-lift wells. Second, Paper III–V discuss model-based control of single gas-lift wells. The final paper (Paper VI) presents results on a dual gas-lift well.

1.5.1 Paper I : Stabilization of Gas Lifted Wells

This publication presents the challenges related to the casing-heading instability phenomenon. A single gas-lift well is presented and the dynamics of an unstable gas-lift well is described. The onset of this work is two studies where a gas-lift well is stabilized by feedback control.

This publication extends previous work by stabilizing a system of two single gas-lift wells sharing a common gas supply source. Two different control structures are applied to stabilize the gas-lift system. In the first control structure the annulus pressure is controlled by manipulating the opening of the production choke. In the second control structure the downhole pressure is controlled by manipulating the opening of the production choke. Both control structures use a control system for distribution of lift-gas between the two wells. This control system decouples the two gas-lift wells. The test case is a full scale model of two gas-lift wells implemented in OLGA 2000. The simulation results indicate that these control structures can be suitable choices for stabilization of unstable gas-lift wells. The simulation study also shows an increased production rate as the system is stabilized.

In addition an analysis of the closed-loop systems is performed for both a stable and an unstable operating point for the gas-lift well. Based upon system identification from closed-loop simulation data Nyquist diagrams of the open-loop system are presented.

1.5.2 Paper II : On Instability in Gas-Lift Wells and Schemes for Stabilization by Automatic Control

In Paper II the work with the single gas-lift well is extended compared to the work performed in Paper I. Based upon the promising results of Paper I, further work is performed by use of a laboratory scale test facility. Analysis, simulations and laboratory experiments are performed for different control structures. These control structures include control of the downhole pressure of the well, control of the casing head pressure of the well and control of the pressure drop across a restriction upstream the production choke. For all control structures the opening of the production choke is selected as the manipulated variable.

A simplified model of the single gas-lift well in laboratory scale is developed. This model is used for controller design and to analyze the dynamics of the gas-lift well in open-loop and closed-loop. The control structures with the designed controllers are initially tested by simulations on the simplified model. Thereafter are these control structures further tested by use of the OLGA 2000 multiphase flow simulator. The simplified model and the OLGA 2000 model of the gas-lift well are both stabilized by the different control structures. Finally, the control structures are verified in the gas-lift laboratory.

These results further substantiate these control structures as possible stabilizing control structures for gas-lift wells. The laboratory experiments also show increased production as a result of stabilization. The presence of three control structures with the possibility of stabilizing the gas-lift well can be applied to obtain a redundant control system for the gas-lift well.

1.5.3 Paper III : Stabilization of Gas Lifted Wells Based on State Estimation

In Paper I and II the downhole pressure measurement is assumed to be present. Downhole conditions are harsh, therefore downhole sensors often fail. Hence, in this paper the use of estimated process measurements as controlled variables are investigated. Two different estimated process measurements are studied, an estimate of the downhole pressure and an estimate of the mass in the system. An extended Kalman filter is applied for the estimation process. The test case selected is an OLGA 2000 model of the single gas-lift well presented in Paper I.

A simplified model of the full scale gas-lift well presented in Paper I is developed. This model is used for estimation of the process by applying the extended Kalman filter. The process measurement available for the Kalman filter is the pressure in the top of the annulus and the pressure in the top of the tubing.

In the first simulation study, where the estimated downhole pressure is controlled by manipulating the opening of the production choke, the gas-lift well is stabilized. In the second simulation study, where the total mass in the system is controlled by manipulating the opening of the production choke, the gas-lift well is also stabilized.

1.5.4 Paper IV : Observer Design for Multiphase Flow in Vertical Pipes with Gas-Lift - Theory and Experiments

Based upon the positive results from Paper II and Paper III, the method for stabilizing the gas-lift well based upon estimated process measurements is tested in laboratory environments. In this study a reduced order observer with stability proof is applied instead of the extended Kalman filter. The structure of the simplified model of the gas-lift laboratory is identical to the one applied in Paper III.

The experiments show that the laboratory scale gas-lift well can be stabilized by control of the estimated downhole pressure. The opening of the production choke is used as the manipulated variable. Increased production is observed as the system is stabilized in the unstable domain.

1.5.5 Paper V : A state feedback controller for a class of nonlinear positive systems applied to stabilization of gas-lifted oil wells

The results from Paper III show that control of the total mass of a gas-lift well can be a possible option. This control structure is further investigated by testing the control structure in laboratory environments.

As the results show, the control structure is able to stabilize the laboratory scale gas-lift well, but this solution seems to be less robust compared to the control structures tested in Paper II and Paper IV.

This publication also contains a detailed description of the theory behind the mass controller applied in the laboratory experiments.

1.5.6 Paper VI : Stabilization of Gas-Distribution Instability in Single-Point Dual Gas Lift Wells

A dual gas-lift well may operate at a low production operating point. In this paper a control structure for the gas distribution of the dual gas-lift well is investigated. The downhole pressure of well 1 is controlled by manipulating the opening of production choke 1. The downhole pressure of well 2 is controlled by manipulating the opening of production choke 2. This paper shows that the dynamics and the control challenges of a dual gas-lift well are different from a single gas-lift well.

A simplified model of the dual gas-lift well is presented in this publication. This model is used for analysis of the dual well system and for simulation of the system in advance of the laboratory experiments. The results from the simulation study show the control structure's ability to stabilize the simplified model of the dual gas-lift well.

The laboratory experiments of the dual gas-lift well using the stabilizing control structure show the ability to stabilize the gas distribution between the two production strings. Increased production is observed as the system is stabilized in the unstable domain.

1.6 Contributions

The main claimed contributions of this thesis are given in this section.

1.6.1 Model Development

A general simplified model of a single gas-lift well is developed. This model is applied both for a full scale gas-lift well and a laboratory scale gas-lift well. The simplified model of the single gas-lift well is applied in the following publications : Paper II - Paper V. The model development work is motivated by the model development work performed by Imsland (2002) and Dvergsnes (1999). The simplified model of the single gas-lift well is extended to represent a dual gas-lift well. A laboratory scale version of this model is presented in Paper VI.

The model work of the single gas-lift well was started by ABB AS. This work is only briefly mentioned in Dvergsnes (1999), no other publications are available. ABB has developed a three state model, where the states are mass of gas and liquid in the tubing and in the annulus. In Dvergsnes (1999) the work of ABB is continued by including two additional states for the energy, one in the tubing and one in the annulus. In Imsland (2002) the work of Dvergsnes is continued by removing the two energy states and including a more complex pressure calculation. The work of the author uses the model presented by Imsland as a basis. The selection of three states is continued, but the pressure calculation is simplified. The selected pressure calculation is closely related to the pressure calculation performed by ABB and Dvergsnes. The assumption of constant temperature introduced by Imsland is continued. The main difference between the authors model and the limited documented model by ABB seems to be at least the assumption of constant temperature, no use of multiphase flow equation across the production choke and the removal of the tubing section below the gas injection point. A five state model of a dual gas-lift well is developed based upon the same modeling principles as applied for the single gas-lift well. In addition, the simplified models presented by the author are verified by laboratory experiments. In particular is it demonstrated that the the models capture the instability mechanisms of interest in this study very well.

1.6.2 Control Structures

This research work has presented two new control structures for stabilization of the single gas-lift well, and one control structure for gas distribution of the dual gas-lift well.

- Control of the filtered pressure drop across a restriction upstream the production choke by manipulating the opening of the production choke, presented in Paper II.
- Control of an estimate of the downhole pressure by manipulating the opening of the production choke, presented in Paper III and Paper IV.

- A control structure for stabilization of the dual gas-lift well has been presented. The downhole pressure of both strings have been controlled, using an identical setpoint for both control loops. The choke openings are selected as manipulated variables. This control structure is presented in Paper VI.

The control structures presented in this section are supported by simulation studies and experimental results. These results show the stabilizing properties of the control structures in laboratory scale environments.

1.6.3 Model Based Control

The simplified model of the single gas-lift well is used for model based control of the single gas-lift well.

- Experimental results verifying the ability to stabilize the gas-lift well based on control of the estimated downhole pressure. The estimated pressure is obtained from a reduced order observer.

- Experimental results verifying the ability to stabilize the gas-lift well based on control of the total mass of the gas-lift well. The estimate of the mass in the system is obtained from an extended Kalman filter.

1.6.4 Additional Contributions

Experimental results verifying the stabilization of the single gas-lift well by control of the downhole pressure or the casing-head pressure, using the production choke as the manipulated variable.

A software solution for realtime control of the gas-lift laboratory from Matlab is developed. This communication solution uses the OPC standard to exchange process data. OPC is an abbreviation for OLE for Process Control.

Analysis of linearized models in open-loop and closed-loop for the control structure applying the direct measurement of the downhole pressure are performed.

1.6.5 Comments

This section points out two contributions in this work which are not provided by the author. These contributions are:

- The development and the stability proof of the reduced order nonlinear observer presented in Paper IV.
- The development and the theory related to the mass controller presented in Paper V.

1.7 Conclusions

The work of this thesis has investigated six different control structures for stabilization of the single gas-lift well. All six control structures were able to stabilize the gas-lift well. These control structures have been investigated by simulation studies and thereafter verified by experiments in a laboratory scale gas-lift well. A simplified model of the single gas-lift well has been applied for analysis of the open-loop and closed-loop system. Two of these control structures are contributions by the author. The first control structure controls the pressure drop across a restriction upstream the production choke by manipulating the opening of the production choke. The second controls an estimate of the downhole pressure of the well by manipulating the opening of the production choke. The estimate is obtained by use of an extended Kalman filter. The results from stabilizing the gas-lift well, both in simulation studies and in laboratory experiments, showed an increased production rate from the well.

The author has also presented a control structure for gas distribution in a dual gas-lift well. A simplified model of the dual gas-lift well has been developed and applied for simulations and analysis of the system. Simulation studies and laboratory experiments have been performed to verify the control structure. Both in simulation studies and in laboratory experiments an increased production rate from the dual gas-lift well was obtained.

1.8 Directions for Further Work

Based upon the work presented in this thesis some directions for further work can be given.

- The experimental results presented in Paper II showed working control structures for the laboratory scale gas-lift well. These control structures can be further tested in full scale production wells.
- A method for selection of the pressure setpoint for the stabilizing control loop can be developed. In this research this setpoint selection is performed by trial and error.
- Perform further comparison of the casing-heading slugging and the pipeline/riser slugging. Compare the stabilization solutions for the pipeline/riser system with the solutions applied for the gas-lift well.
- Investigate further improvements on the performance of the model based control structures.
- Perform nonlinear analysis of the single and the dual gas-lift well systems.
- Perform further analysis of the control structure for gas distribution in the dual gas-lift well.
- Investigate if and how a dual gas-lift well can be stabilized by only topside measurements.

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Paper I

Stabilization of Gas Lifted Wells

G.O. Eikrem, B.A. Foss, L.S. Imsland, B. Hu and M. Golan

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STABILIZATION OF GAS LIFTED WELLS

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Abstract: Oil wells with an oscillating production rate have a lower production compared to oil wells producing at a constant rate. This study looks at instability caused by the casing-heading phenomenon. Control is applied to achieve stable production rate. Two realistic gas lifted systems for oil and gas production are investigated, using the multiphase flow simulator OLGA2000. Different control structures are evaluated, and linear stability analysis is used to substantiate open loop simulation results. The study shows that substantial production improvement can be achieved by applying control to the above mentioned system.

Keywords: Process Control, Oil Production, Multiphase Flow, Gas Lifted Wells

1. INTRODUCTION

Hydrocarbons are produced from wells that penetrate geological formations rich on oil and gas. The wells are perforated in the oil and gas bearing zones. The hydrocarbons can flow to the surface provided the reservoir pressure is high enough to overcome the back pressure from the flowing fluid column in the well and the surface facilities. Detailed information on wells and well completion can be found in Golan and Whitson (1991).

Gas lift is a technology to produce oil and gas from wells with low reservoir pressure by reducing the hydrostatic pressure in the tubing. Gas is injected into the tubing close to the bottom of the well and mixes with the fluid from the reservoir, see Figure 2. The gas lifts oil out of the tubing and reduces the density of the fluid in the tubing. The lift gas is routed from the surface and into the annular conduit (annulus) between the casing and the tubing. The gas enters the tubing through a valve, an injection orifice, at the wellbore.

Gas lift can result in highly oscillating well flow when the pressure drop in the tubing is gravity dominated and there is a large annulus volume filled with compressible gas. In this case the pressure buildup inside the tubing under no-flow or low-flow conditions is faster than the pressure buildup in the annulus. If the pressure in the annulus is able to overcome the pressure in the tubing at a later point, the gas will flow into the tubing and the oil and gas will be lifted out of the tubing. After the fluid is removed from the tubing a new pressure buildup period starts. This type of oscillations is described as casing-heading instability and is shown in the first part of Figure 3. More information can be found in Xu and Golan (1989).

Figure 1 shows an example of a gas lift production curve. The produced oil and gas rates, assuming stable flow conditions, is a function of gas injected into the well at steady state. The curve also shows in which areas the well exhibits stable and highly oscillating flow, respectively. The region of optimum lift gas utilization may lie in the unstable region. Figure 1 is not valid for zero gas injection.

A requirement for casing-heading instability is pressure communication between the tubing and the casing, i.e. the pressure in both the tubing and the casing will influence the flow rate through the injection orifice. There are in principle three ways to eliminate highly oscillating well flow. First,

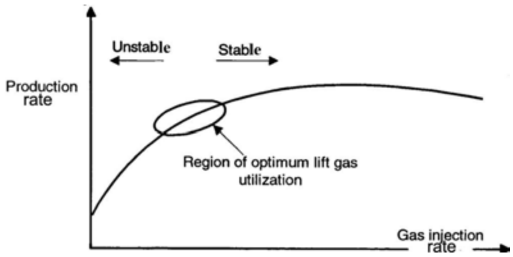

Fig. 1. The gaslift curve with the region of optimum lift gas utilization.

operating conditions can change to achieve stable condition. This can be done by increasing the gas flow rate and/or by reducing the opening of the production choke downstream the well. Both remedies reduces well efficiency. Second, the injection orifice may be a valve with critical flow, meaning that the flow through the injection orifice is constant. This is a solution that has achieved industrial interest. Third, the use of control is a method to stabilize well flow. This is the scope of the present study.

Large oscillations in the flow rate causes poor oil/water separation downstream, limits the production capacity and causes flaring. Hence, a reduction of the oscillations will result in increased processing capacity because of the reduced need for buffer capacity in the process equipment.

Control has to a limited degree been studied for single well systems. Some earlier work has been reported, (Jansen *et al.*, 1999) and (Kinderen, 1998). This earlier work looked only at single well systems, while this study also considers a realistic two-well system where one gas source supplies two gas lifted wells.

We will use the transient multiphase flow simulator OLGA®2000 (Scandpower, 2000), commonly used in the petroleum industry. This simulator has been used to study a set of realistic wells. The controllers are implemented in Matlab (The MathWorks, 2000).

A detailed well system model is prepared in OLGA. It includes the geometry of the well system, initial conditions and boundary conditions. OLGA is based on a modified two-fluid two-phase flow model. It uses semi-implicit time integration, which allow relatively long time steps. OLGA contains specific parametrized models for the production chokes.

The scope of the paper is to develop and assess a control strategy for the above problem and investigate alternative solutions depending on the availability of downhole online measurements. We believe that the paper introduces a new field for process control technology with a substantial potential.

2. SYSTEM DESCRIPTION

2.1 *Single well system*

The basis for this study is a realistic gas lifted well model, see well 1 in Figure 2. The parameters of this vertical well are given as:

- Well parameters:
  - · 2048 m vertical well
  - · 5 inch tubing
  - · 2.75 inch production choke
  - · 0.5 inch injection orifice
- Reservoir parameters
  - · P_R = 160 bara
  - · T_R = 108 ^{o}C
  - · PI = 2.47E-6 kg/s/Pa
- Separator inlet pressure
  - · 15 bara
- Gas injection into annulus
  - · 0.6 kg/s
  - · 120 bara
  - · 60 ^{o}C

The productivity index, PI, is defined by:

$$PI = \frac{\dot{m}}{\Delta P} \quad (1)$$

Where $\dot{m}$ is the total mass flow rate from the reservoir to the well and ΔP is the pressure difference between the reservoir and the well. This index relates the mass flow from the reservoir and into the well to the corresponding pressure drop. The PI is assumed constant.

We assume that there is no water in the produced fluids, only oil and gas. The gas/oil ratio, GOR, is 80 Sm^3/Sm^3. GOR is defined by:

$$GOR = \frac{\dot{q}_{gas}}{\dot{q}_{oil}} \quad (2)$$

Hence the gas-oil-ratio, GOR, is defined as the ratio between the volumetric gas rate and the volumetric oil rate at standard temperature and pressure.

2.2 *Two well system*

This study advances compared to earlier studies by focusing on a two well system with a common gas supply source. The two-well system in this study is shown in Figure 2. Well 1 is defined above, and well 2 is identical to well 1 except that it has a higher productivity index, PI = 3.00E-6 kg/s/Pa. The two wells produce the same reservoir fluid and connect to the same downstream separator. It is assumed that the separator is located close to the wellheads.

The two wells share the same gas source. The total gas injection rate is 1.1 kg/s. This study reflects the case when there is a shortage of gas supply for gas lift operation.

Since there is a limited gas supply, the question becomes how to use this limited lift gas to maximize the oil production. Oil has a much higher

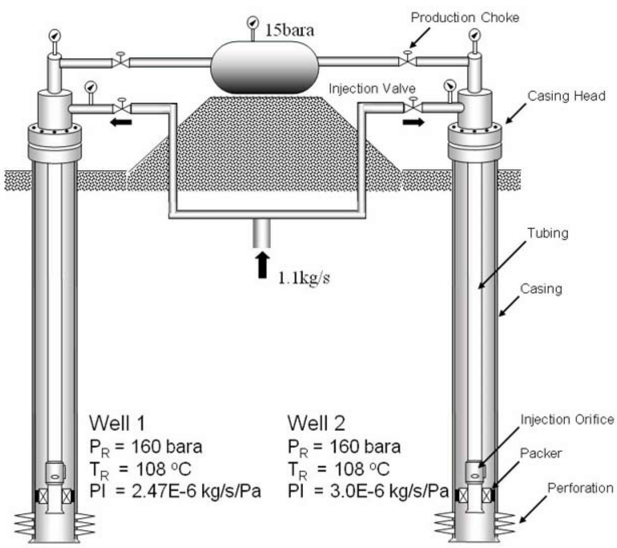

Fig. 2. Two-well system with common gas supply source.

sales value compared to gas, hence maximizing oil production is the vital point. Steady state analysis of the system (i.e. optimization on a static model) shows that the total oil production is maximized if the gas injection rate equals 0.5 kg/s to well 1 and 0.6 kg/s to well 2.

2.3 *OLGA and Matlab*

For this study the multiphase flow simulator OLGA is used for the well and gas supply simulations, while Matlab is used for controller development and implementation. Matlab will read the process outputs from OLGA, calculate new process inputs and return them to OLGA. The connection between OLGA and Matlab is managed by the OSI (OLGA Server Interface) toolbox for Matlab (ABB, 1998).

3. CONTROL OF SINGLE WELL SYSTEM

The measurements which are assumed available in this study are pressure at the wellhead, downhole and at the casing head, and mass flow through the production chokes and the injection chokes. The pressure measurement downhole is often not reliable and hence it can be disadvantageous to make the control structure dependent on this measurement.

The process inputs which can be used to control the one-well system are the production choke and the injection choke. The single well system is defined by well 1, with a gas lift supply rate of 0.6 kg/s.

3.1 *Control structure*

Two available control structures are studied. The first structure controls the downhole pressure, using the production choke. This control structure is the same as for well 1 in Figure 6. The second structure controls the pressure in the annulus, this control structure is the same as for well 1 in Figure 7. A PI-controller is used in both cases to control the pressure. The controllers are tuned using a combination of process knowledge and iterative simulations. The valve models include saturation and limitations on the valve opening/closing rate. The sampling time for the controller is 30 seconds.

3.2 *Simulation results*

The results from the simulations of the one-well system are given in Figure 3. These simulations are noise free. The system was run in open loop for about 4 hours before the controller was activated. The valve opening of the production choke in this period was 80 %. The simulation study shows that control stabilizes the system and increases the amount of produced oil by 10-15%. The valve settles at 55 % at the end of this simulation.

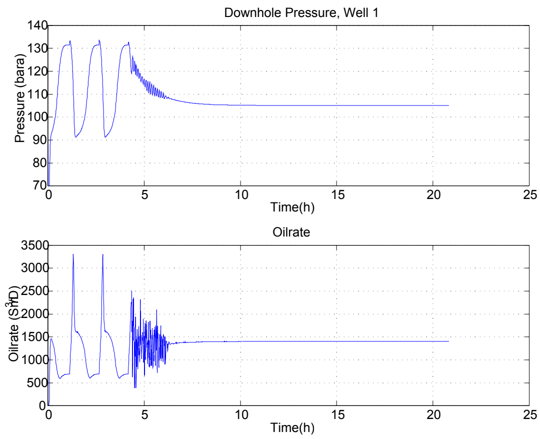

Fig. 3. Simulation results of single well system using pressure in annulus as controlled variable.

The oscillations during the open loop part of the simulations result from the casing-heading instability.

3.3 *Stability analysis*

As mentioned earlier an oscillating well can be stabilized by reducing the opening of the production choke. With low opening of the choke the pressure drop of the flow is dominated by friction. The fluid flow of well 1 in Figure 2 is stable at a high downhole pressure, for instance 122 bara, which corresponds to an opening of the production choke of 16 %. This means that it is no prerequisite to use control, because the low valve opening makes it possible to run the system in an open loop stable condition. This observation is supported by linear stability analysis. The Nyquist plot of the loop transfer function in Figure 4 is computed on the basis of the downhole pressure mentioned above (122 bara). The system is stable in closed loop, which is showed by the Nyquist curve, the curve is not encircling the critical point (-1,0).

At high openings of the production choke the system becomes unstable. This is seen in Figure

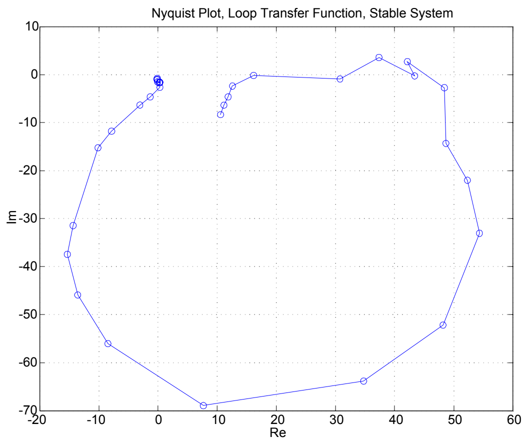

Fig. 4. Nyquist plot of loop transfer function for a stable system. The downhole pressure is 122 bara and the production choke opening is 16 %.

3 where the valve opening is 80 % in open loop. The pressure drop in the flow is now not friction dominated, but gravity dominated. This unstable behavior can also be seen from the Nyquist plot in Figure 5, here is the downhole pressure 105 bara and the production choke is 55 % open. The curve will encircle the critical point (-1,0) twice if the frequency runs from -∞ to ∞. This indicates that the open loop system has two poles in the right half plane.

The Nyquist plots have been generated from simulations of the closed loop well system. The reason why the identification has to be run in closed loop is that the well system is unstable for high choke openings. The setpoint for the closed loop system is a sinusoidal signal, and simulations with different frequencies for this sinusoidal signal have been run.

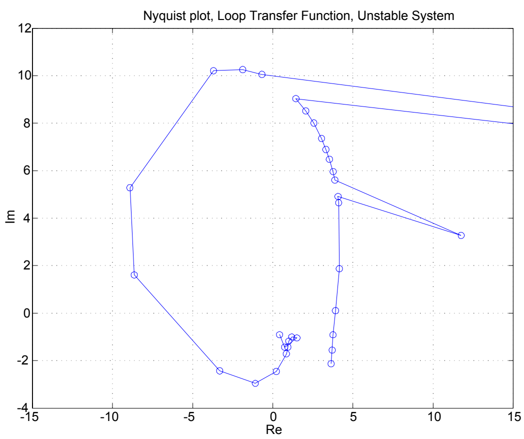

Fig. 5. Nyquist plot of loop transfer function for an unstable system. The downhole pressure is 105 bara and the production choke opening is 55 %.

4. CONTROL OF TWO-WELL SYSTEM

The oil flow rate from the two wells should be maximized, and at the same time be stable to prevent downstream handling problems. A means to achieve this is to keep the downhole pressure constant at the lowest possible level. This will result in stable inflow of gas from the annulus and high inflow of oil from the reservoir.

4.1 *Control of downhole pressure by choking production*

To be able to use the above mentioned production strategy the pressure in the well has to be controlled and the lift gas distributed with an optimal ratio between the two wells. To achieve these optimal gas rates it is necessary to include a control structure on the distribution system of the gas. The control structure in Figure 6 is proposed to achieve this.

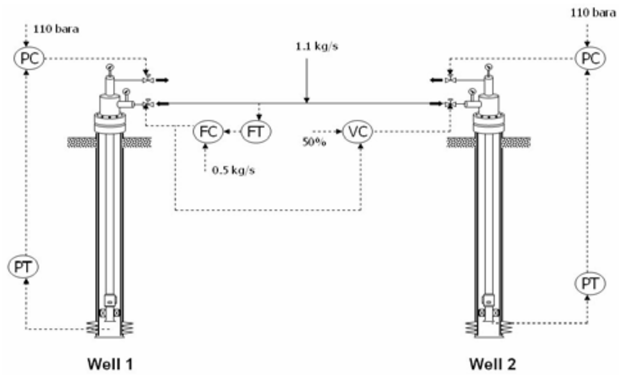

Fig. 6. Control stucture for stabilizing downhole pressure with use of production chokes.

This control structure focuses on controlling the downhole pressure. The pressure transmitters are located downhole in the wells. A stable pressure at this location is a requirement for a constant inflow of hydrocarbons from the reservoir to the well.

The downhole pressure is influenced by changes in the opening of the production choke. An opening of the valve results in a reduced pressure drop over the valve and this gives a reduced back pressure for the well. The well is hence able to increase its production. With a reduction in the opening of the production choke, the pressure drop over the valve is increased. This results in a lower mass flow from the well because of higher back pressure.

Since the supply of gas is constant at 1.1 kg/s, it is sufficient to control the gas flow rate to one of the wells. The mass flow rate to well 1 is controlled with the use of one PID-controller. To avoid saturation of injection choke 1, a controller is connected to injection choke 2. This valve is connected to a PD controller and is adjusted until the opening of injection choke 1 is about 50%. This second control loop is significantly slower than the flow rate control loop. This is a variant of a parallel control structure as found in e.g. Balchen and Mummé (1988).

4.2 *Control of pressure in annulus by choking production*

The advantage of using the pressure in the top of the annulus is the easier access to measurements.

Downhole pressure measurements are more rare and generally not regarded reliable by the industry. The proposed control structure for this setup is given in Figure 7.

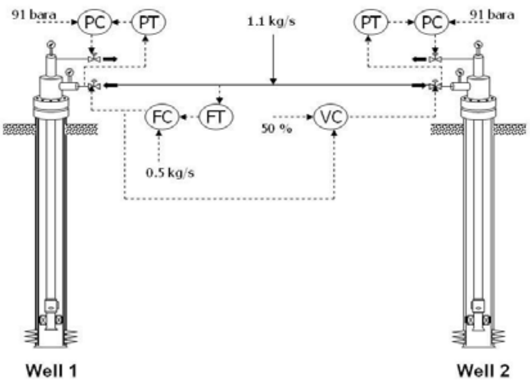

Fig. 7. Control of pressure in annulus with use of production chokes.

This control structure controls the two-well system by measuring the pressure at the top of the annulus. When the rates of injection gas to each well is constant, a constant pressure in the top of the annulus means a constant mass flow of gas from the annulus into the tubing. Since it can be argued that the instability is caused by compressibility of the gas in the annulus, the idea is that controlling the pressure in the annulus will stabilize the system.

4.3 *Controller development*

To control the two-well system, it was decided to use conventional PI controllers to control the downhole pressure and the pressure in the annulus. The controllers have been tuned in an iterative way, and the valve models includes saturation and limitations on the valve opening/closing rate. The sampling time for the controllers is 30 seconds.

4.4 *Simulations and measurements*

The simulations run in open loop for 4 hours before the loop is closed. The initial values equal steady-state conditions.

4.5 *Results from control of downhole pressure*

The results from the simulations with the control structure in Figure 6 are given in Figure 8.

The results from the open-loop simulations show that the downhole pressure in well 1 is stabilized at 96 bara while well 2 is stabilized at 132 bara. This is because all the injection gas is routed to well 1. The gas flows to the well with the lowest counter pressure, which is well 1, because this well has the lowest productivity index, PI. The productivity index is the only difference between the two wells. The well with the highest PI therefore gets the highest liquid level in the tubing and thereby the highest hydrostatic pressure. Well 2 produces at a low rate because of lack of lift gas, while

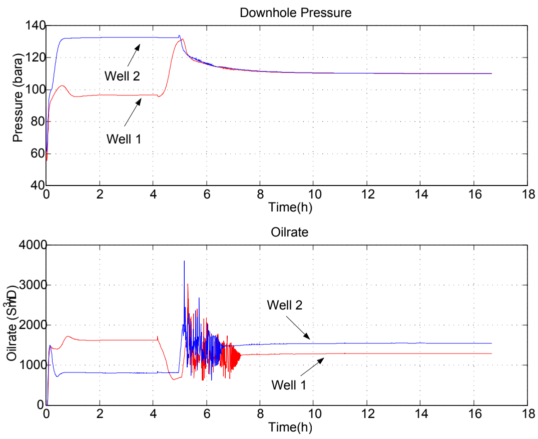

Fig. 8. Simulation results from control of downhole pressure in well.

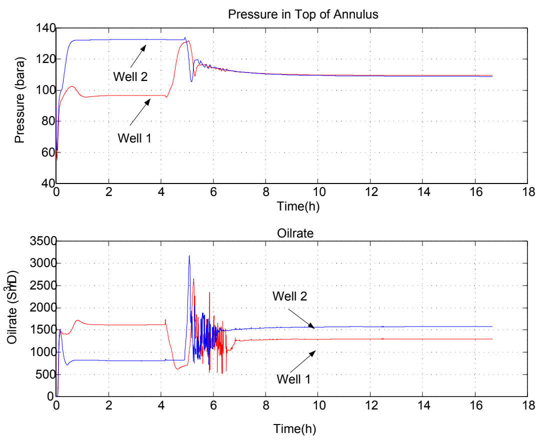

Fig. 9. Simulation results from control of pressure in top of annulus.

well 1 produces at a high rate because of the low downhole pressure. This shows the need for a control structure for the allocation of injection gas. Note that the gas lift flow rate to well 1 is 1.1 kg/s in open-loop, as opposed to 0.6 kg/s in section 3. This is why well 1 exhibits stable flow condition in open-loop; the pressure drop becomes friction dominant.

After the control loops have been closed, it takes almost 2 hours before the system is stabilized at the desired setpoints. This is roughly the time it is takes to build up the pressure in the annulus. There is a significant increase in production of oil from the two wells when the injection of gas is allocated between the two wells in an optimal ratio. The production is increased by 20%, see Figure 8. The choke openings at the end of the simulation are 40 % for well 1 and 60 % for well 2.

4.6 *Results from control of pressure in annulus*

The results from the simulations using the pressure in the annulus as the controlled variable, see Figure 7, show that this control structure also is able to stabilize the two-well system, see Figure 9.

5. DISCUSSION

A high rate of injection gas will stabilize the well, as seen in the simulations, but not at an optimal operating point. A fixed choke opening will also stabilize the well, provided the opening of the choke is reduced until the flow from the well is stable. The reason why an increased amount of lift gas and/or a reduced choke opening gives stable flow is that the flow in the tubing changes from gravitational dominant to friction dominant flow. An improved solution is to stabilize the well system in the unstable region with feedback control.

The reason why the system behaves differently in open loop for the one-well gas lifted system and the two-well gas lifted system, is the rate of the injected lift gas. The one-well system has a lift gas rate (0.6 kg/s) which results in an oscillating system because the pressure drop is gravity dominant, while the two-well system is stable because one well receives all the lift gas (1.1 kg/s) and thereby the pressure drop of the flow becomes friction dominant. The other well is stable due to the complete lack of lift gas.

The control structure which is applied to the distribution system for the lift gas between the wells in the two-well system can be seen as decoupling the two wells. This means that the operation of one well will not affect the other, they act as independent systems. The control structure for allocation of lift gas is required for the two-well system to be able to reach its setpoints. If all lift gas is going to one well, control of the pressure alone will not be able to redistribute the lift gas.

This study describes two possible control structures for stabilizing the two-well system. Both these structures have advantages and disadvantages. The structure which uses measurements of the pressure downhole has a shorter dynamic lag between the control input and output, compared to the measurement in the top of the annulus. However, the pressure measurement downhole in the well is not always reliable, because of the harsh conditions in a well. The second control structure used the pressure in the top of the annulus as measurement, this measurement is easy available and reliable.

The difference between the two control structures regarding stabilization of the well systems is small. The reason is that the pressure waves move so quickly that the dynamic lag through the casing is small for this type of system.

To elaborate on the controller tuning, this was done iteratively based upon process understanding. The setpoints for the two wells were reduced as much as possible, while maintaining a reasonable operating range for the production chokes, 40-60% opening. A high choke opening implies loss of controllability, since changes in the opening in the area 70-100% hardly affect the pressure drop across the valve. It shall be notified that it was not necessary to resort to nonlinear control, for instance gain scheduling. Linear controllers could handle the operating range in question.

6. CONCLUSION

This paper shows how control can improve the performance of gas lifted wells by stabilizing the well flow. Different control structures are available for this well stabilization. Finally, the study substantiates that there is a substantial economical benefit from controlling the pressure in the well, and thereby stabilizing production.

7. ACKNOWLEDGEMENT

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Paper II

On Instability in Gas-Lift Wells and Schemes for Stabilization by Automatic Control

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On Instability in Gas-Lift Wells and Schemes for Stabilization by Automatic Control

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Abstract

In this paper, we present a simple nonlinear dynamic model which is shown to capture the essential dynamics of the casing-heading instability in gas-lift wells despite the complex nature of two-phase flow. Using the model, stability maps are generated showing regions of stable and unstable settings for the production valve governing the produced flow of oil and gas from the tubing. Optimal steady state production is shown to lie well within the unstable region, corresponding to an oil production rate that cannot be sustained without automatic control. Three simple control structures are suggested that successfully stabilize the casing-heading instability in simulations, and more importantly in laboratory experiments.

1 Introduction

Artificial lift is a common technique to increase tail-end production from mature fields, and injection of gas rates among the most widely used such methods. Gas is injected into the tubing, as deep as possible, and mixes with the fluid from the reservoir (see **Fig. 1**). Since the gas has lower density than the reservoir fluid, the density of the fluid in the tubing, and consequently the downhole pressure decreases. As the downhole pressure decreases the production from the reservoir increases. The lift gas is routed from the surface and into the annulus, which is the volume between the casing and the tubing, and enters the tubing through a valve, or an injection orifice. Backflow from the tubing into the annulus is not permitted by this valve. Gas-lift can induce severe production flow oscillations because of casing-heading instability, a phenomenon which originates from dynamic interaction between injection gas in the casing and the multiphase fluid in the tubing. The fluctuating flow typically has an oscillation period of a few hours and is distinctly different from short-term oscillations caused by hydrodynamic slugging. The casing-heading instability introduces two production-related challenges. Average production is decreased as compared to a stable flow regime, and the highly oscillatory flow puts strain on downstream equipment. Reports from industry as well as academia suggest that automatic control (feedback control) is a powerful tool to eliminate casing-heading instability and increase production from gas-lift wells[1-7].

Understanding and predicting under which conditions a gas-lift well will exhibit flow instability is important in every production planning situation. This problem has been adressed by several authors by constructing stability maps, ie. a 2D diagram which shows the regions of stable and unstable production of a well[6,8,9]. The axes may define the operating conditions in terms of gas-injection rate, and production choke opening or wellhead pressure.

In this paper, we present three different control structures for stabilizing casing-heading instability in gas-lift wells. Stability is analyzed for each controller, and it is shown how feedback control stabilizes performance, at least locally around some operating point. The performance of the controllers is demonstrated in simulations, but more importantly, stabilization is also achieved in laboratory experiments. The control structure uses the production choke as the manipulated variable. An alternative input could be the gas-lift choke.

The paper is organized as follows: Section 2 describes the laboratory facilities which are used in this work. Thereafter the dynamics of casing-heading instability are discussed, and suitable models for analysis and design are proposed. The proposed control structures are presented in Section 4, along with stability analysis, closed-loop simulations and experimental results. The paper ends with some conclusions in Section 5.

2 The Gas-Lift Laboratory at TU Delft

Realistic tests of control structures for gas-lift wells are performed using the gas-lift well laboratory setup at TU Delft[♣]. The laboratory installation represents a dual gas-lift well, using compressed air as lift-gas and water as produced fluid. It is shown

♣ The experimental setup is designed and implemented by Shell International Exploration and Production B.V., Rijswijk, and is now located in the Kramers Laboratorium voor Fysische Technologie, Faculty of Applied Sciences, Delft University of Technology.

in **Fig. 2**. Only the long tubing is used in the experiments (single gas-lift well), hence only this part is described. The production tube is transparent, facilitating visual inspection of the flow phenomena occurring as control is applied. It measures 18 m in height and has an inner diameter of 20 mm, see **Fig. 2a**. The fluid reservoir is represented by a tube of the same height, but with the substantially larger inner diameter of 80 mm. The reservoir pressure is given by the static height of the fluid in the reservoir tube. A gas bottle represents the annulus, see **Fig. 2b**. In the experiments run in this study, gas is fed into the annulus at a constant rate of 0.6×10^{-3} kg/s. Input and output signals to and from the installation are handled by a microcomputer system, see **Fig. 2c**, to which a laptop computer is interfaced for running the control algorithm and presenting output. The sampling time is 1 second.

3 Casing-Heading Instability

This section discusses the casing-heading instability, and presents a nonlinear dynamic model suitable for analysis.

3.1 Explaining the Phenomenon

The dynamics of highly oscillatory flow in single point injection gas-lift wells can be described as follows:

1. Gas from the annulus starts to flow into the tubing. As gas enters the tubing the pressure in the tubing falls, accelerating the inflow of lift-gas.
2. If there is uncontrolled gas passage between the annulus and tubing, the gas pushes the major part of the liquid out of the tubing, while the pressure in the annulus falls dramatically.
3. The annulus is practically empty, leading to a negative pressure difference over the injection orifice blocking the gas flow into the tubing. Due to the blockage, the tubing becomes filled with liquid and the annulus with gas.
4. Eventually, the pressure in the annulus becomes high enough for gas to penetrate into the tubing, and a new cycle begins.

For more information on this type of instability, also termed severe slugging, the reader is referred to Xu and Golan[10]. The oscillating production causes problems for downstream processing equipment, and may be unacceptible in operations. The traditional remedy is to choke back to obtain a non-oscillating flow. As mentioned in the Introduction, automatic control is a powerful approach to eliminate oscillations. Moreover, reports also show that this technology increases production[1-7]. Another approach is to fit a gas-lift valve which secures critical flow. This decouples the dynamics of the casing and tubing volumes and thereby eliminates casing-heading instabilities.

3.2 A Nonlinear Dynamic Model

The process is modelled by three states: x_1 is the mass of gas in the annulus; x_2 is the mass of gas in the tubing, and; x_3 is the mass of oil above the gas injection point in the tubing. Looking at Fig. 1, we have

$$\dot{x}_1 = w_{gc} - w_{iv}, \quad (1)$$

$$\dot{x}_2 = w_{iv} + w_{rg} - w_{pg}, \quad (2)$$

$$\dot{x}_3 = w_{ro} - w_{po}, \quad (3)$$

where $\bullet$ denotes differentiation with respect to time, and w_{gc} is a constant mass flow rate of lift gas into the annulus, w_{iv} is the mass flow rate of lift gas from the annulus into the tubing, w_{rg} is the gas mass flow rate from the reservoir into the tubing, w_{pg} is the mass flow rate of gas through the production choke, w_{ro} is the oil mass flow rate from the reservoir into the tubing, and w_{po} is the mass flow rate of produced oil through the production choke. The flows are modeled by

$$w_{gc} = \text{constant flow rate of lift gas}, \quad (4)$$

$$w_{iv} = C_{iv}\sqrt{\rho_{a,i}\max\{0, p_{a,i} - p_{wi}\}}, \quad (5)$$

$$w_{pc} = C_{pc}\sqrt{\rho_m \max\{0, p_{wh} - p_s\}}f_{pc}(u), \quad (6)$$

$$w_{pg} = \frac{x_2}{x_2 + x_3}w_{pc}, \quad (7)$$

$$w_{po} = \frac{x_3}{x_2 + x_3}w_{pc}, \quad (8)$$

$$w_{ro} = f_r(p_r - p_{wb}), \quad (9)$$

$$w_{rg} = r_{go}w_{ro} \quad (10)$$

C_{iv} and C_{pc} are constants, u is the production choke setting ($u(t) \in [0,1]$), $\rho_{a,i}$ is the density of gas in the annulus at the injection point, $p_{a,i}$ is the pressure in the annulus at the injection point, ρ_m is the density of the oil/gas mixture at the well head, p_{wh} is the pressure at the well head, p_{wi} is the pressure in the tubing at the gas injection point, p_{wb} is the pressure at the well bore, p_s is the pressure in the manifold, p_r is the reservoir pressure far from the well, and r_{go} is the gas-to-oil-ratio (based on mass flows, at actual conditions) of the flow from the reservoir.

The function f_{pc} is valve specific and represents a possibly nonlinear scaling of the flow as a function of the choke setting u. f_r is a case specific, possibly nonlinear, mapping from the pressure difference between the reservoir and the well bore to the fluid flow from the reservoir. The manifold pressure, p_s, is assumed to be held constant by a control system, and the reservoir pressure, p_r, and gas-to-oil-ratio, r_{go}, are assumed to be varying slowly and are therefore treated as constants. Note that the fluid flow rates through the valves are restricted to be positive. The densities are modelled as follows

$$\rho_{a,i} = \frac{M}{RT_a} p_{a,i}, \qquad (11)$$

$$\rho_m = \frac{x_2 + x_3}{L_w A_w}, \qquad (12)$$

and the pressures as follows

$$p_{a,i} = \left(\frac{RT_a}{V_a M} + \frac{gL_a}{V_a} \right) x_1, \qquad (13)$$

$$p_{wh} = \frac{RT_w}{M} \frac{x_2}{L_w A_w - v_o x_3}, \qquad (14)$$

$$p_{wi} = p_{wh} + \frac{g}{A_w}(x_2 + x_3), \qquad (15)$$

$$p_{wb} = p_{wi} + \rho_o g L_r. \qquad (16)$$

M is the molar weight of the gas, R is the gas constant, T_a is the temperature in the annulus, T_w is the temperature in the tubing, V_a is the volume of the annulus, L_a is the length of the annulus, L_w is the length of the tubing, A_w is the cross sectional area of the tubing above the injection point, L_r is the length from the reservoir to the gas injection point, A_r is the cross sectional area of the tubing below the injection point, g is the gravity constant, ρ_o is the density of the oil, and v_o is the specific volume of the oil. The oil is considered incompressible, so $\rho_o = 1/v_o$ is constant. The temperatures T_a and T_w are varying slowly and therefore treated as constants. By modeling the pressure in the top of the tubing and the top of the annulus by use of the ideal gas law, the masses of gas in the annulus and the tubing are expected to be underestimated. The model presented above is a slight extension of an existing model for gas-lift wells[7]. This model as well as an extended version for single point dual gas-lift wells[6] have both been validated experimentally.

3.3 A Linearized Dynamic Model

A linear version of the nonlinear model presented above will be derived in this section.

Given a production choke opening $u = u^*$, we look for a steady state solution $x \overset{\Delta}{=} [x_1 \quad x_2 \quad x_3] = x^*$ that solves equations (1)-(3) when the time derivatives on the left hand side are set equal to zero. This solution has a corresponding steady oil production flow. This flow, however, may be unstable so that it cannot be sustained in practice when various disturbances are present. Equations (1)-(3) are linearized around (x^*, u^*) to obtain the system

$$\dot{\Delta x} = A\Delta x + B\Delta u, \qquad (17)$$

where $\Delta x = x - x^*$, $\Delta u = u - u^*$, and A and B are $3x3$ and $3x1$ matrices, respectively.

4 Automatic Control

In this section, control structures will be proposed and studied using tools for stability analysis. In addition, the performance of the proposed controllers will be demonstrated in simulations using the nonlinear model presented in Section 3.2, as well as using OLGA 2000, which represents the state-of-the-art in multiphase flow simulation. Finally, constituting the main findings of this paper, results from laboratory experiments will be presented that prove the feasibility of applying the proposed control strategies in practice. In all cases presented in the following sections, a PI-controller will be employed. Its transfer function is given by

$$H_C(s) = K_P(1 + \frac{1}{\tau_I s}), \qquad (18)$$

where K_P is the proportional gain, τ_I is the integral time and s is the Laplace variable. The controller includes intergrator windup to limit the effect of the integral term when the control input, u, saturates.

4.1 Stabilization using Downhole Pressure

4.1.1 Controller Design and Linear Analysis

In this section, a possible control structure for stabilization of gas-lift wells based on measuring the downhole pressure, y_{tub}, is investigated. As in all cases studied, the means of actuation is the production choke, giving the control structure marked "tub" in **Fig. 3**. Given a desired setpoint y^*_{tub}, the corresponding steady state is denoted (x^*, u^*). A linearized output

$$\Delta y = C_{tub}\Delta x, \qquad (19)$$

can be derived where $\Delta x = x - x^*$, $\Delta y = y_{tub} - y^*_{tub}$ and C_{tub} is a $1x3$ matrix.

The system analysis takes place in the frequency domain, making use of the transfer function from control input Δu to output Δy. The transfer function is obtained from (17) and (19) by noting that $\Delta\dot{x} = s\Delta x$.

$H_{y,u^*}(s) = C_{tub}(sI - A)^{-1}B$ ………………..…….(20)

The subscript u^* reflects the fact that the transfer function can be parameterized by the steady-state production choke opening.

Here, the setpoint is chosen as $y^*_{tub} = 1.7$ bara which corresponds to u^* =82.5%. With the transfer function (20) established, the next step is to design the parameters of the PI controller. The stabilizing component of the controller is its proportional gain K_P. Thus, the integral part is disregarded for now, noting that for sufficiently large τ_I, which should be ensured by proper design, it will only have an effect for very low frequencies. For the design of K_P, the Nyquist stability criterion is employed. This well-known tool is an application of the argument principle of complex analysis, and lends itself particularly useful for choosing K_P. It states that the graph of the complex loop transfer function $H_{y,u^*}(j\omega)$ should encircle the point $-\frac{1}{K_P}$ on the real axis N=Z-P times in the clockwise direction as ω runs from -∞ to ∞ (*j* is the imaginary unit). Z and P are the number of zeros, and respectively poles, of $H_{y,u^*}(s)$ that lie in the right half of the complex plane.[11] The Nyquist plot for this case is shown in **Fig. 4**, where the solid line represents the time-continuous case, while the dashed line represents the time-discrete case with a sampling interval of 1 second. The × identifies the point $-\frac{1}{K_P}$ for $K_P = 1$. As can be seen from the figure, the interval (approximately) (-1.2,0) is encircled twice by the solid line, and the interval (-1.2,-0.2) is encircled twice by the dashed line. It follows that for stability, K_P must lie in (approximately) [0.8,5]. Notice that the lower bound on K_P comes from the properties of the system, while the upper bound on K_P is a result of discretization, and can be altered by changing the sampling interval. The controller gain for this case is selected as $K_P = 3$. With the controller gain selected, the integral time is selected to obtain a desired rate at which to compensate for steady state error. Simulations suggest that $\tau_I = 200\,\text{sec}$ is a reasonable choice. **Fig. 5** shows the location of closed-loop poles as a function of K_P. For large values of K_P, all poles are located in the left half of the complex plane. As K_P decreases, they move to the right in the plot. When $K_P \approx 0.8$, two poles cross into the right half of the complex plane, causing instability. This is consistent with the Nyquist plot. **Fig. 6** investigates the capability of the controller designed for u*=82.5% to locally stabilize other steady states. It shows the location of closed-loop poles as functions of steady state given in terms of production choke opening u*. As the steady state production choke opening is increased, two poles (complex conjugated) move towards the right, and cross into the right half of the complex plane at $u^* \approx$96%. One intepretation of this result is that the system is not controllable for large valve opening because the pressure drop over the production choke becomes too small. Hence, there is not sufficient control authority to stabilize the casing-heading instability. A second interpretation is that the controller designed for u*=82.5% fails to provide local stability for all u* due to nonlinear effects, indicating that gain scheduling may be in order.

4.1.2 Simulation Results

The analysis of the closed-loop system indicates that the designed controller will stabilize the unstable gas-lift well. Based on these results, simulations of the closed-loop system are performed in this section.

Simplified Model

Simulations using the simplified model presented in Section 3.2 have been performed using the model parameters given in Table 6, and the controller parameters given in Table 2. Gain scheduling is used to improve performance. In this case gain scheduling means that the gain K_P changes according to the production choke opening,

The simulation sequence is as follows (see also Table 1): After keeping the choke at 50% for 10 minutes, the downhole pressure is at 1.97 bara and total production is at 5.3 kg/min (both almost steady), see **Figs. 7 and 8**. When the controller is turned on after 10 minutes, the downhole pressure is reduced from 1.97 bara to 1.70 bara as the controller gently opens the production choke from 50% to 82.5%. The total production from the gas-lift well increases from 5.3 kg/min to 5.9 kg/min due to the reduced downhole pressure. The casing head pressure is reduced from 2.01 bara to 1.74 bara, while the tubing head pressure is reduced from 1.27 bara to 1.04 bara in the same time period, see

Fig. 9. Since the gas supply into the annulus is kept at a constant rate, the increase in total production between the two steady states u*=50% and u*=82.5% actually represents an increase in oil production. Notice in the figures that the system goes into severe slugging when the controller is turned off and a small disturbance is introduced after 35 minutes. This is consistent with the results from the linear analysis as shown in **Fig. 4**.

Additional simulations were carried out in order to test robustness of the control system. It was observed that the controller was able to stabilize the system even from severe slugging operation, indicating that choking back to obtain steady flow before turning the controller on is not really necessary in this case.

OLGA 2000 Model

Although the simulation study using the simplified gas-lift model in the previous section gave promising results for the chosen control structure, a more realistic test will be carried out in this section by applying the control structure to a OLGA 2000 model of the laboratory scale gas-lift well. The simulation of the closed-loop system uses the control sequence given in Table 1, and the controller parameters given in Table 3. In order to achieve satisfactory performance, the controller parameters had to be tuned: A higher gain was needed in the OLGA 2000 simulations.

The simulation results are given in **Figs. 10-12.** Choking back to 55% over a period of 10 minutes brings the system into a steady state with a total production of about 2 kg/min. As the controller gently increases the choke opening from 50% to 88%, the downhole pressure is reduced from 2.7 bara to 2.2 bara, causing the total production to increase from 2 kg/min to about 4 kg/min. The tubing head pressure is reduced from 1.75 bara to 1.23 bara, and the casing head pressure is reduced from 1.7 bara to 1.25 bara in the same time period. Notice that the system quickly goes into severe slugging when the controller is turned off after 35 minutes, confirming that the chosen steady state is open-loop unstable.

Experimental Results

Motivated by the promising results obtained in simulations, experimental tests were performed in the gas-lift laboratory. The experiment uses the control sequence shown in Table 4, and the control parameters shown in Table 5. The control parameters were retuned to improve performance.

A representative set of results from the laboratory experiments are given in **Figs. 13-15**. As the controller opens the production choke from 55 % to 82.5 %, the downhole pressure is reduced from 2.53 bara to 2.18 bara. As a result, the total production is increased from about 2 kg/min to about 4.5 kg/min. The tubing head pressure is reduced from 1.51 bara to 1.19 bara, and the casing head pressure is reduced from 2.65 bara to 2.3 bara in the same time period. The controller clearly achieves regulation to the desired setpoint, and the setpoint represents an open-loop unstable steady state as shown by the oscillations appearing when the controller is turned off after 25 minutes. In conclusion, the experiment shows that the controller performs well, and that it has the potential of significantly increasing production.

4.2 Stabilization using Casing Head Pressure

In this section a control structure based on measuring the casing head pressure, y_{cas}, is investigated. Again the means of actuation is the production choke, giving the control structure marked "cas" in **Fig. 3**. For a given setpoint y_{cas}*, a linearized output as in (19) and a corresponding transfer function (20) can be derived.

The results using the casing head pressure follow the results using the downhole pressure. Linear analysis shows that there is a lower and upper bound on the gain K_P, and nonlinearities imply that it is advantageous to apply gain scheduling control. Further, the gains chosen for the simplified model need to be adjusted when applied to the OLGA 2000 simulator and in experiments. To provide a flavour for the results, a representative result is shown in **Fig. 16**. This experiment again follows the sequence outlined in Table 4. The casing head pressure settles at its setpoint value of 2.3 bara roughly 15 min after the control loop has been closed. Further, it is easily seen how casing-heading oscillations appear shortly after the controller is deactivated.

4.3 Stabilization using Differential Pressure

The outset for the third control structure is the use of a differential pressure measurement as a means to stabilize casing-heading instability. This may be viewed as a way of controlling the production flow rate. The control structure is marked “dif” in **Fig. 3**. Some comments need to be made related to experimental results. First, the differential pressure measurement y_{dif} is measured across a restriction in front of the production choke. A typical result is shown in **Fig. 17**. Again the controller stabilizes the flow. It should however be noted that the variation in the production choke opening is much larger than for the other controllers, see **Fig. 13** and **Fig. 16.** The reason for this difference is a higher noise level on the differential pressure measurement compared to the downhole pressure menasurement and the casing head pressure measurement. The noise level can be reduced using a lowpass filter with a lower cutoff frequency than the currently implemented lowpass filter. The filter can be implemented without reducing controller performance since the bandwidth of the controller is in the order of several minutes and the sampling time is 1 second.

5 Conclusions

This investigation has shown that automatic control is a feasible option for optimizing production from gas-lift wells suffering from casing-heading instability. In particular, it has been shown that different control structures can be applied to the same well with similar performance. This is a positive result in the sense that it is possible to switch from one control structure to another in the event of sensor failure, for instance by switching from downhole pressure to casing head pressure. Hence, the design of a backup strategy is straightforward. Moreover, the algorithms used in the different cases are equal; simple PI-controller algorithms. This means that a change from one control structure to another only implies the change of a few controller parameters.

The results indicate that gain scheduling improves performance. This is commonly used in industry. It may, however, be possible to alleviate the need for gain scheduling by including a cascaded loop where the stabilizing controller sees the setpoint of an inner flow loop as its control input rather than the production choke opening.

This study hopefully contributes to the understanding of casing-heading instability through its emphasis on a mix of analysis and trials. The consistency of the findings, in particular between linear analysis and experimental results, indicates that a mixed approach could be useful also for other similar applications.

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Nomenclature

A_r Cross sectional area of tubing below the gas injection point, [L^2], m^2

A_w Cross sectional area of tubing above the gas injection point, [L^2], m^2

C_{iv} Valve constant for gas injection valve, [L^2], m^2

C_{pc} Valve constant for production valve, [L^2], m^2

C_1 Valve constant for reservoir valve, [L^2], m^2

Δt Time step, [t], s

e Regulation error, [m/Lt^2], Pa

g Acceleration of gravity, [L/t^2], m/s^2

K_P Controller gain

L_a Length of annulus, [L], m

L_r Length of tubing below gas injection point, [L], m

L_w Length of tubing above gas injection point, [L], m

M Molar weight of gas, [m/n], kg/mol

ν_o Specific volume of oil, [L^3/m], m^3/kg

p_a Pressure at the gas injection point in the annulus, [m/Lt^2], Pa

p_r Pressure in reservoir, [m/Lt^2], Pa

p_s Pressure in the manifold, [m/Lt^2], Pa

p_{wh} Pressure at well head, [m/Lt^2], Pa

p_{wb} Pressure at well bore, [m/Lt^2], Pa

p_{wi} Pressure at gas injection point in tubing, [m/Lt^2], Pa

R Universal gas constant, [mL^2/nTt^2], J/Kmol

r_{go} Gas-to-oil ratio in flow from reservoir

$\rho_{a,i}$ Density of gas at injection point in annulus, [m/L^3], kg/m^3

ρ_m Density of mixture at well head, [m/L^3], kg/m^3

ρ_o Density of oil, [m/L^3], kg/m^3

t Time, [t], s

T_a Temperature in annulus, [T], K

T_w Temperature in tubing, [T], K

τ_I Controller integral action [t], s

u Setting of production valve

V_a Volume of annulus, [L^3], m^3

w_{gc} Flow of gas into annulus, [m/t], kg/s

w_{iv} Flow of gas from annulus into tubing, [m/t], kg/s

w_{pc} Flow of mixture from tubing, [m/t], kg/s

w_{po} Flow of oil from tubing, [m/t], kg/s

w_{pg} Flow of gas from tubing, [m/t], kg/s

w_{ro} Flow of oil from reservoir into tubing, [m/t], kg/s

w_{rg} Flow of gas from reservoir into tubing, [m/t], kg/s

x_1 Mass of gas in annulus, [m], kg

x_2 Mass of gas in tubing, [m], kg

x_3 Mass of oil in tubing, [m], kg

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SI Metric Conversion Factors

bar	×	1.0*	E+05	=	Pa
bara		(bara + 1) ×1.0*	E+05	=	Pa
bbl	×	1.589 873	E-01	=	m^3
Btu	×	1.055 056	E+00	=	kJ
ft	×	3.048*	E-01	=	m
ft^2	×	9.290 304*	E-02	=	m^2
ft^3	×	2.831 685	E-02	=	m^3
°F		(°F+459.67)/1.8		=	K
lbm	×	4.535 924	E-01	=	kg

*Conversion factor is exact.

Table 1. Control sequence for simulation trials.

Time slot	Control mode	Choke opening
0min – 10min	Open loop	50.0% (simple) 55.0% (OLGA)
10min-35min	Closed loop	Controlled
35min – 45min	Open loop	82.5% (simple) 88.0% (OLGA)

Table 2. Controller parameters for PI-controller used in simulations with the simplified model.

Choke opening	Gain K_P	Integral time τ_I
$50\% \le u \le 75\%$	2	200 sec
$75\% \le u \le 100\%$	3	200 sec

Table 3. Controller parameters for PI-controller used in simulations with OLGA 2000.

Choke opening	Gain K_P	Integral time τ_I
$50\% \le u \le 75\%$	5	100 sec
$75\% \le u \le 100\%$	9	100 sec

Table 4. Control sequence for experiments.

Time slot	Control mode	Choke opening
(-5)min – 0.5min	Open loop	55.0%
0.5min - 25min	Closed loop	Controlled
25min – 35min	Open loop	82.9%

Table 5. Controller parameters for PI-controller used in the experiments.

Choke opening	Gain K_P	Integral time τ_I
$55\% \le u \le 75\%$	0.5	100 sec
$75\% \le u \le 83\%$	1.0	100 sec
$83\% \le u \le 100\%$	1.5	150 sec

Table 6: Numerical coefficients.

Parameter	Value	Unit
M	0.028	kg/mol
R	8.31	J/Kmol
g	9.81	m/s^2
T_a	293	K
L_a	0.907	m
V_a	22.3×10^{-3}	m^3
ρ_o	1000	kg/m^3
p_s	1×10^5	Pa
w_{gc}	0.6×10^{-3}	kg/s
p_r	2.9×10^5	Pa
T_w	293	K
L_w	14	m
L_r	4	m
A_w	0.314×10^{-3}	m^2
A_r	0.314×10^{-3}	m^2
C_{iv}	1.60×10^{-6}	m^2
C_{pc}	0.156×10^{-3}	m^2
C_r	12×10^{-6}	m^2
r_{go}	0	-

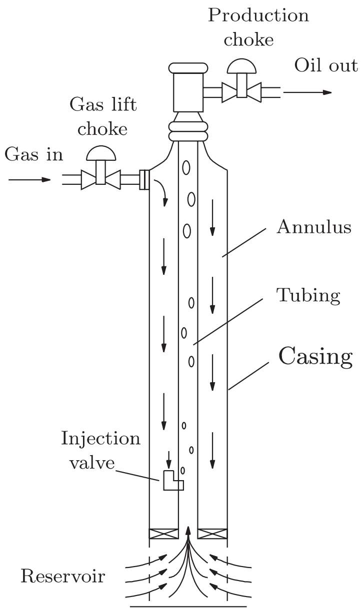

Fig. 1: A gas-lift oil well.

a) The production tubes.

b) The annulus volume.

c) The microcomputer.

Fig. 2: Sketch of the gas-lift laboratory.

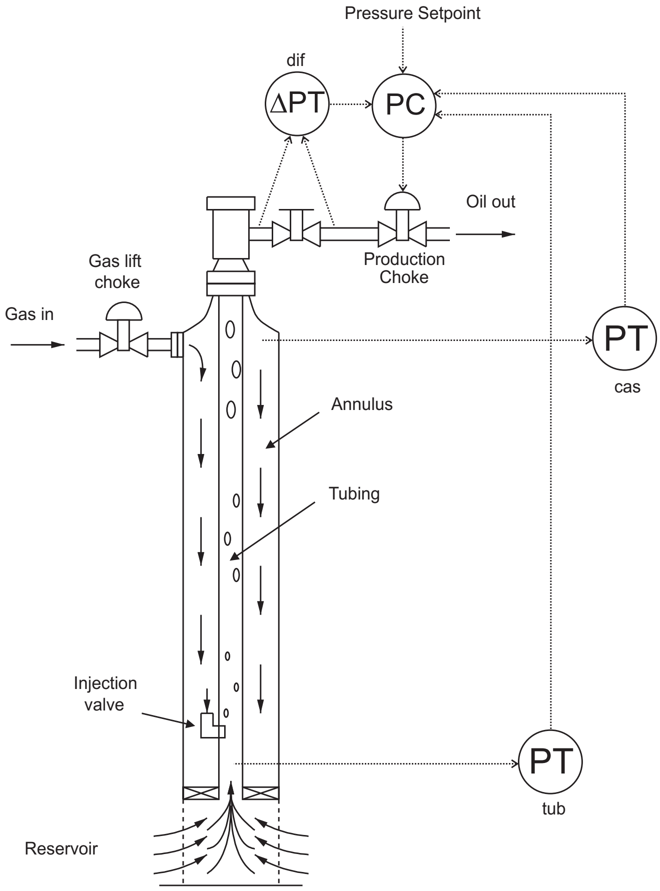

Fig. 3: The three control structures.

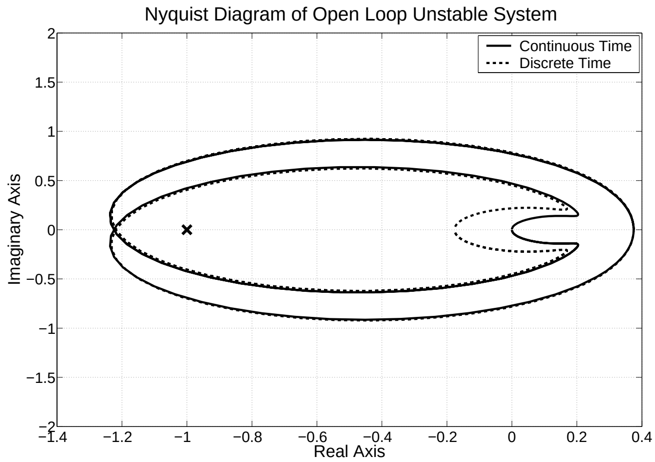

Fig. 4: The Nyquist diagram of the open-loop transfer function for u^*=82.5%.

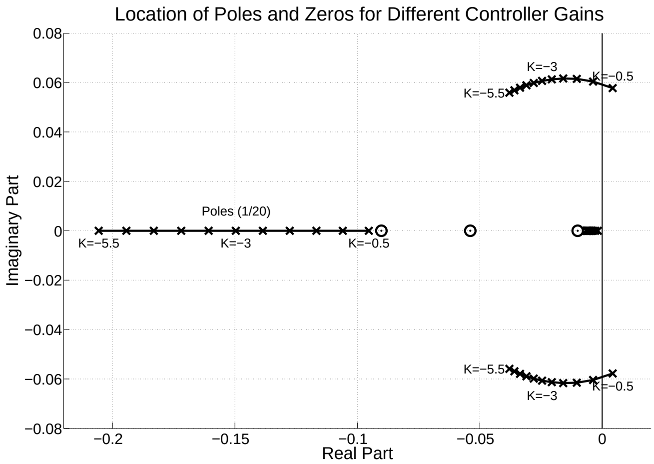

Fig. 5: The location of the closed loop poles for different controller gains K_P. u^*=82.5%.

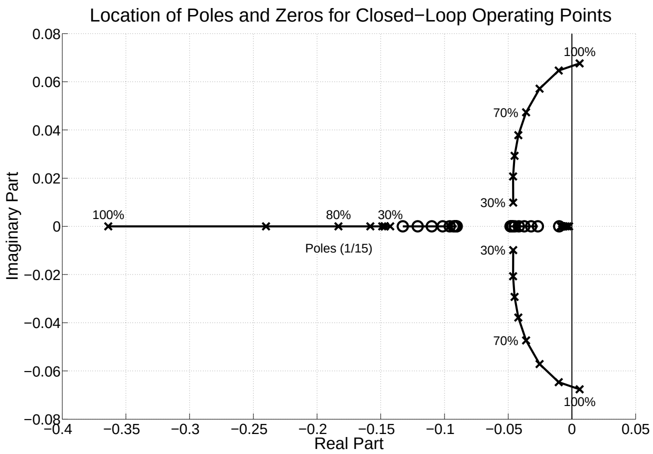

Fig. 6: The location of the closed loop poles for different operating points u^*. Controller parameters are $K_P = 3$ and $\tau_I = 100$.

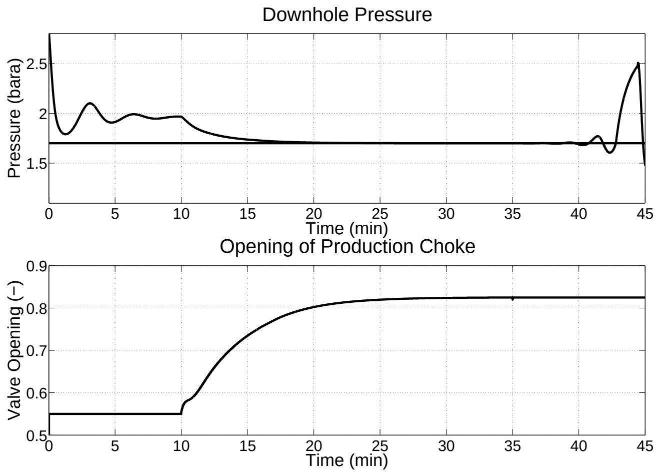

Fig. 7: Downhole pressure and production choke opening for simulation with the simplified model.

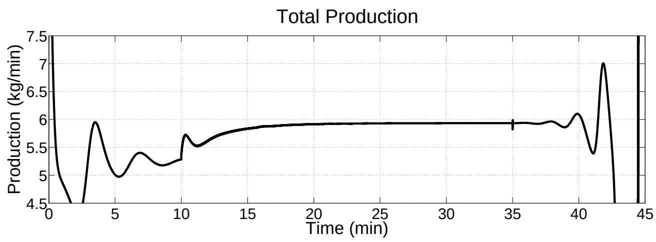

Fig. 8: Total production for simulation with the simplified model.

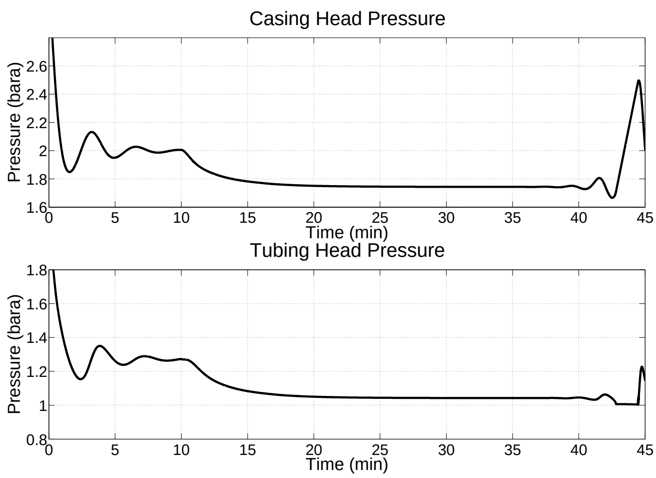

Fig. 9: Casing head pressure and tubing head pressure for simulation with the simplified model.

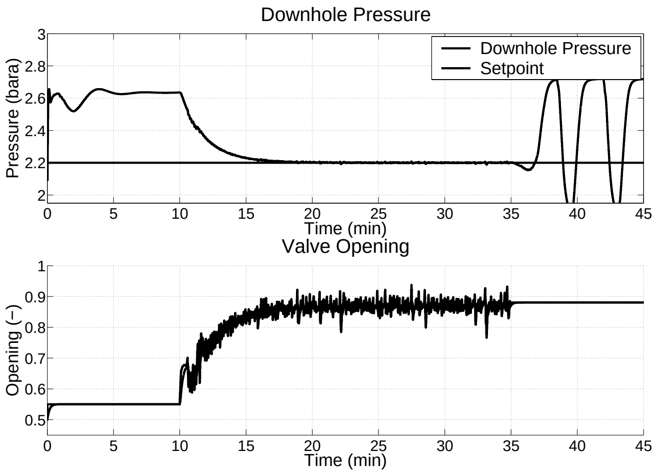

Fig. 10: Downhole pressure and production choke opening for simulation with OLGA 2000.

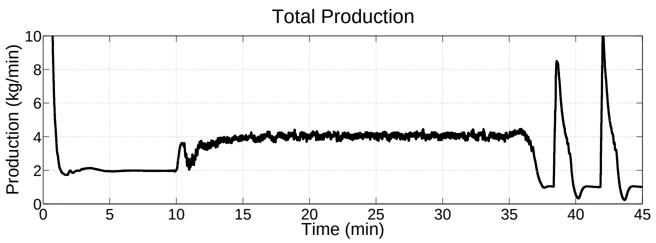

Fig. 11: Total production for simulation with OLGA 2000.

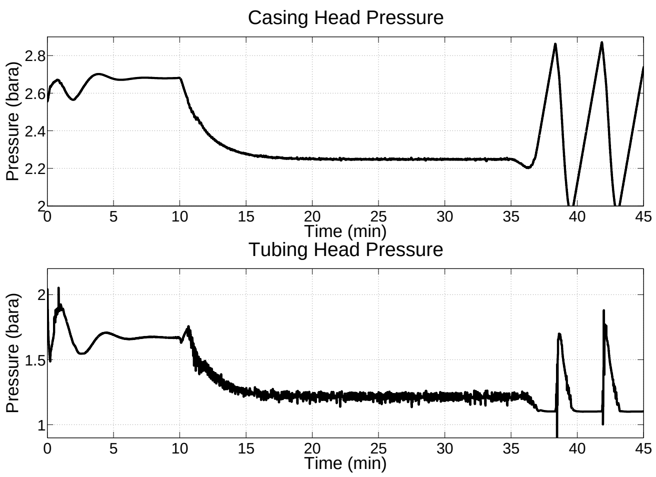

Fig. 12: Casing head pressure and tubing head pressure for simulation with OLGA 2000.

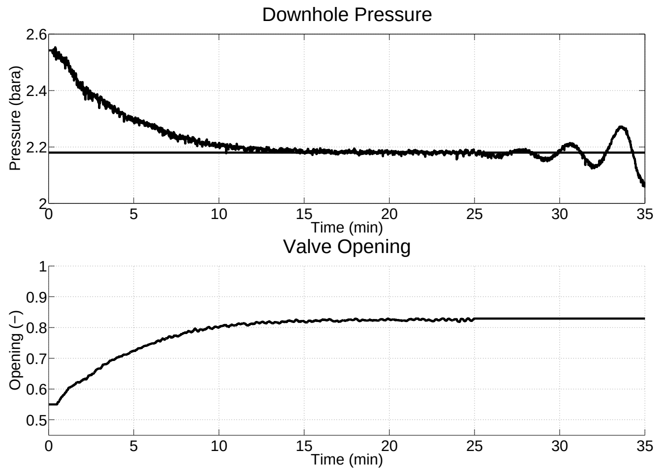

Fig. 13: Downhole pressure and production choke opening for laboratory experiment.

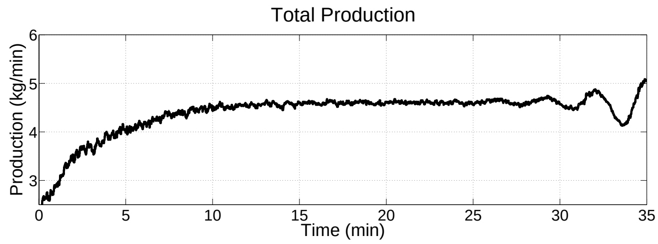

Fig. 14: Total production for laboratory experiment.

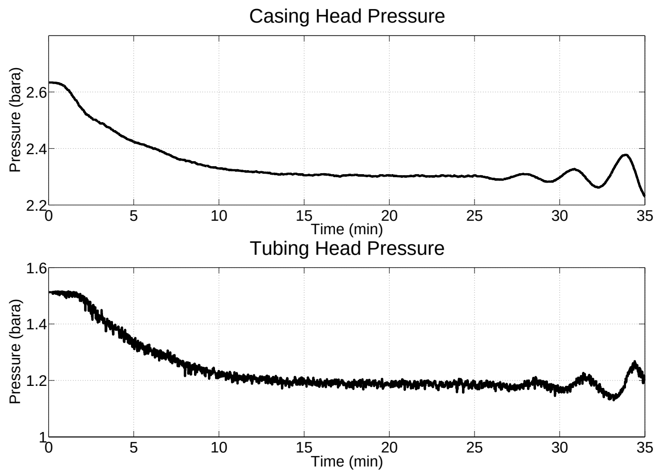

Fig. 15: Casing head pressure and tubing head pressure for laboratory experiment.

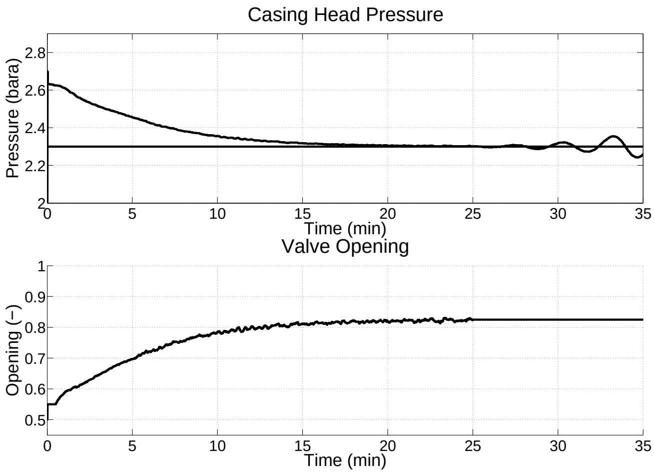

Fig. 16: Experimental results controlling casing head pressure.

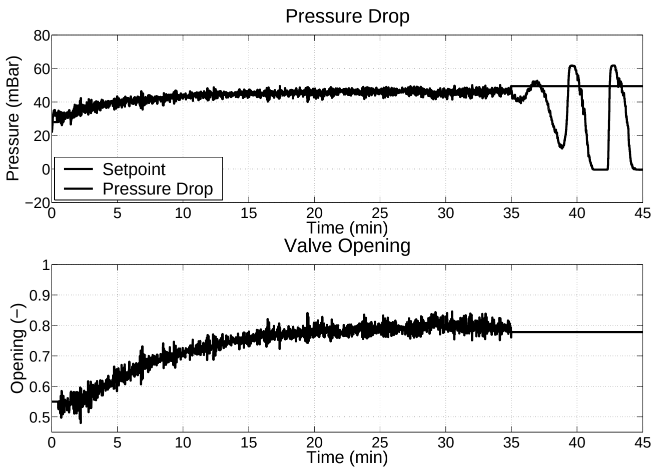

Fig. 17: Experimental results controlling differential pressure.

Paper III

Stabilization of Gas Lifted Wells Based on State Estimation

G.O. Eikrem, L.S. Imsland and B.A. Foss

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Hong Kong, China, January 11–14, 2004

STABILIZATION OF GAS LIFTED WELLS BASED ON STATE ESTIMATION

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Abstract: This paper treats stabilization of multiphase flow in a gas lifted oil well. Two different controllers are investigated, PI control using the estimated downhole pressure in the well, and nonlinear model based control of the total mass in the system. Both control structures rely on the use of a state estimator, and are able to stabilize the well flow with or without a downhole pressure measurement available. In both cases stabilization of gas lifted wells increases total production significantly.

Keywords: State estimation with Kalman filter, control, multiphase flow, positive systems.

1. INTRODUCTION

The use of control in multiphase flow systems is an area of increasing interest for the oil and gas industry. Oil wells with highly oscillatory flow are a significant problem in the petroleum industry. Several different instability phenomena related to oil and gas wells exist, in this study unstable gas lifted wells will be the area of investigation.

Gas lift is a technology to produce oil and gas from wells with low reservoir pressure by reducing the hydrostatic pressure in the tubing. Gas is injected into the tubing, as deep as possible, and mixes with the fluid from the reservoir, see Figure 2. The gas reduces the density of the fluid in the tubing, which reduces the downhole pressure, DHP, and thereby increases the production from the reservoir. The lift gas is routed from the surface and into the annulus, the volume between the casing and the tubing. The gas enters the tubing through a valve, an injection orifice.

The dynamics of highly oscillatory flow in a gas lifted well can be described as follows:

(1) Gas from the casing starts to flow into the tubing. As gas enters the tubing the pressure in the tubing falls. This accelerates the inflow of gas.
(2) The gas pushes the major part of the liquid out of the tubing.
(3) Liquid in the tubing generates a blocking constraint downstream the injection orifice. Hence, the tubing gets filled with liquid and the annulus with gas.
(4) When the pressure upstream the injection orifice is able to overcome the pressure on the downstream side, a new cycle starts.

This type of oscillation is described as casing-heading instability and is shown in the first part of Figure 5 and 6. More information can be found in Xu and Golan (1989).

There are in principle two approaches to eliminate highly oscillating well flow in gas lifted wells: The first approach is to increase the pressure drop

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caused by friction; either by increasing the gas flow rate, reducing the opening of the production choke or reducing the size of the gas orifice. The second method is the use of active control to stabilize the well flow, which is the subject of this study.

Figure 1 shows a conceptual gas lift production curve. The produced oil and gas rate is a function of the flow rate of gas injected into the well. The curve shows under which conditions the well exhibits stable or highly oscillatory flow. It is important to note that the average production rate may be significantly lower with unstable, see the line "open loop production", compared to stable well flow, see the line "theoretical production". The region of optimum lift gas utilization may lie in the unstable region.

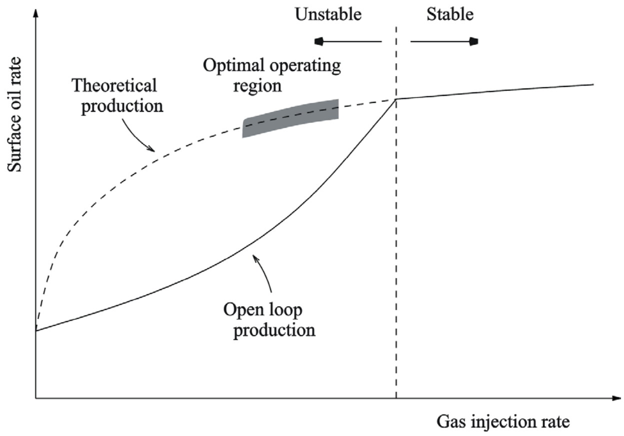

Fig. 1. The gas lift curve with the region of optimum lift gas utilization.

Large oscillations in the flow rate from the well causes lower total production, poor downstream oil/water separation, limits the production capacity and causes flaring. A reduction of the oscillations gives increased processing capacity because of the reduced need for buffer capacity in the process equipment.

Control has to a limited degree been studied for single well systems, see Jansen *et al.* (1999), Kinderen and Dunham (1998) and Dalsmo *et al.* (2002). In addition a two-well simulation study was investigated in Eikrem *et al.* (2002).

The scope of this paper is to study the use of state estimation and control as a tool for stabilizing highly oscillatory well flow in gas lifted wells. Further, earlier work with state feedback for nonlinear positive systems is extended to a realistic output feedback case.

This paper is structured as follows: The system and models are described in Section 2 and 3. A brief theoretical basis is outlined in Section 4 and 5, while the results are shown in Section 6. The paper ends with a discussion and some concluding remarks.

2. SYSTEM DESCRIPTION

2.1 Single Well System

The basis for this study is a realistic gas lifted well, see Figure 2. Reservoir fluid flows through a perforated well, into the wellbore, upwards through the tubing, through the production choke, before it enters downstream equipment which typically will be a manifold and an inlet separator. Gas is injected into the annulus and enters the tubing close to the bottom of the well. The gas mixes with the reservoir fluid to reduce the density of the fluid in the tubing.

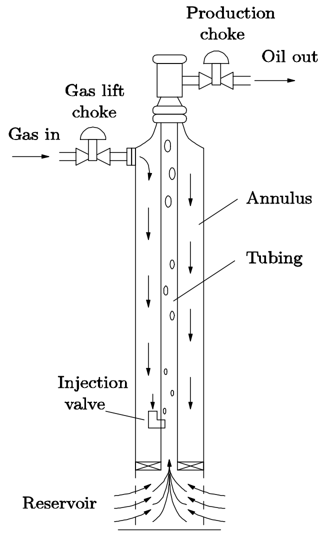

Fig. 2. A gas lifted oil well

The well is described by the following parameters:

- Well parameters
  - · 2048 m vertical well
  - · 5 inch tubing
  - · 2.75 inch production choke
  - · 0.5 inch injection orifice
- Reservoir parameters
  - · P_R = 160 bara
  - · T_R = 108 ^{o}C
  - · PI = 2.47E-6 kg/s/Pa
- Separator inlet pressure
  - · 15 bara
- Gas injection into annulus
  - · 0.8 kg/s
  - · 160 bara
  - · 60 ^{o}C

The productivity index, PI, is defined by

$$PI = \frac{\dot{m}}{\Delta P}$$

where $\dot{m}$ is the total mass flow rate from the reservoir to the well and ΔP is the pressure difference between the reservoir and the bottom of the well. This index relates the mass flow from the reservoir and into the well to the corresponding pressure drop. The PI is assumed constant.

It is assumed that there is no water in the produced fluids, only oil and gas. The gas/oil ratio, GOR, is 80 Sm^3/Sm^3. GOR is defined by:

$$GOR = \frac{\dot{q}_{gas}}{\dot{q}_{oil}}$$

Hence the GOR is defined as the ratio between the volumetric gas rate and the volumetric oil rate at standard temperature and pressure.

The valve model for the production choke includes limitation for the actuator speed, closing time for the valve is 420 sec.

2.2 Simulator

The transient multiphase flow simulator OLGA 2000 [2], commonly used in the petroleum industry, is selected as a platform for the simulations. The state estimator and the controllers are implemented in Matlab [3]. OLGA 2000 and Matlab are connected using a Matlab-OLGA link [4].

OLGA 2000 is a modified two-fluid model, i.e. separate continuity equations for the gas, liquid bulk and liquid droplets are applied. Two momentum equations are used, one for the continuous liquid phase and one for the combination of gas and possible liquid droplets. Entrainment of liquid droplets in the gas phase is given by a slip relation. One mixture energy equation is applied. This yields six conservation equations to be solved in each volume (Scandpower, 2001).

The OLGA 2000 model developed for the gas lifted well is built upon the description given in Section 2. The OLGA 2000 model consists of an annulus divided into 25 volumes, and a tubing divided into 25 volumes. The fluid used in the simulations consists of two phases, oil and gas. The inflow of oil and gas from the reservoir is modelled by use of the productivity index, as defined in section 2. The injection rate of lift gas to the annulus is fixed, a fast and well tuned flow controller is assumed used. Fixed boundary conditions for the tubing is assumed, i.e. a fixed reservoir pressure and a fixed separator pressure downstream the production choke.

3. A SIMPLE GAS LIFT MODEL

To be able to develop a state estimator, a simplified model of the gas lifted well is required. This model uses the same boundary conditions as the OLGA 2000 model, but has no mass transfer between the phases, only one volume for the annulus and only one volume for the tubing. The flow between the volumes, into the system and out of the system is controlled by general valve models:

$$w = \begin{cases} C\sqrt{\rho(p_2 - p_1)} & \text{if } \; p_2 \geq p_1 \\ 0 & \text{else} \end{cases} \tag{1}$$

where w is the mass flow, C is the valve parameter, ρ is the density, while the $p_2 - p_1$ is the pressure drop across the restriction. C takes on different values for each restriction.

The pressures in the system are calculated from the mass in the volumes and the pressure drop through the volumes. The pressure at the top of the annulus is calculated by use of the ideal gas law. The pressure at the bottom of the annulus is given by adding the pressure drop from the gas column to the pressure at the top of the annulus. The pressure at the top of the tubing is calculated by the ideal gas law. The volume of the gas in the tubing is given by the volume which is not occupied by oil. The pressure at the bottom of the tubing is given by adding the pressure drop from the fluid column to the pressure at the top of the tubing. Based upon the pressure calculations of the system, the mass flows in and out of the volumes are given by the valve equation (1). The model parameters are tuned based upon OLGA simulations.

To summarize, the following mass balances are assumed to describe the dynamics of the gas lifted well:

$$\begin{aligned} \dot{x}_1 &= w_{iv}(x) - w_{gc}(x) && \text{Mass of gas in annulus} \\ \dot{x}_2 &= w_{gc}(x) - w_{pg}(x,u) && \text{Mass of gas in tubing} \\ \dot{x}_3 &= w_r(x) - w_{po}(x,u) && \text{Mass of oil in tubing} \end{aligned}$$

The symbols are described in Table 1.

Table 1. Symbols

Symbol	Description
$w_{iv}(x)$	Gas flow from source into annulus
$w_{gc}(x)$	Gas flow from annulus into tubing
$w_{pg}(x,u)$	Gas flow out of tubing
$w_r(x)$	Oil flow from reservoir into tubing
$w_{po}(x,u)$	Oil flow out of tubing
u	Production choke
M	Total mass in system
λ	Mass control parameter
w_{ref}	Setpoint for flow controller

The simplified model herein is a modified version of the simplified gas lift well model given in Imsland (2002).

[2] Scandpower AS, Norway
[3] The Mathworks Inc
[4] ABB AS, Norway

4. THEORETICAL BASIS

4.1 State Estimation

A standard extended Kalman filter based on the simplified model is developed. Numerical derivation of the simplified model is used to derive a linear model at each time step, corresponding to the current operating point. The covariance matrices for the process and measurement noise are diagonal matrices. The measurement noise matrix is designed based upon the uncertainty of the measurement devices. This matrix is scaled to account for differences in the range of the measurements. The process noise matrix is tuned to obtain a reasonable bandwidth for the state estimator.

4.2 Positive Systems and Feedback Control

Positive systems are dynamical systems which are described by ODEs where the state variables are non-negative. Since mass is an inherently positive quantity, systems modelled by mass balances are a natural example of positive systems, see e.g. Bastin (1999). In Imsland (2002) and Imsland *et al.* (2003) a state feedback controller that exploits positivity is developed. Further, it is shown that the controller exhibits robust stability properties. This work is extended by applying this method in a realistic output feedback setting.

The purpose of the controller is to stabilize the total mass in the system. This is achieved by linearizing the total mass dynamics and exploiting the positivity of the system. The controller calculates the setpoint for an "inner" PI mass flow control loop, and this setpoint is given by:

$$w_{ref} = \max\{0, w_r(x) + w_{iv}(x) + \lambda[M^* - M(x)]\} \quad (2)$$

where M^* is the total mass setpoint. The symbols are described in Table 1.

5. CONTROL STRUCTURES

Several control structures for stabilization of gas lifted wells are available. The possibilities of stabilizing a gas lifted well by use of the measured downhole pressure or the measured casing head pressure have been shown in e.g. Eikrem *et al.* (2002).

5.1 Kalman Filter and Measurements

The Kalman filter uses the available process measurements for correction of the states in the simplified model, in this case the masses in the system. The selected measurements are the pressure at the top of the tubing, the pressure at the top of the casing and the pressure at the bottom of the well. These are realistic measurements from an industrial point of view. Since the downhole pressure measurement is located in a harsh and quite inaccessible location, the effect of failure of this measurement will be investigated.

The Kalman filter includes a check on positivity of the state variables in the sense that state estimates always will be positive.

5.2 Pressure Control and DHP Measurement Failure

The first control structure uses the opening of the production choke as the manipulated variable and the estimated downhole pressure as the controlled variable. The PI-controller is tuned on the basis of process knowledge and iterative simulations. The controller, including the state estimator, is shown in Figure 3.

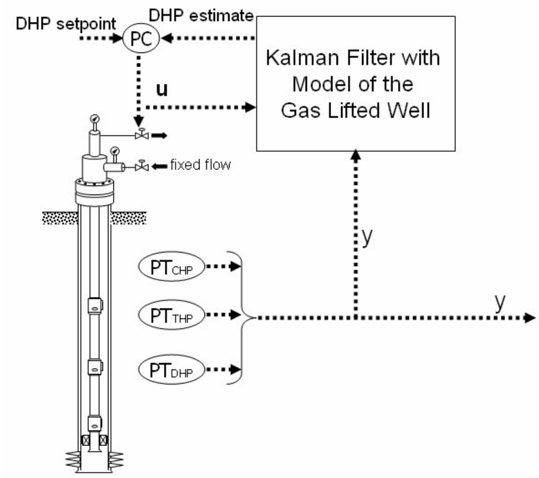

Fig. 3. Control structure for stabilization of a gas lifted well, by controlling the estimated downhole pressure.

5.3 Mass Control and DHP Measurement Failure

The second control structure uses the opening of the production choke as the manipulated variable and the total mass in the system as the controlled variable. In a cascade-manner, the setpoint for the "inner" flow control loop is given by w_{ref}, see (2). The controller including the state estimator is shown in Figure 4.

5.4 Simulation Scenario

The simulations follow the same scenario:

- Timeslot 1, (0-4 h): The well simulator is run in open loop with 50% choke opening.

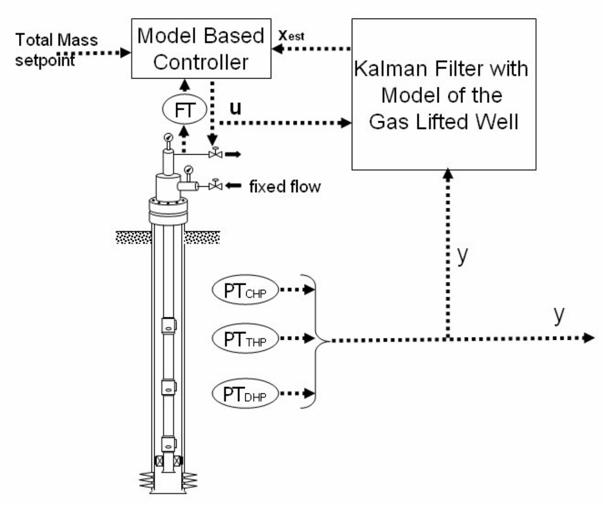

Fig. 4. Control structure for stabilization of gas lifted well, by controlling the total mass in the system.

- Timeslot 2, (4-10 h): The well is stabilized by use of a small choke opening, 20%.
- Timeslot 3: (10-19h): Control using estimated variable, with DHP measurement available.
- Timeslot 4: (19-25h): Control using estimated variable, without DHP measurement available.
- Timeslot 5 (25-30 h): Open loop simulation.

6. SIMULATION RESULTS

6.1 Pressure Control and DHP Measurement Failure

The result from the stabilization of the gas lifted well based upon estimated downhole pressure is given in Figure 5.

The highly oscillatory behaviour is clearly observed during Timeslot 1. The flow is stabilized by closing the valve to 20%, i.e. by increasing the pressure drop caused by friction. The flow is well behaved during Timeslot 3. There is a major disturbance due to the loss of the DHP measurement at 19 hours. This is reasonable since the DHP estimate is heavily influenced by the DHP measurement. The important issue, however, is the fact that the flow is stable. Moreover, the flow becomes highly oscillatory after the controller has been deactivated during Timeslot 5. It should be mentioned that the controller showed a close-to-identical behaviour during Timeslot 3, when the DHP estimate was replaced by the DHP measurement.

The values for the controller parameters for the PI controller are K_p = -0.1 and T_i = 7200 sec. The pressure measurement is given in bara.

The production of oil and gas is given in Figure 6. The stabilization of the gas lifted oil well gives a significant increase in the produced amount of oil and gas. This is particularly pronounced by comparing the production during Timeslot 4 and 5, and this agrees with Figure 1. In the unstable region, the average production rate is 6 kg/s, while the stabilized region gives a production of 15 kg/s.

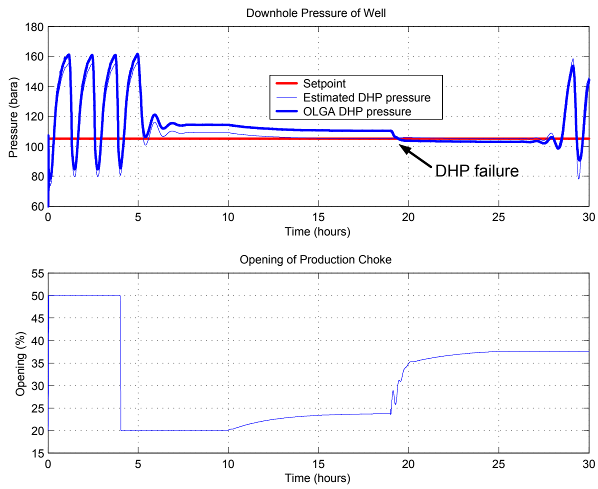

Fig. 5. The estimated and the OLGA downhole pressure for the well. The OLGA well is stabilized by stabilizing the estimated downhole pressure.

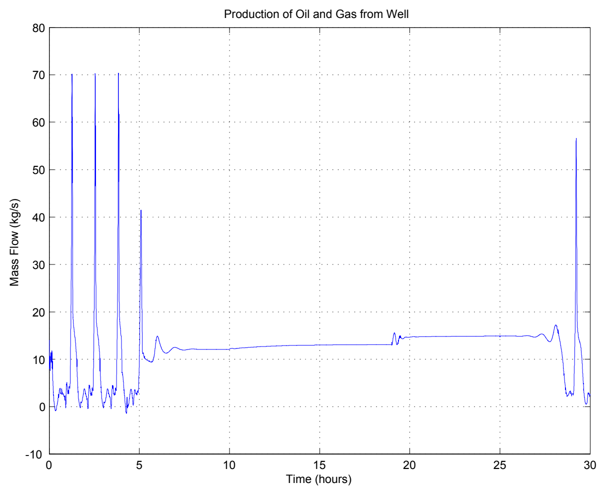

Fig. 6. The oil production from the well when the well is unstable and when it is stabilized.

6.2 Mass Control and DHP Measurement Failure

The result from the stabilization of the gas lifted well based upon mass estimation is given in Figure 7. The downhole pressure measurement fails after 19 hours. The description of the simulation results is identical to the description in the previous case, see Section 6.1. Note again the disturbance when the DHP measurement fails.

Figure 7 reveals that the controller quickly takes the system mass to the desired value, or close to, due to model and estimation error. It can be observed that the input continues to move afterwards. This can be explained by the fact that the point on the "constant mass"-manifold which the system initially converges to, is not the closed loop equilibrium. The slow dynamics on the manifold takes the system to this equilibrium.

The controller parameters for the "inner" mass flow control loop are $K_p = 0.004$ and $T_i = 5$ sec, while the parameter for the "outer" total mass control loop is $\lambda = 0.003$. The mass is given in kg.

The production of oil and gas for the system in open and closed loop is similar to the results given in Figure 6.

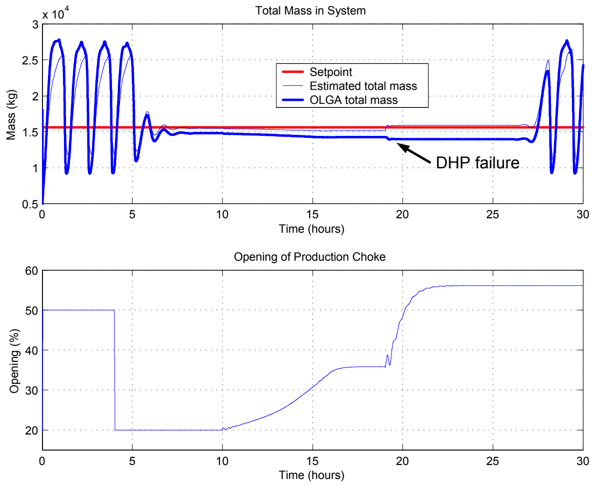

Fig. 7. The estimated and the OLGA total mass for the well. The system is stabilized by controlling the estimated total mass.

7. DISCUSSION AND CONCLUDING REMARKS

This study shows how a low order model in a Kalman filter can provide useful information for control purposes. In particular it is shown how a state estimator can alleviate the common situation in which a difficult accessible downhole measurement fails. It is further shown that stabilization of gas lifted wells is important since it gives an increased production, in this case the production of oil and gas is more than doubled, see Figure 6.

The state estimator functions well both with the traditional PI-controller and the nonlinear controller for positive systems. In this paper the controllers are not pushed to the limit to assess the potential of the nonlinear controller. The possible merit of the nonlinear controller has been showed in a realistic output feedback application.

The simplified model needs to be well tuned to reflect the dynamics of the real system as the DHP measurement fails. The main challenge is related to the estimation of downhole conditions upon the loss of the DHP measurement. In practice the tuning is done by adjusting the valve parameters, see (1). Typically they have been changed ±25% compared to their original values. It should be mentioned that the low order model is observable at all times.

An alternative to the current approach is the use of an augmented Kalman filter in which model parameters are tuned online.

To re-iterate there are alternative control structures, that do not involve downhole measurements nor estimation, that are able to stabilize the highly oscillatory flow for this particular well. There is still considerable value in the problem addressed in this paper since other more complex well completions may require measurements or estimates of downhole conditions.

8. ACKNOWLEDGEMENT

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Paper IV

Observer design for multiphase flow in vertical pipes with gas-lift - theory and experiments

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Observer design for multiphase flow in vertical pipes with gas-lift—theory and experiments ☆

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Abstract

Unstable regimes occurring for multiphase flow in vertical risers have successfully been stabilized using conventional linear control techniques. However, these control systems rely on downhole measurements which are at best unreliable, if at all available. In this paper, we design a nonlinear observer for the states of the multiphase flow that relies on topside measurements only, and apply it to estimate downhole pressure for feedback control. A key feature of the design is that it exploits the structure of the model to obtain robustness with respect to the internal flows in the system. Combining the nonlinear observer with conventional PI control of the downhole pressure, we demonstrate in laboratory experiments the potential for increasing production from gas-lift wells by stabilizing the multiphase flow.

Keywords: Nonlinear observer design; Multiphase flow; Petroleum technology

1. Introduction

Pipelines and oil wells with highly oscillatory flow constitute a significant problem in the petroleum industry, and efforts to find inexpensive solutions based on automatic control have increased both in academia and industry [8,5,12,6,15,13,11]. Several different instability phenomena related to oil and gas pipelines and wells exist. This study will investigate one such phenomenon: unstable gas-lift wells. Gas-lift is a technology that reduces the hydrostatic pressure in the tubing, facilitating production from wells with low reservoir pressure. Gas is injected into the tubing, as deep as possible, and mixes with the fluid from the reservoir (see Fig. 1). Since the gas has lower density than the reservoir fluid, the density of the fluid in the tubing, and consequently the downhole pressure (DHP), decrease. When the downhole pressure decreases, the production from the reservoir increases. The lift gas is routed from the surface and into the annulus, which is the volume between the casing and the tubing, and enters the tubing through a valve, or an injection orifice. Backflow from the tubing into the annulus is not permitted by this valve. The dynamics of highly oscillatory flow in gas-lift wells can be described as follows:

(1) Gas from the annulus starts to flow into the tubing. As gas enters the tubing the pressure in the tubing falls, accelerating the inflow of lift-gas.
(2) The gas pushes the major part of the liquid out of the tubing, while the pressure in the annulus falls dramatically.
(3) The annulus is practically empty, and the gas flow into the tubing is blocked by liquid accumulating in the tubing. Due to the blockage, the tubing becomes filled with liquid and the annulus with gas.

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(4) Eventually, the pressure in the annulus becomes high enough for gas to penetrate into the tubing, and a new cycle starts.

For more information on this type of instability, often termed *casing-heading instability*, leading to *severe slugging*, see [16].

There are in principle two approaches to eliminate highly oscillating flow in gas-lift wells. The first approach is to increase the pressure drop caused by friction. Here it is possible to increase the gas flow rate, reduce the opening of the production choke, reduce the size of the gas-lift valve, or decouple the dynamics of the annulus and tubing by obtaining supercritical flow through the injection valve. The second approach is the use of active control to stabilize the well flow, which is the subject of this study. Fig. 2 shows a conceptual gas-lift production curve. The produced oil rate is a function of the flow rate of gas injected into the well. The curve shows conditions under which the well exhibits stable or highly oscillatory flow. It is important to note that the average production rate may be significantly lower with unstable well flow, than with stable well flow. This is illustrated by contrasting the "open loop production" to the "theoretical production" curves in Fig. 2. The region of optimum lift gas utilization may lie in the unstable region. In addition to causing lower oil production, large oscillations in the flow rate from the well complicate downstream gas/oil/water separation, and may even cause flaring.

Stabilization of gas-lift wells using conventional control techniques has been studied for single well systems in [1,9,10], and for a two-well system in [3]. In [7], a state feedback control law was designed using Lyapunov theory, and the controller was used in an output feedback setting with an extended Kalman filter in [4]. In this paper, we design a nonlinear observer for the states of the multiphase flow in the tubing, and apply it to estimate downhole pressure for feedback control. The design exploits the structure of the model to obtain robustness with respect to the internal flow between the annulus and the tubing.

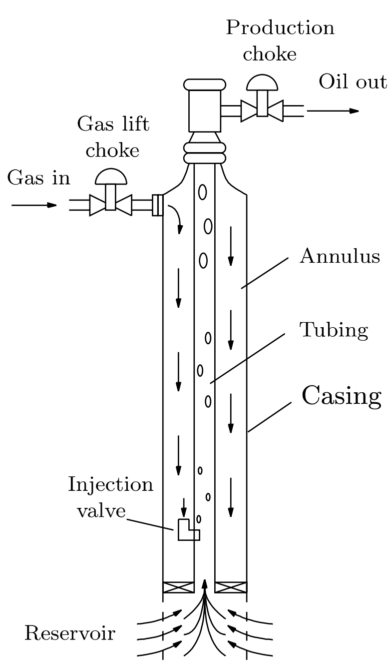

Fig. 1. A gas-lift oil well.

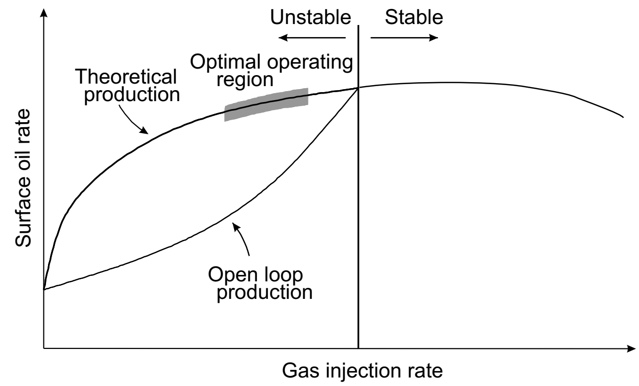

Fig. 2. Gas-lift curve with the region of optimum lift gas utilization.

The paper is organized as follows: In Section 2 we present a mathematical model of the gas-lift well due to [4,7]; in Section 3 we design the observer and apply it in open-loop simulations; in Section 4 an output feedback stabilization scheme combining the nonlinear observer with PI control of the estimated downhole pressure is proposed, and; Section 5 presents experimental results using this controller. Concluding remarks are offered in Section 6.

2. Mathematical model

The process described in Section 1, and sketched in Fig. 1, is modelled mathematically by three states: x_1 is the mass of gas in the annulus; x_2 is the mass of gas in the tubing, and; x_3 is the mass of oil in the tubing. Looking at Fig. 1, we have

$$\dot{x}_1 = w_{gc} - w_{iv}, \tag{1}$$

$$\dot{x}_2 = w_{iv} - w_{pg}, \tag{2}$$

$$\dot{x}_3 = w_r - w_{po}, \tag{3}$$

where w_{gc} is a constant mass flow rate of lift gas into the annulus, w_{iv} is the mass flow rate of lift gas from the annulus into the tubing, w_{pg} is the mass flow rate of gas through the production choke, w_r is the oil mass flow rate from the reservoir into the tubing, and w_{po} is the mass flow rate of produced oil through the production choke. The flows are modeled by

$$w_{gc} = \text{constant flow rate of lift gas}, \tag{4}$$

$$w_{iv} = C_{iv}\sqrt{\rho_{a,i}\max\{0, p_{a,i} - p_{t,i}\}}, \tag{5}$$

$$w_{pc} = C_{pc}\sqrt{\rho_m \max\{0, p_t - p_s\}}u, \tag{6}$$

$$w_{pg} = \frac{x_2}{x_2 + x_3} w_{pc}, \tag{7}$$

$$w_{po} = \frac{x_3}{x_2 + x_3} w_{pc}, \tag{8}$$

$$w_r = C_r(p_r - p_{t,b}). \tag{9}$$

C_{iv}, C_{pc}, and C_r are constants, u is the production choke opening ($u(t) \in [0,1]$), $\rho_{a,i}$ is the density of gas in the annulus at the injection point, ρ_m is the density of the oil/gas mixture at the top of the tubing, $p_{a,i}$ is the pressure in the annulus at the injection point, $p_{t,i}$ is the pressure in the tubing at the gas injection point, p_t is the pressure at the top of the tubing, p_s is the pressure in the separator, p_r is the pressure in the reservoir, and $p_{t,b}$ is the pressure at the bottom of the tubing. The separator pressure, p_s, is assumed to be held constant by a control system, and the reservoir pressure, p_r, is assumed to be slowly varying and therefore treated as constant. Note that flow rates through the valves are restricted to be positive. The densities are modelled as follows

$$\rho_{a,i} = \frac{M}{RT_a} p_{a,i}, \tag{10}$$

$$\rho_m = \frac{x_2 + x_3 - \rho_o L_r A_r}{L_t A_t}, \tag{11}$$

and the pressures as follows

$$p_{a,i} = \left(\frac{RT_a}{V_a M} + \frac{gL_a}{V_a}\right) x_1, \tag{12}$$

$$p_t = \frac{RT_t}{M} \frac{x_2}{L_t A_t + L_r A_r - v_o x_3}, \tag{13}$$

$$p_{t,i} = p_t + \frac{g}{A_t}(x_2 + x_3 - \rho_o L_r A_r), \tag{14}$$

$$p_{t,b} = p_{t,i} + \rho_o g L_r. \tag{15}$$

M is the molar weight of the gas, R is the gas constant, T_a is the temperature in the annulus, T_t is the temperature in the tubing, V_a is the volume of the annulus, V_t is the volume of the tubing, L_a is the length of the annulus, L_t is the length of the tubing, A_t is the cross-sectional area of the tubing above the injection point, L_r is the length from the reservoir to the gas injection point, A_r is the cross-sectional area of the tubing below the injection point, g is the gravity constant, ρ_o is the density of the oil, and v_o is the specific volume of the oil. The molar weight of the gas, M, the density of oil, ρ_o, and the temperatures, T_a and T_t are assumed slowly varying and therefore treated as constants.

In summary, the model covers the following case:

- Two-phase flow in the tubing, treating oil and water as a single phase;
- No flashing effects;
- Low gas-to-oil ratio (GOR), reflected in the fact that the flow from the reservoir is modelled as pure oil, and;
- Slowly varying components of gas and oil.

The dynamics of the model has been compared to that of the OLGA 2000 [1] multiphase flow simulator in [7], and found to be in satisfactory agreement. It should be noted, however, that the simplicity of the model is a result of the modelling objective, which is to adequately capture the casing-heading instability. A number of other instabilities may occur in gas-lifted oil wells, for instance tubing-heading instability, tubing–reservoir interactions, and hydrodynamic slugging, but these are not captured by this model.

3. State estimation

In practice, measurements downhole in the tubing or annulus will in general not be available. If they are available, they must be considered unreliable due to the harsh conditions in which the sensors operate, and the fact that maintenance of the sensors is virtually impossible (a very high failure rate of installed pressure sensors is reported by Statoil [14]). Thus, we will in this work assume that we have measurements at the top of the annulus and tubing, only. The main challenge is how to deal with the multiphase flow in the tubing, whereas the single phase flow in the annulus can accurately be estimated based on one pressure measurement and one temperature measurement. Thus, we will assume that x_1 is measured. For estimation of the two remaining states, we measure the pressure at the top of the tubing, and either the flow through the production choke *or* the density at the top of the tubing. Our measurements are therefore

$$y_1(t) = x_1(t), \quad y_2(t) = p_t(t), \quad \text{and}$$
$$y_3(t) = w_{pc}(t) \quad \text{or} \quad y_3(t) = \rho_m(t). \tag{16}$$

3.1. Reduced order observer design

Since the mass of gas in the annulus can be considered a measurement, we design a reduced order observer

[1] OLGA 2000 is a state-of-the-art multiphase flow simulator available from Scandpower AS. http://www.olga2000.com.

for the remaining two states. Before we state our main result, we state key assumptions and intermediate results needed in the convergence proof for the observer.

Assumption 1. The production choke is not allowed to close completely. That is,

$$u \geqslant \delta_u > 0, \quad \forall t \geqslant 0. \tag{17}$$

Assumption 2. The states are bounded away from zero, and the part of the tubing below the gas injection point is filled with oil. More precisely,

$$\begin{aligned} &x_1 \geqslant \delta_1 > 0, \quad x_2 \geqslant \delta_2 > 0, \quad \text{and} \\ &x_3 \geqslant \rho_o L_r A_r + \delta_3 > \rho_o L_r A_r, \qquad \forall t \geqslant t_0. \end{aligned} \tag{18}$$

Assumption 3. The gas in the tubing has lower density than the oil. More precisely,

$$L_t A_t + L_r A_r - v_o(x_3 + x_2) \geqslant \delta_g > 0, \quad \forall t \geqslant 0. \tag{19}$$

Remark 4. Assumptions 1–3 are not restrictive. Since the production choke opening is a control input, Assumption 1 can be satisfied by the construction of the control law. Of course, δ_u has consequences for the solvability of the state feedback regulation problem, and must therefore be sufficiently small. The first condition in (18) is always satisfied in practice, since there is a steady flow of gas into the annulus. The second condition in (18) is imposed to deal with the fact that the model of the compressible multiphase flow in the tubing is invalid when the fluid is incompressible, which corresponds to $x_2 = 0$, since the pressure calculation is based on the ideal gas law. From a practical point of view there will always be some gas in the tubing, so this assumption is no restriction. The third condition in (18) states that the reservoir pressure must be high enough for oil to rise above the gas injection point in the tubing. The last assumption, Assumption 3, imposes an upper bound on the gas density in the tubing. For practical gas-lift oil wells, the density of gas will always be less than the density of oil under normal operation.

Lemma 5 [2, Lemma 2]. *Let $x = 0$ be an equilibrium point for the nonlinear system*

$$\dot{x} = f(t, x), \quad x(t_0) = x_0, \tag{20}$$

where $f : [t_0, \infty) \times \mathbb{R}^n \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x. Let $V : [t_0, \infty) \times \mathbb{R}^n \to \mathbb{R}_+$ be a continuously differentiable function such that

$$k_1 \|x\|^c \leqslant V(t, x) \leqslant k_2 \|x\|^c, \tag{21}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) \leqslant -k_3 \|x\|^c + g(\|x\|)\sigma(\|x(t_0)\|, t - t_0) \tag{22}$$

$\forall t \geqslant t_0$, $\forall x \in \mathbb{R}^n$, where k_1, k_2, k_3, and c, are strictly positive constants, $g : \mathbb{R}_+ \to \mathbb{R}$ is continuous, and σ is a class $\mathcal{KL}$ function satisfying

$$\int_{t_0}^{\infty} \sigma(r, s)\,ds \leqslant \sigma_\infty r \tag{23}$$

for some constant σ_∞. Suppose that there exist constants $k > 0$, and $r \geqslant 0$ such that $k\|x\|^c \geqslant g(\|x\|)$, $\forall \|x\| \geqslant r$. Then, the equilibrium point $x = 0$ of (20) is globally uniformly asymptotically stable. Moreover, solutions of (20) satisfy

$$\|x(t)\| \leqslant C e^{-\frac{k_3}{c k_1}(t - t_0)}, \tag{24}$$

where C depends on the initial state and σ_∞.

Lemma 6. *Solutions of system (1)–(3) are bounded in the sense that there exists a constant B, depending on the initial state, such that*

$$x_i \leqslant B(x(t_0)), \quad i = 1, 2, 3, \ \forall t \geqslant 0. \tag{25}$$

In particular,

$$x_3 < \rho_o(L_t A_t + L_r A_r), \quad \forall t \geqslant 0. \tag{26}$$

Proof. It is easily shown that the Lyapunov function candidate $V = 2x_1 + x_2 + x_3$ is strictly negative for sufficiently large V. □

The following theorem states the main result when the flow through the production choke is measured, that is $y_3(t) = w_{pc}(t)$.

Theorem 7. *Solutions $\hat{x}(t) = (\hat{x}_2(t), \hat{x}_3(t))$ of the observer*

$$\dot{\hat{z}}_1 = w_{gc} - \frac{\hat{z}_1 - y_1}{\hat{z}_2 - y_1} y_3 + k_1(\hat{z}_1, \hat{z}_2, y_1, y_2), \tag{27}$$

$$\begin{aligned} \dot{\hat{z}}_2 = {} & w_{gc} \\ & + C_r\left(p_r - \rho_o g L_r + \frac{A_r}{A_t}\rho_o g L_r + \frac{g}{A_t} y_1 - y_2 - \frac{g}{A_t}\hat{z}_2\right) \\ & - y_3 + k_2(\hat{z}_2, u, v, y_1, y_2), \end{aligned} \tag{28}$$

$$\hat{z}_1 \geqslant \delta_2 + y_1, \quad \text{and} \quad \hat{z}_2 \geqslant \rho_o L_r A_r + \delta_3 + \hat{z}_1, \tag{29}$$

$$\hat{x}_2 = \hat{z}_1 - y_1, \tag{30}$$

$$\hat{x}_3 = \hat{z}_2 - \hat{z}_1, \tag{31}$$

where the output injections, k_1 and k_2, are given by

$$\begin{aligned} k_1(\hat{z}_1, \hat{z}_2, y_1, y_2) = c_1\bigg(& \frac{M}{R T_t}(L_t A_t + L_r A_r - v_o(\hat{z}_2 - \hat{z}_1)) y_2 \\ & - (\hat{z}_1 - y_1)\bigg), \end{aligned} \tag{32}$$

$$k_2(\hat{z}_2, u, y_1, y_2, y_3) = c_2\left(\left(\frac{y_3}{C_{\mathrm{pc}}u}\right)^2 - \frac{\hat{z}_2 - y_1 - \rho_{\mathrm{o}}L_{\mathrm{r}}A_{\mathrm{r}}}{L_{\mathrm{t}}A_{\mathrm{t}}}(y_2 - p_{\mathrm{s}})\right), \tag{33}$$

converge to the actual state $x(t) = (x_2(t), x_3(t))$ *exponentially fast in the following sense*

$$\|x(t) - \hat{x}(t)\| \leqslant C\mathrm{e}^{-\gamma x(t-t_0)}, \tag{34}$$

where C depends on initial conditions, and

$$\gamma = \min\left\{c_1\frac{\delta_{\mathrm{g}}}{L_{\mathrm{t}}A_{\mathrm{t}} + L_{\mathrm{r}}A_{\mathrm{r}}}, \frac{C_{\mathrm{r}}g}{A_{\mathrm{t}}} + c_2\frac{\delta_{\mathrm{p}}}{L_{\mathrm{t}}A_{\mathrm{t}}}\right\}. \tag{35}$$

$\delta_p \geqslant 0$ *is a constant satisfying* $\max\{0, p_t - p_s\} \geqslant \delta_p$ *for all* $t \geqslant t_0$.

Proof. Define $z_2 = x_1 + x_2 + x_3$, which is the total amount of mass in the system. From (1)–(3), (9), (15), (14), its time derivative is

$$\dot{z}_2 = w_{\mathrm{gc}} + C_{\mathrm{r}}\left(p_{\mathrm{r}} - \rho_{\mathrm{o}}gL_{\mathrm{r}} + \frac{A_{\mathrm{r}}}{A_{\mathrm{t}}}\rho_{\mathrm{o}}gL_{\mathrm{r}} + \frac{g}{A_{\mathrm{t}}}y_1 - y_2 - \frac{g}{A_{\mathrm{t}}}z_2\right) - y_3. \tag{36}$$

We estimate z_2 by $\hat{z}_2$, which is governed by

$$\dot{\hat{z}}_2 = w_{\mathrm{gc}} + C_{\mathrm{r}}\left(p_{\mathrm{r}} - \rho_{\mathrm{o}}gL_{\mathrm{r}} + \frac{A_{\mathrm{r}}}{A_{\mathrm{t}}}\rho_{\mathrm{o}}gL_{\mathrm{r}} + \frac{g}{A_{\mathrm{t}}}y_1 - y_2 - \frac{g}{A_{\mathrm{t}}}\hat{z}_2\right) - y_3 + k_2(\cdot), \tag{37}$$

where $k_2(\cdot)$ is an output injection term to be determined. The observer error, $e_2 = z_2 - \hat{z}_2$, is governed by

$$\dot{e}_2 = -\frac{C_{\mathrm{r}}g}{A_{\mathrm{t}}}e_2 - k_2(\cdot). \tag{38}$$

Take the Lyapunov function candidate $V_2 = \frac{1}{2}e_2^2$. Its time derivative along solutions of (38) is

$$\dot{V}_2 = e_2\left(-\frac{C_{\mathrm{r}}g}{A_{\mathrm{t}}}e_2 - k_2(\cdot)\right). \tag{39}$$

Selecting

$$k_2(\hat{z}_2, u, y_1, y_2, y_3) = c_2\left(\left(\frac{y_3}{C_{\mathrm{pc}}u}\right)^2 - \frac{\hat{z}_2 - y_1 - \rho_{\mathrm{o}}L_{\mathrm{r}}A_{\mathrm{r}}}{L_{\mathrm{t}}A_{\mathrm{t}}} \times \max\{0, y_2 - p_{\mathrm{s}}\}\right), \tag{40}$$

where $c_2 > 0$, and inserting (40) into (39), we get

$$\dot{V}_2 = -\left(\frac{C_{\mathrm{r}}g}{A_{\mathrm{t}}} + c_2\frac{\max\{0, y_2 - p_{\mathrm{s}}\}}{L_{\mathrm{t}}A_{\mathrm{t}}}\right)e_2^2. \tag{41}$$

So we obtain

$$\|e_2(t)\| \leqslant \|e_2(t_0)\|\mathrm{e}^{-\frac{C_{\mathrm{r}}g}{A_{\mathrm{t}}}(t-t_0)}. \tag{42}$$

Next, define $z_1 = x_1 + x_2$, which is the total mass of gas in the system. From (1) and (2), its time derivative is

$$\dot{z}_1 = w_{\mathrm{gc}} - \frac{z_1 - y_1}{z_2 - y_1}y_3. \tag{43}$$

We estimate z_1 by $\hat{z}_1$, which is governed by

$$\dot{\hat{z}}_1 = w_{\mathrm{gc}} - \frac{\hat{z}_1 - y_1}{\hat{z}_2 - y_1}y_3 + k_1(\cdot), \tag{44}$$

where $k_1(\cdot)$ is an output injection term to be determined. The observer error, $e_1 = z_1 - \hat{z}_1$, is governed by

$$\dot{e}_1 = -\frac{z_1 - y_1}{z_2 - y_1}y_3 + \frac{\hat{z}_1 - y_1}{\hat{z}_2 - y_1}y_3 - k_1(\cdot). \tag{45}$$

Notice that the observer error dynamics (38) and (45), is in a cascaded form, where the dynamics of e_2 is independent of e_1. Since e_2 converges to zero, we will seek to apply Lemma 5. Towards that end, we take the Lyapunov function candidate $V_1 = \frac{1}{2}e_1^2$. Its time derivative along solutions of (45) is

$$\dot{V}_1 = -\frac{y_3}{z_2 - y_1}e_1^2 + y_3\frac{\hat{z}_1 - y_1}{(z_2 - y_1)(\hat{z}_2 - y_1)}e_1e_2 - e_1k_1(\cdot). \tag{46}$$

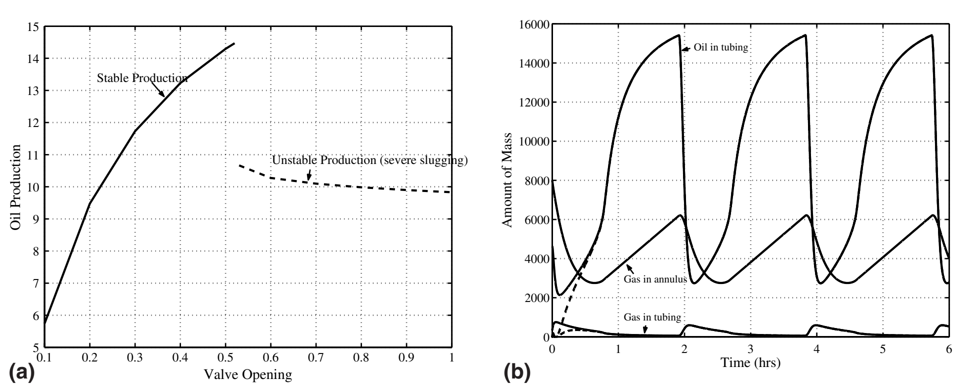

Fig. 3. (a) Mean oil production versus production choke opening. Solid line indicates stable production and dashed line indicates severe slugging. (b) States (solid line) and their estimates (dashed line) for the system during severe slugging.

We now select

$$k_1(\hat{z}_1,\hat{z}_2,y_1,y_2) = c_1\left(\frac{M}{RT_t}(L_tA_t + L_rA_r - v_o(\hat{z}_2-\hat{z}_1))y_2 - \hat{z}_1 + y_1\right), \tag{47}$$

where $c_1 > 0$, and obtain

$$\dot{V}_1 = -\left(\frac{y_3}{z_2-y_1} + c_1\frac{L_tA_t+L_rA_r-v_o(z_2-y_1)}{L_tA_t+L_rA_r-v_o(z_2-z_1)}\right)e_1^2 + \left(y_3\frac{\hat{z}_1-y_1}{(z_2-y_1)(\hat{z}_2-y_1)} - c_1\frac{v_o(z_1-y_1)}{L_tA_t+L_rA_r-v_o(z_2-z_1)}\right)e_1e_2. \tag{48}$$

Using Lemma 6, Assumptions 2 and 3, and noticing that $(\hat{z}_1-y_1)/(\hat{z}_2-y_1) < 1$, we obtain

$$\dot{V}_1 \leqslant -\left(\frac{y_3}{2B} + c_1\frac{\delta_g}{L_tA_t+L_rA_r}\right)e_1^2 + \left(\frac{y_3}{\delta_2+\delta_3} + c_1\frac{Bv_o}{\delta_g}\right)\|e_1\|\|e_2\|. \tag{49}$$

We can now apply Lemma 5 with $V = (e_1^2+e_2^2)/2$,

$$k_3 = \min\left\{c_1\frac{\delta_g}{L_tA_t+L_rA_r}, \frac{C_rg}{A_t} + c_2\frac{\delta_p}{L_tA_t}\right\}, \tag{50}$$

$$g(\|e\|) = \left(\frac{y_3}{\delta_2+\delta_3} + c_1\frac{Bv_o}{\delta_g}\right)\|e_1\|, \tag{51}$$

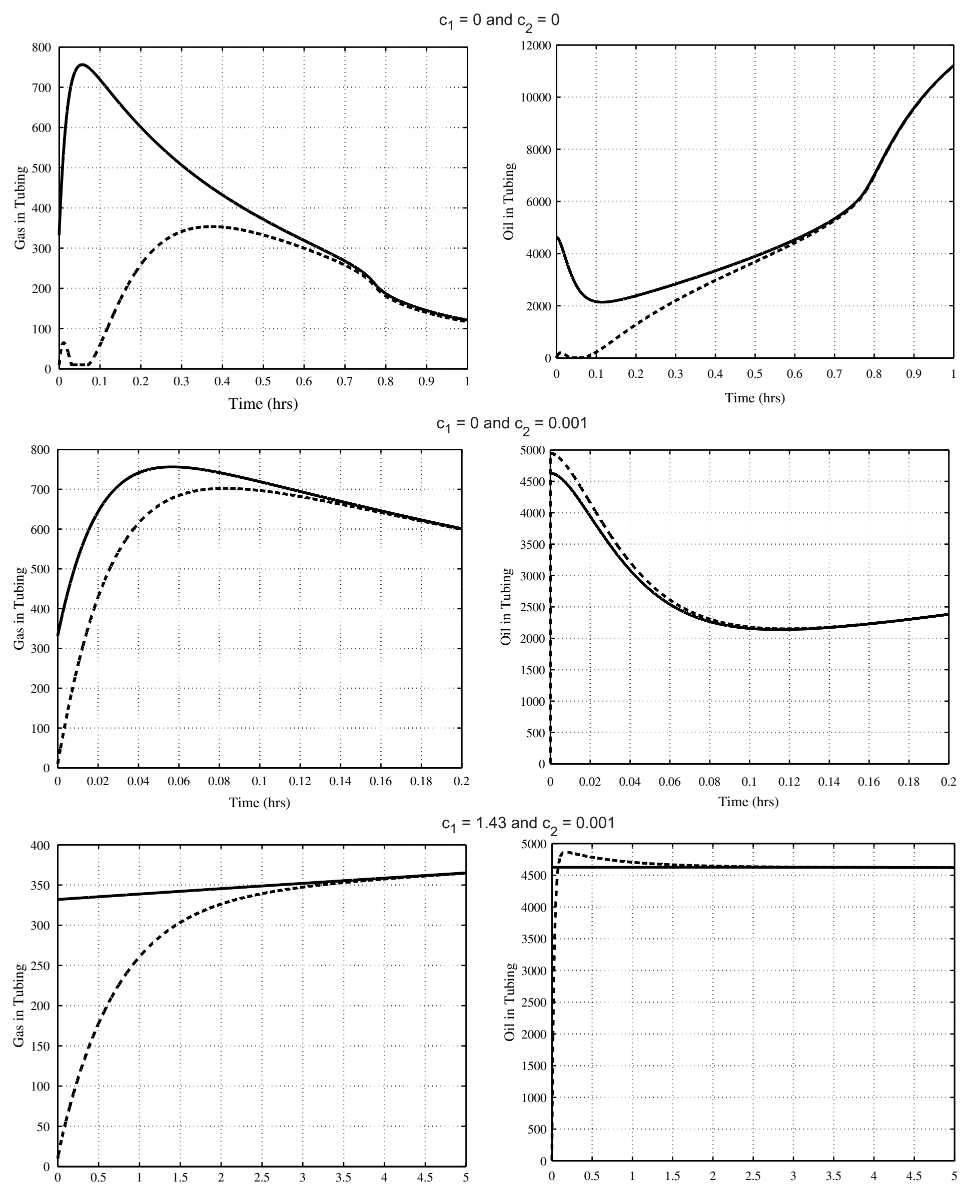

Fig. 4. Details of estimates for various choices of c_1 and c_2. Notice the different time scales in the three cases.

$$\sigma(\|e(t_0)\|, t - t_0) = \|e_2(t_0)\| \mathrm{e}^{-\frac{C_{\mathrm{T}}g}{A_{\mathrm{t}}}(t-t_0)}, \tag{52}$$

to achieve the desired result, and in particular the estimate (35). □

In the case when the density is measured, that is $y_3(t) = \rho_{\mathrm{m}}(t)$, we can simply replace y_3 with

$$C_{\mathrm{pc}}\sqrt{y_3 \max\{0, y_2 - p_{\mathrm{s}}\}}u \tag{53}$$

in (27), (28) and (33).

A key feature of the observer design is that it is independent of the flow of gas from the annulus to the tubing. This is important, because it provides robustness with respect to modelling errors of this internal flow. It is due to the coordinate change $x_1 + x_2 \to z_1$, $x_1 + x_2 + x_3 \to z_2$, and the fact that the dynamics of (z_1, z_2) is independent of (5).

3.2. Open-loop simulations

The numerical coefficients used in the simulations of system (1)–(3) are taken from a full-scale gas-lift well of depth approximately 2 km. For this case, simulations have been performed to calculate mean oil production as a function of production choke opening. The result is presented in Fig. 3a. The production is stable for small choke openings and increases as the choke opening is increased. At a choke opening of about 0.52, the flow becomes unstable and goes into severe slugging, leading to a dramatic loss of production. The increasing trend of the production for small choke openings, suggests that a higher production is possible for large choke openings if the flow can be stabilized. This is shown to be the case in the next section. In this section, we will illustrate the performance of the observer by running open-loop simulations for the nominal case of perfect model. Fig. 3b shows the states along with the estimates for the tubing over a 6 h simulation with $c_1 = 0$ and $c_2 = 0$. The flow is clearly in the state of severe slugging, and the estimates converge to the actual states. The first row of graphs in Fig. 4 shows the details over the first hour for this case. Estimates are good within 0.7 h. In the second row of graphs in Fig. 4, $c_1 = 0$ and $c_2 = 0.001$, and convergence is much faster. Estimates are good within 8 min. Setting $c_1 = 1.43$ (keeping $c_2 = 0.001$) increases the convergence rate further, as the third row of graphs in Fig. 4 shows. Estimates are in this case good within 3 s. It is clear that the convergence rate estimate γ, as defined in (35) is very conservative, since it is equal to 0 in the first two cases ($c_1 = 0$). However, looking at inequality (49), the flow through the production choke, v, defines a better bound for the estimation convergence rate, and explains why the observer converges with $c_1 = 0$. Although γ is a very conservative estimate for the convergence rate, (35) tells us that our observer can achieve any desired convergence rate by increasing c_1 and c_2. From the proof of Theorem 7, we see that c_2 governs the convergence rate of the estimate for the total mass in the system (gas and oil), whereas c_1 governs the convergence rate of the estimate for the total mass of gas in the system, but with an upper bound governed by c_2.

4. Anti-slug control by output feedback

It has been shown in [3] that severe slugging can be attenuated by stabilizing the downhole pressure using a control law of the form

$$u = u^* + K(p_{\mathrm{t,b}} - p^*_{\mathrm{t,b}}), \tag{54}$$

where u^* and $p^*_{\mathrm{t,b}}$ are some appropriate constants. [2] This control configuration is sketched in Fig. 5a. The downhole pressure is in general not available as a measurement, and neither are the individual states in the tubing. However, we may replace the states in (15) by their estimates generated by the observer designed in the previous section, to obtain an estimate of the downhole pressure. The controller obtained by using this estimate in place of $p_{\mathrm{t,b}}$ in (54), is called the certainty equivalence controller. For linear systems, stability of the closed loop using the certainty equivalence controller is guaranteed by the separation principle of linear systems. For general nonlinear systems, however, not even an exponentially convergent observer in conjunction with an exponentially stabilizing state feedback control law can guarantee stability of the closed loop system. Stability of the closed loop system obtained by combining our observer with some state feedback control law must therefore be checked in each specific case. Since the topic of this paper is observer design, we will not persue a mathematically rigorous proof of stability for the closed loop system using our observer in conjunction with (54). We will point out, however, that (54) becomes practically implementable by means of our observer, and that the resulting controller successfully stabilizes the closed loop system in laboratory experiments, results from which are presented next.

5. Laboratory experiments

Realistic tests of control structures for gas-lift wells are performed using the gas-lift well laboratory setup at TU Delft. [3] Results from prior experiments show that stabilization of the multiphase flow in the tubing can be achieved by a conventional PI controller adjusting the

[2] Usually, $p^*_{\mathrm{t,b}}$ is the setpoint chosen by the operator, while u^* is the resulting steady state choke opening adapted to by adding slow integral action to (54).

[3] The experimental setup is designed and implemented by Shell International Exploration and Production B.V., Rijswijk, and is now located in the Kramers Laboratorium voor Fysische Technologie, Faculty of Applied Sciences, Delft University of Technology.

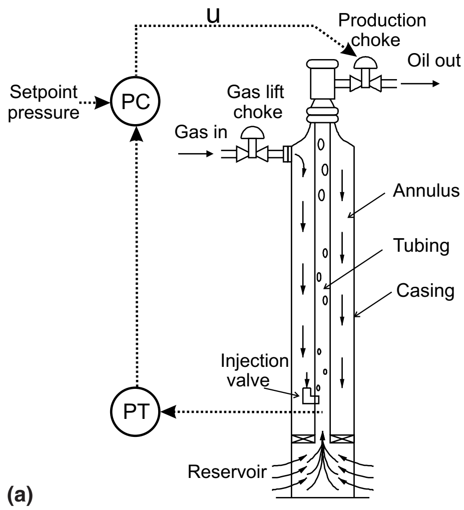

Fig. 5. Conventional control structure with downhole pressure measurement (a), and observer-based control structure with top-side information, only (b).

production choke opening based on measurement of the downhole pressure. The control structure applied in the laboratory experiments controls the estimated downhole pressure, given by the observer, and manipulates the opening of the production choke. The control structure is given in Fig. 5b.

5.1. Experimental setup

The laboratory installation represents a gas-lift well, using compressed air as lift gas and water as produced fluid. The production tube is transparent, facilitating visual inspection of the flow phenomena occurring as control is applied. The production tube measures 18 m in height and has an inner diameter of 20 mm. The fluid reservoir is represented by a tube of the same height and an inner diameter of 80 mm. The reservoir pressure is given by the static height of the fluid in the reservoir tube. A 30 l gas bottle represents the annulus, with the gas injection point located at the bottom of the production tube. In the experiments run in this study, gas is fed into the annulus at a constant rate of 11 l/min (under standard conditions: 25 °C, 1 *bara*). Input and output signals to and from the installation are handled by a microcomputer system, to which a laptop computer is interfaced for running the control algorithm and presenting output.

5.2. PI controller and observer

In the laboratory, density in the top of the tubing, $\rho_m(t)$, is available as a measurement. [4] Therefore, the observer given in Theorem 7, with y_3 replaced by (53), was used for estimation of downhole pressure. The estimated downhole pressure of the gas-lift well was stabilized using the digital PI controller

$$\Delta u_k = K_c\left[(e_k - e_{k-1}) + \frac{\Delta t}{\tau_I} e_k\right]. \tag{55}$$

The gains and integral times applied by the controller are given in Table 1, and the observer gains are given in Table 2. The PI-controller has varying gain. Note, however, that hysteresis is implemented to prevent frequent gain changes due to noise in the pressure estimate.

5.3. Experimental results

The laboratory experiment followed the control sequence given in Table 3. The multiphase flow is initially open-loop stabilized by applying a small choke opening, causing the pressure drop to be friction dominated. When steady-state is reached the controller is turned on. It gradually increases the choke opening and moves the system into the open-loop unstable domain in order to reach the pressure setpoint. When steady-state is reached again, this time at a large choke opening, the controller is turned off, leaving the choke with the last controlled opening. This is why oscillations appear at the end of the time series below, confirming that the new operating point is indeed in the open-loop unstable domain.

Table 1
Gain scheduling

Valve opening	Gain	Integral time
$55\% \leqslant u < 65\%$	−0.5	150 s
$65\% \leqslant u < 73\%$	−1.5	150 s
$73\% \leqslant u < 100\%$	−2.0	200 s

[4] The density measurement is derived from two pressure measurements in the upper half of the tubing, and is therefore an approximation to the actual density in the top of the tubing.

Table 2
Gains–observer

Total mass of gas, c_1	Total mass, c_2
0.5	0.01

Table 3
The control sequence

Time slot	Control	Valve opening
(−5) min–0 min	Open loop	55%
0 min–50 min	Closed loop	Controlled
50 min–55 min	Open loop	83.2%

The estimated downhole pressure from the observer and the measured downhole pressure are shown in Fig. 6a. The results show that the estimated downhole pressure is stabilized, with the steady-state production choke opening at 79% (see Fig. 6b). The valve opening fluctuates somewhat due to noise in the downhole pressure estimate. After 43 min a large external disturbance to the gas supply source of the system is introduced, as can be seen from the peaks in the experimental results. The controller successfully attenuates the disturbance, indicating robustness.

The measurements used by the observer, that is the casing head pressure, the tubing head pressure, and the fluid density, are shown in Figs. 7 and 8a. The measurements of the tubing head pressure and the density are low-pass filtered, using a cut-off period of $T_{c,\mathrm{THP}} = 20\,\mathrm{s}$ and $T_{c,\rho} = 10\,\mathrm{s}$, respectively.

The flow rate from the well is shown in Fig. 8b, and corresponds to an average production choke opening of 79%. The average open-loop liquid production as a function of production choke opening has been produced by a series of experiments and is shown in Fig. 9. For valve openings less than about 70%, the production is open-loop stable, while for openings larger than 70%, the production is open-loop unstable, and is significantly lower compared to the production at 70% valve opening. It is interesting to notice that the production curve from the gas-lift laboratory shows the same qualitative behaviour as the one predicted by system (1)–(3) for a full-scale gas-lift oil well (recall Fig. 3a).

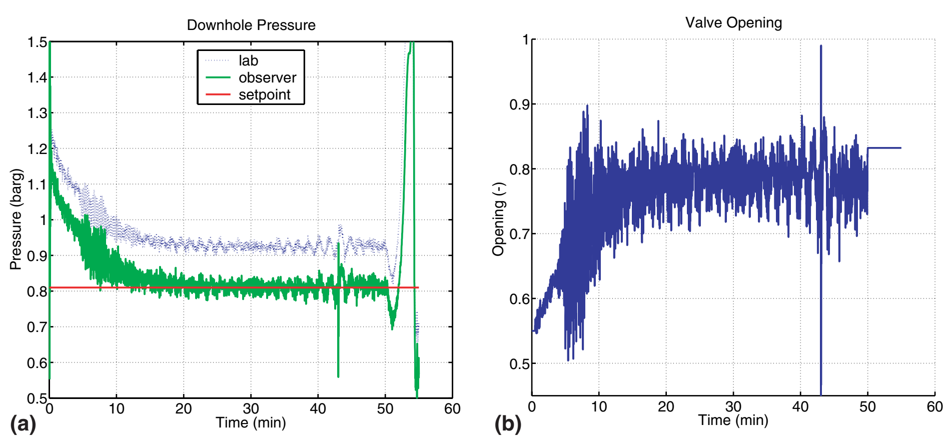

Fig. 6. (a) Estimated downhole pressure given by the observer and measured downhole pressure. (b) Opening of the production choke.

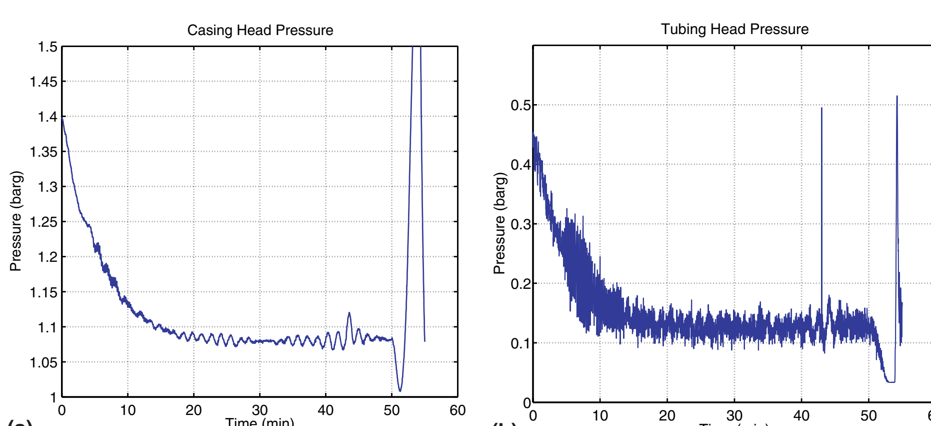

Fig. 7. (a) Pressure in the annulus. (b) Pressure in the top of the tubing.

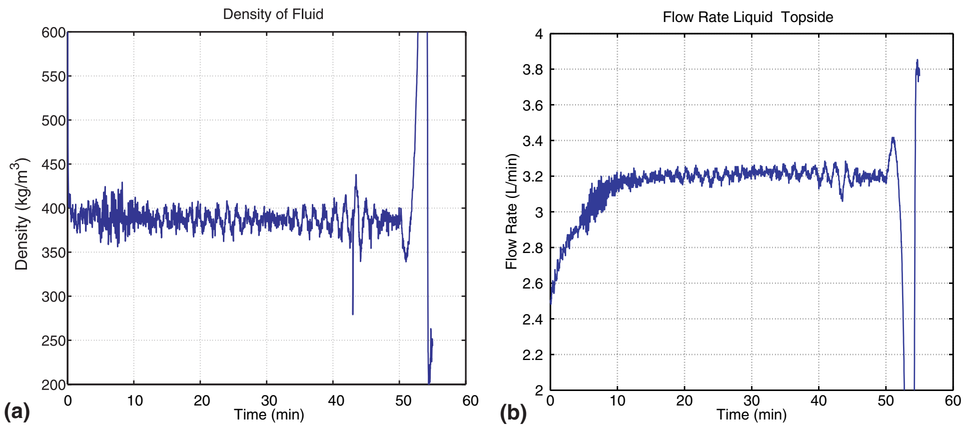

Fig. 8. (a) Fluid density in the top of the tubing. (b) Total fluid production.

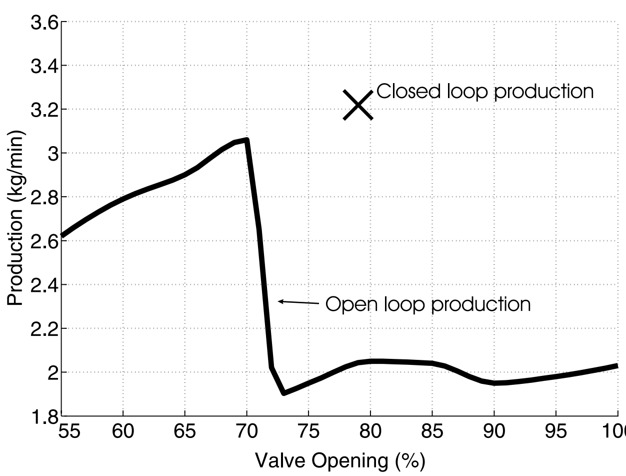

Fig. 9. Production achieved from the gas-lift laboratory by closed loop compared to open loop production.

The cross at 79% choke opening in Fig. 9 gives the production for the controlled experiment. The key information in the figure, and the main result of the experimental part of this paper, is in comparing the production for the controlled experiment with the maximum achievable open-loop production obtained for 70% choke opening: The increase in produced fluid gained by stabilizing the gas-lift well at 79%, by feedback control, is about 5%.

6. Conclusions

In this paper, we have designed and analyzed a reduced order nonlinear observer for the states of the multiphase flow in the tubing. The observer relies on topside measurements, only. A key feature of the design is that it exploits the structure of the model to obtain robustness with respect to the internal flow between the annulus and the tubing. The performance of the observer was demonstrated in simulations. The practical applicability of a control scheme consisting of our observer in conjunction with conventional control techniques was demonstrated in laboratory experiments, where the multiphase flow in the tubing was successfully stabilized using top-side information, only. The resulting production rate was 5% higher than the open-loop maximum production rate.

The results of this paper clearly show that there is a potential for increasing production from gas-lift oil wells by installing a relatively simple control system.

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Paper V

A state feedback controller for a class of nonlinear positive systems applied to stabilization of gas-lifted oil wells

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A state feedback controller for a class of nonlinear positive systems applied to stabilization of gas-lifted oil wells

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Abstract

A state-feedback controller is proposed for a certain class of multi-input nonlinear positive systems. The controller achieves closed loop convergence to a set, which in many cases implies convergence to an equilibrium. The controller is applied to the casing heading problem, an important instability problem occurring in the petroleum industry. Both simulations and a laboratory trial illustrate the merits of the controller.

Key words: nonlinear systems, positive systems, set stability, casing heading

1 Introduction

When modeling systems for control based on first principles, one often obtains nonlinear ordinary differential equations where the state variables (mass, pressure, level, energy, etc.) are *positive*. In addition, the control input will also often be positive (valve openings, amount of inflow, heat input, etc.). Hence, the class of positive systems (systems with nonnegative states and inputs) is a natural class of systems to consider in a control setting.

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Since mass is an inherently positive quantity, systems modeled by mass balances [1] are perhaps the most natural example of positive systems. Another example is the widely studied class of compartmental systems [7,11], used in biomedicine, pharmacokinetics, ecology, etc. Compartmental systems, which are often derived from mass balances, are (nonlinear or linear) systems where the dynamics are subject to strong structural constraints. Each state is a measure of some material in a compartment, and the dynamics consists of the flow of material into (*inflow*) or out of (*outflow*) each compartment. If these flows fulfill certain criteria, the system is called compartmental.

Similar to compartmental systems, we will herein assume that each state can be interpreted as the "mass" (or measure of mass; concentration, level, pressure, etc.) of a compartment. However, we do not make the same strong assumptions on the structure on the flows between the compartments. Instead, we make other (strong) assumptions related to the system being controllable according to the control objective under input saturations. Furthermore, we assume that the compartments that constitute the state vector can be divided into groups of compartments, which we will call *phases*. Each phase will have a controlled inflow or outflow associated with it. The control objective will be to steer the mass of each phase (the sum of the compartment masses in that phase) to a constant, prespecified value.

For this model class we propose a state feedback controller, providing closed loop convergence of the sum of the states of each phase. The controller is inspired by [2], but the class of systems is larger, especially since the phase concept allows us to consider multiple input systems. Furthermore, we also allow saturated inputs and outflow-controlled systems. Similar to [2], we assume that the inputs are positive. The controller in [2] can be viewed as a special case of the controller herein. A related controller for a (similar) class of systems exhibiting *first integrals* is developed in [4]. Instead of controlling the sum of system states (the total mass), it is controlling (more general) first integrals to a specified value. The controller of [4] is different from the one considered herein, for instance the input can take on negative values. However, the stability properties of the closed loop are similar, in the sense that they both achieve convergence to a certain *set*.

The paper is structured as follows: In Section 2 the system class is presented. The controller and a convergence result from a general invariant domain of attraction are presented in Section 3. In Section 4, a simple example illustrates some issues related to the phase concept and stability of the closed loop. Finally, Section 5 provides a semi-realistic application to stabilization of gas-lifted oil wells, including simulations on a de-facto industry standard simulator and lab trials.

In the following, $\mathbb{R}_+ = [0, \infty)$ and $\mathbb{R}_+^n = \{x = [x_1, x_2, \dots, x_n]^\mathsf{T} \mid x_i \in \mathbb{R}_+\}$. Further, $\mathrm{blockdiag}(A_1, \dots, A_r)$ denotes a block diagonal matrix with the matrices $A_1, \ldots, A_r$ on the "diagonal".

2 Model class

We consider nonlinear systems

$$\dot{x} = f(x, u), \tag{1a}$$

where the state is positive ($x \in \mathbb{R}^n_+$), and the input is positive and upper bounded, $u \in U := \{u \in \mathbb{R}^m_+ \mid 0 \le u_j \le \bar{u}_j\}$. Each state can be interpreted as the "mass" (amount of material, or some measure of amount) in a compartment [2] . The controller we will propose exploits system structure, thus we assume the model equations to be on the following form:

$$f(x, u) = \Phi(x) + \Psi(x) + B(x)u. \tag{1b}$$

Loosely speaking, $\Phi(x)$ represents "interconnection structure" between compartments, $\Psi(x)$ represents *uncontrolled* external inflows to and outflows from compartments and $B(x)u$ represents *controlled* external inflows to and outflows from compartments.

Furthermore, we will assume that the state vector can be divided into m different parts, which will be denoted *phases*. Phase j will consist of r_j states, and have the control u_j associated with it, corresponding to *either* controlled inflow *or* outflow to compartments of that phase. The states in phase j will be denoted χ^j, such that $x = [(\chi^1)^\mathsf{T}, (\chi^2)^\mathsf{T}, \ldots, (\chi^m)^\mathsf{T}]^\mathsf{T}$, and it follows that necessarily, $\sum_{j=1}^m r_j = n$. Corresponding to this structure, the vector functions $\Phi(x)$, $\Psi(x)$ and the matrix function $B(x)$ are on the form

$$\begin{aligned}
\Phi(x) &= \left[\phi^1(x)^\mathsf{T}, \phi^2(x)^\mathsf{T}, \ldots, \phi^m(x)^\mathsf{T}\right]^\mathsf{T} \\
\Psi(x) &= \left[\psi^1(x)^\mathsf{T}, \psi^2(x)^\mathsf{T}, \ldots, \psi^m(x)^\mathsf{T}\right]^\mathsf{T} \\
B(x) &= \text{blockdiag}\left(b^1(x), b^2(x), \ldots, b^m(x)\right).
\end{aligned}$$

Note that element j is (in general) a function of x, not (only) χ^j. Also note that the partitioning into phases need not be unique.

We will state the assumptions on these functions on the set $D \subseteq \mathbb{R}^n_+$. In the case of global results, $D = \mathbb{R}^n_+$.

[2] The word compartment *does not* imply that the system class we look at is compartmental [11]. However, it enjoys strong similarities with compartmental systems.

A1. **(Interconnection structure)** The function $\Phi : D \to \mathbb{R}^n$ is locally Lipschitz, $\phi_i^j(x) \geq 0$ for $\chi_i^j = 0$, and

$$\sum_{i=1}^{r_j} \phi_i^j(x) = 0, \; j = 1, \ldots, m.$$

A2. **(Controlled external flows)** The block diagonal matrix function $B(x) : D \to \mathbb{R}^{n \times m}$ is locally Lipschitz and satisfies:
 a. Phase j has controlled inflow:

$$b_i^j(x) \geq 0 \text{ for all } x \in D$$
$$\sum_{i=1}^{r_j} b_i^j(x) > 0 \text{ for all } x \in D$$

 b. Phase j has controlled outflow:

$$b_i^j(x) \leq 0 \text{ for all } x \in D$$
$$\chi_i^j = 0 \Rightarrow b_i^j(x) = 0 \text{ (if } \{x \mid \chi_i^j = 0\} \cap D \neq \emptyset)$$
$$\sum_{i=1}^{r_j} b_i^j(x) < 0 \text{ for all } x \in D$$

These assumptions says that there is zero net contribution to phase mass from the "interconnection structure", and that the controlled flows really are inflow and outflow (and in the outflow-controlled case, that no mass can flow when a state is zero).

The uncontrolled external flows must satisfy some "controllability" assumption in relation to the controlled flows. Before we define this, it is convenient to define the "mass" of each phase, being the sum of the compartment masses (the states) of that phase:

$$M_j(x) := \sum_{i=1}^{r_j} \chi_i^j.$$

Our control objective will be to control $M_j(x)$ to some prespecified (positive) desired mass of phase j, denoted M_j^*, from initial conditions in D. For the control problem to be meaningful, the intersection of the set where $M_j(x) = M_j^*$ and D should be nonempty.

A3. **(Uncontrolled external flows)** For given $M^* = [M_1^*, M_2^*, \ldots, M_m^*]^\mathsf{T}$, $\Psi(x) : D \to \mathbb{R}^n$ is locally Lipschitz and satisfies $\psi_i^j(x) \geq 0$ for $\chi_i^j = 0$ (if $\{x \mid \chi_i^j = 0\} \cap D \neq \emptyset$), and in addition, if:
 a. Phase j has controlled inflow:
  1. For $x \in \{x \in D \mid M_j(x) > M_j^*\}$, $\sum_{i=1}^{r_j} \psi_i^j(x) \leq 0$ and the set $\{x \in D \mid \sum_{i=1}^{r_j} \psi_i^j(x) = 0 \text{ and } M_j(x) > M_j^*\}$ does not contain an invariant set.

2. For $x \in \{x \in D \mid M_j(x) < M_j^*\}$, $-\sum_{i=1}^{r_j} \psi_i^j(x) < \sum_{i=1}^{r_j} b_i^j(x)\bar{u}_j$.

b. Phase j has controlled outflow:

1. For $x \in \{x \in D \mid M_j(x) < M_j^*\}$, $\sum_{i=1}^{r_j} \psi_i^j(x) \geq 0$ and the set $\{x \in D \mid \sum_{i=1}^{r_j} \psi_i^j(x) = 0 \text{ and } M_j(x) < M_j^*\}$ does not contain an invariant set.
2. For $x \in \{x \in D \mid M_j(x) > M_j^*\}$, $\sum_{i=1}^{r_j} \psi_i^j(x) < -\sum_{i=1}^{r_j} b_i^j(x)\bar{u}_j$.

Assumption A3.a.1 (A3.b.1) means that when the phase mass is large (small), the outflow of this phase is dominantly outflow (inflow). The "no invariant set" part plays the same role as the assumption of "zero state detectability" through (the equivalent of) $\sum_{i=1}^{r_j} \psi_i^j(x)$ in [2]. The assumption A3.a.2 (A3.b.2) means that when the phase mass is small (large) and the controller is saturating, the outflow (inflow) must be smaller than the (saturated) inflow (outflow)

Proposition 1 (Positivity) *For $x(0) \in \mathbb{R}_+^n$, the state of the system (1) fulfilling A1-A3 with $D = \mathbb{R}_+^n$, satisfies $x(t) \in \mathbb{R}_+^n$, $t > 0$.*
Proof It suffices to notice that for $x_i = 0$, $\dot{x}_i \geq 0$. □

3 Stabilizing state feedback controller

In this section, the state feedback controller is defined, and a convergence result is given for a general invariant set D that (is a subset of the set that) A1-A3 hold on. The set D could then be considered a region of attraction.

As mentioned in the previous section, our control objective is to control the total mass $M_j(x)$ of each phase to a prespecified value M_j^*. To this end, the following constrained, positive state feedback control law is proposed:

$$u_j(x) = \begin{cases} 0 & \text{if } \tilde{u}_j(x) < 0 \\ \tilde{u}_j(x) & \text{if } 0 \leq \tilde{u}_j(x) \leq \bar{u}_j \\ \bar{u}_j & \text{if } \tilde{u}_j(x) > \bar{u}_j \end{cases} \tag{2}$$

where

$$\tilde{u}_j(x) = \frac{1}{\sum_{i=1}^{r_j} b_i^j(x)} \left(-\sum_{i=1}^{r_j} \psi_i^j(x) + \lambda_j (M_j^* - M_j(x)) \right) \tag{3}$$

and λ_j is a positive constant. Assumption A2 ensures that $\sum_{i=1}^{r_j} b_i^j(x) \neq 0$ always, such that $\tilde{u}_j$ is always defined.

Define the set

$$\Omega = \{x \in \mathbb{R}_+^n \mid M_1(x) = M_1^*, \ldots, M_m(x) = M_m^*\}.$$

Assumption 1 *There exists a set D that is invariant for the dynamics (1) under the closed loop with control (2), and has a nonempty intersection with Ω.*
Assumption 2 *For $x \in \Omega \cap D$, $0 < \tilde{u}_j(x) < \bar{u}_j$.*

Under the given assumptions, the convergence properties of the controller are summarized as follows:

Theorem 1 *Under the given assumptions, the state of the system (1), controlled with (2) and starting from some initial condition $x(0) \in D$, stays bounded and converges to the set $\Omega \cap D$ which is positively invariant.*
Proof The set D is by Assumption 1 invariant, hence Assumptions A1-A3 hold along closed loop trajectories.

Define the positive semidefinite function

$$V(x) := \frac{1}{2} \sum_{j=1}^{m} \left(M_j(x) - M_j^* \right)^2, \tag{4}$$

with time derivative

$$\dot{V}(x) = \sum_{j=1}^{m} \left[M_j(x) - M_j^* \right] \left(\sum_{i=1}^{r_j} \psi_j^i(x) + \sum_{i=1}^{r_j} b_j^i(x) u_j(x) \right).$$

For $M_j(x) \neq M_j^*$, we have one of the following cases:

(1) If $0 \leq \tilde{u}_j \leq \bar{u}_j$, summand j is

$$\left[M_j(x) - M_j^* \right] \left(\sum_{i=1}^{r_j} \psi_j^i(x) + \sum_{i=1}^{r_j} b_j^i(x) u_j(x) \right) = -\lambda_j \left[M_j(x) - M_j^* \right]^2 < 0.$$

(2) If $\tilde{u}_j < 0$, then $u_j(x) = 0$ and summand j is

$$\left[M_j(x) - M_j^* \right] \sum_{i=1}^{r_j} \psi_j^i(x).$$

Assumption A3.a.1 and A3.b.1 ensures that this is negative for both inflow and outflow controlled phases.

(3) If $\tilde{u}_j \geq \bar{u}_j$, then $u_j(x) = \bar{u}_j$ and summand j is

$$\left[M_j(x) - M_j^* \right] \left(\sum_{i=1}^{r_j} \psi_j^i(x) + \sum_{i=1}^{r_j} b_j^i(x) \bar{u}_j \right).$$

Assumption A3.a.2 and A3.b.2 ensures that this is negative for both inflow and outflow controlled phases.

For the details of point 2 and 3, check [10]. We can conclude that $\Omega \cap D$ is invariant, since D is invariant, and by Assumption 2 and a continuity argument, case (1) (and thus $\dot{V}(x) < 0$) holds in the intersection between a neighborhood of $\Omega \cap D$ and D.

Moreover, since $\dot{V}(x) \leq 0$, $V(x(t)) \leq V(x(t_0))$ along system trajectories. From the construction of $V(x)$ it is rather easy [10] to see that for $x \in \mathbb{R}^n_+$, $\|x\| \to \infty$ if and only if $V(x) \to \infty$, hence $V(x(t))$ bounded implies that $\|x(t)\|$ is bounded. This allows us to conclude from LaSalle's invariance principle that $x(t)$ converges to the largest invariant set contained in $\{x \mid \dot{V}(x) = 0\} \cap D$. By the above and Assumption A3.a.1 and A3.b.1, there is no other invariant set for which $\dot{V}(x) = 0$ larger than $\Omega \cap D$. □

To use this theorem, we need to find invariant sets D. In some cases, the assumptions might hold globally in the sense that $D = \mathbb{R}^n_+$. However, in the general case, constructing a set D might be hard. Typically, one would look for sets of the shape $D = D_1$ or $D = D_2$, where

$$D_1 := \{x \in \mathbb{R}^n_+ | M_j^* - \underline{c}_j \leq M_j(x) \leq M_j^* + \overline{c}_j,\ j = 1, \ldots, m\}$$

that is, a "Lyapunov level set"-type region, and

$$D_2 := \{x \in \mathbb{R}^n_+ \mid \underline{\chi}_j^i \leq \chi_j^i \leq \overline{\chi}_j^i,\ i = 1, \ldots, r_j \text{ and } M_j^* - \underline{c}_j \leq M_j(x) \leq M_j^* + \overline{c}_j,\ j = 1, \ldots, m\}.$$

For further details and examples, we refer to [10].

Although this theorem merely shows convergence to the set Ω, it is possible to prove that Ω is asymptotically (set) stable [9]. In many applications, stability of equilibria is arguably more interesting. It is thus interesting to note that the controller (2) often (but not always, as the counterexample in [10] reveals) leads to a stable equilibrium. A sufficient condition for an asymptotically stable equilibrium can be found from the theory of semidefinite Lyapunov functions. Here, we state the following theorem which can be proved in a similar way as Theorem 5 in [4]:

Theorem 2 *Let the conditions of Theorem 1 hold. If the closed loop (1) has a single equilibrium in the interior of $\Omega \cap D$ that is asymptotically stable with respect to initial conditions in $\Omega \cap D$ and attractive for all initial conditions in $\Omega \cap D$, the equilibrium is asymptotically stable for the closed loop with a region attraction (of at least) D.*

We conclude this section with a brief remark about robustness: The proposed feedback scheme is independent of the interconnection structure and hence robust [3] to model uncertainties in $\Phi(x)$ (as long as Assumption A1 holds). This is the most important robustness property. As mentioned in [2], the interconnection terms are in practical examples often the terms that are hardest

[3] Robust in the sense that convergence to Ω still holds. Note that changes in $\Phi(x)$ will typically move the equilibria in Ω.

to model. However, the unconstrained controller also has some (weaker) robustness properties with respect to bounded uncertainties in $\Psi(x)$ and $B(x)$. For further details on this, we refer to [10].

4 Simple example: Tanks in series

This example illustrates the versatility of the phase concept, in addition to shedding light on how to compute a region of attraction and decide stability of equilibria. Further simulation studies, including a benchmark Van der Vusse reactor, can be found in [10].

Consider a system with three tanks in series,

$$\begin{aligned}\dot{x}_1 &= u_1 - \alpha_1\sqrt{x_1}\\ \dot{x}_2 &= \alpha_1\sqrt{x_1} - \alpha_2\sqrt{x_2}\\ \dot{x}_3 &= \alpha_2\sqrt{x_2} - \alpha_3\sqrt{x_3}u_2,\end{aligned}$$

where the states are the level (or mass, or pressure) in each tank. The inflow to the first tank and the outflow of the third thank can be controlled, and are bounded, $0 \leq u_i \leq \bar{u}_i$. The system is obviously positive.

According to the model structure in Section 2, there are several different control structures that can be chosen, depending on how we divide the state into phases, and which inputs we choose to control.

i) <u>One phase, inflow controlled:</u> Choosing u_1 as control, setting $u_2 = u_2^* > 0$ constant, and total mass $M(x) = x_1 + x_2 + x_3$ gives

$$\tilde{u}_1 = \alpha_3\sqrt{x_3}u_2^* + \lambda_1(M^* - M(x)).$$

The (single) phase is inflow controlled. Assumption A3.a.1 obviously holds globally, and Assumption A3.a.2 (which translates to $\alpha_3\sqrt{x_3}u_2^* < \bar{u}_1$ for $M(x) < M^*$) holds on $\mathbb{R}^3_+$ if M^* is chosen such that $\alpha_3\sqrt{M^*}u_2^* < \bar{u}_1$. Then, by Theorem 1 with $D = \mathbb{R}^3_+$, the state converges to $\Omega = \{x \mid M(x) = M^*\}$ from any (positive) initial condition.

ii) <u>One phase, outflow controlled:</u> Choosing u_2 as control, setting $u_1 = u_1^* > 0$ constant, and total mass $M(x) = x_1 + x_2 + x_3$ gives

$$\tilde{u}_2 = \frac{-1}{\alpha_3\sqrt{x_3}}\left[-u_1^* + \lambda_2(M^* - M(x))\right].$$

The single phase is outflow controlled. Assumption A3.b.1 holds globally, but Assumption A3.b.2 ($u_1^* < \alpha_3\sqrt{x_3}\bar{u}_2$ for $M(x) > M^*$) does not hold globally for any combination of M^* and $\bar{u}_2$ (consider e.g. an initial condition with $x_3(0) = 0$ and $M(x(0)) > M^*$).

Let $D_2 = \{x \in \mathbb{R}^3_+ \mid x_1 \geq a, x_2 \geq b, x_3 \geq c\}$ with a, b and c positive constants satisfying $a < \left(\frac{u_1^*}{\alpha_1}\right)^2$, $b < \left(\frac{\alpha_1}{\alpha_2}\right)^2 a$ and $c < \left(\frac{\alpha_2}{\alpha_3}\right)^2 b$. Assumption A3.b.2 hold on D_2 if $\bar{u}_2$ satisfies $u_1^* < \alpha_3\sqrt{c}\bar{u}_2$. Since D_2 is invariant, convergence to Ω (if the intersection between Ω and D_2 is nonempty) from initial conditions in D_2 follows from Theorem 1.

iii) Two phases, case 1: Phase 1 consist of x_1 and x_2 ($M_1(x) = x_1 + x_2$), phase 2 of x_3 ($M_2(x) = x_3$):

$$\tilde{u}_1 = \alpha_2\sqrt{x_2} + \lambda_1(M_1^* - M_1(x)),$$
$$\tilde{u}_2 = \frac{-1}{\alpha_3\sqrt{x_3}}\left[-\alpha_2\sqrt{x_2} + \lambda_2(M_2^* - M_2(x))\right].$$

Phase 1 is inflow controlled. Assumption A3.a.1 holds, and Assumption A3.a.2 ($\alpha_2\sqrt{x_2} < \bar{u}_1$ for $M_1(x) < M_1^*$) holds globally if $\alpha_2\sqrt{M_1^*} < \bar{u}_1$. Phase 2 is outflow controlled. Assumption A3.b.1 holds globally, but Assumption A3.b.2 ($\alpha_2\sqrt{x_2} < \alpha_3\sqrt{x_3}\bar{u}_2$ for $M_2(x) > M_2^*$) does not hold globally. However, since $\alpha_2\sqrt{x_2} < \alpha_3\sqrt{x_3}\bar{u}_2$ for $M_2(x) > M_2^*$ holds when $x_2 < \left(\frac{\alpha_3}{\alpha_2}\right)^2 M_2^*\bar{u}_2$, we can have $D_1 = \{x \in \mathbb{R}^3_+ \mid 0 \leq x_1 + x_2 \leq \left(\frac{\alpha_3}{\alpha_2}\right)^2 M_2^*\bar{u}_2\}$ (obviously, with $M_1^* < \left(\frac{\alpha_3}{\alpha_2}\right)^2 M_2^*\bar{u}_2$) and convergence to Ω is guaranteed from initial conditions in D_1 from Theorem 1.

iv) Two phases, case 2: Phase 1 consist of x_1 ($M_1(x) = x_1$), phase 2 of x_2 and x_3 ($M_2(x) = x_2 + x_3$):

$$\tilde{u}_1 = \alpha_1\sqrt{x_1} + \lambda_1(M_1^* - M_1(x)),$$
$$\tilde{u}_2 = \frac{-1}{\alpha_3\sqrt{x_3}}\left[-\alpha_1\sqrt{x_1} + \lambda_2(M_2^* - M_2(x))\right].$$

Phase 1 is inflow controlled. Assumption A3.a.1 holds globally, and Assumption A3.a.2 ($\alpha_1\sqrt{x_1} < \bar{u}_1$ for $M_1(x) < M_1^*$) holds if $\alpha_1\sqrt{M_1^*} < \bar{u}_1$. Phase 2 is outflow controlled. Assumption A3.b.1 holds globally, but Assumption A3.b.2 ($\alpha_1\sqrt{x_1} < \alpha_3\sqrt{x_3}\bar{u}_2$ for $M_2(x) > M_2^*$) does not hold globally.

Suppose we specify that initial conditions for x_1 should satisfy $\underline{a} \leq x_1(= M_1(x)) \leq \overline{a}$. Similarly as in ii), there exists b and c such that $D_2 = \{\underline{a} \leq x_1(= M_1(x)) \leq \overline{a}, x_2 \geq b, x_2 \geq c\}$ is (closed loop) invariant. Suppose that $\alpha_1\sqrt{\overline{a}} < \alpha_3\sqrt{c}\bar{u}_2$ holds, then Theorem 1 guarantees convergence to Ω from initial conditions in D_2.

Simulations indicate that the regions of attractions given in iii) and iv) are rather conservative. The reasons for this is that Theorem 1 requires the time derivative of $V_j(x)$ to be negative at all times. In both cases, if there is a large amount of mass in the first phase compared to the second phase, due to the saturation of the outflow, it is impossible to avoid the situation where the mass in the second phase increases. However, since the inflow to the first phase is also restricted, after a while the mass is distributed such that the masses in both phases decrease.

Let us analyze the dynamics in Ω in i) above (when the input $u_1 = \alpha_3\sqrt{x_3}u_2^*$). The dynamics in Ω can be parameterized by x_1 and x_2, with $x_3 = M^* - x_1 - x_2$:

$$\dot{x}_1 = \alpha_3\sqrt{M^* - x_1 - x_2}u_2^* - \alpha_1\sqrt{x_1} =: f_1(x)$$
$$\dot{x}_2 = \alpha_1\sqrt{x_1} - \alpha_2\sqrt{x_2} =: f_2(x).$$

Note that there is a unique equilibrium in Ω, which by linearization is (asymptotically) stable. Furthermore, we see that

$$\frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} = -\frac{\alpha_3 u_2^*}{2\sqrt{M^* - x_1 - x_2}} - \frac{\alpha_1}{2\sqrt{x_1}} - \frac{\alpha_2}{2\sqrt{x_2}}$$

has the same sign for all x_1, x_2 such that $M^* - x_1 - x_2 > 0$ (and thus, in the interior of Ω). This allows us to use Bendixon's Criterion (see e.g. [13]) to conclude that no periodic orbits can exist in Ω. Moreover, the generalized Poincaré-Bendixson Theorem tells us that in the two-dimensional case, the only possibilities for trajectories confined to a compact set, are convergence to an equilibrium or a periodic orbit (or a graphic). This means that all trajectories in Ω converge to the (single) asymptotically stable equilibrium in Ω, and we may conclude from Theorem 2 that the equilibrium in Ω is globally (with respect to the positive orthant) asymptotically stable (for the total system).

A very similar analysis for case ii) above shows that D_1 is then a region of attraction for the unique equilibrium in Ω. In case iii) (and case iv)), the dynamics in Ω can be parameterized by $\dot{x}_2 = \alpha_1\sqrt{M_1^* - x_2} - \alpha_2\sqrt{x_2}$ ($\dot{x}_2 = \alpha_1\sqrt{M_1^*} - \alpha_2\sqrt{x_2}$) with unique equilibrium $x_2 = \alpha_1^2 M_1^*/(\alpha_1^2 + \alpha_2^2)$ ($x_2 = M_1^*\alpha_1^2/\alpha_2^2$). In both cases, $\dot{x}_2$ is strictly decreasing in x_2, and asymptotic stability of the equilibrium in Ω is immediate.

5 Stabilization of flow in gas-lifted oil wells

This section presents an application to an important instability problem in the petroleum industry. First, the problem and a simple mass-balance model with two manipulated variables, suitable for control design, is presented. A two input version of the controller is then applied to a stylistic (but realistic) well, analyzed for the simple model and assessed with simulations using the rigouros (de facto industry standard) multiphase flow simulator OLGA 2000. Thereafter, a one-input version (the second input is kept constant) is considered for a laboratory scale well.

5.1 *Gas-lifted oil wells*

The use of hydrocarbons is essential in modern every-day life. In nature, hydrocarbons are typically found in petroleum-bearing geological formations (*reservoirs*) situated under the earth's crust, and hydrocarbons from these reservoirs are produced by means of an *oil well*. An oil well is made by drilling a hole (wellbore) into the ground. A metal pipe (casing) is placed in the wellbore to secure the well, before "downhole well completion" is performed by running the production pipe (tubing), packing and possibly valves and sensors into the well and perforate the casing to make the reservoir fluid flow into the well. Detailed information on wells and well completion can e.g. be found in [8], see also Figure 1.

If the reservoir pressure is high enough to overcome the back pressure from the flowing fluid column in the well and the surface (topside) facilites, the reservoir fluid can flow to the surface. In some cases, the reservoir pressure is not high enough to make the fluid flow freely, at least not at the desired rate. A remedy is then to inject gas close to the bottom of the well, which will mix with the reservoir fluid, see Figure 1. The gas is transported from the topside through the gas-lift choke into the annulus (the space between the casing and the tubing), and enters the tubing through the injection valve close to the bottom of the well. The gas will help to "lift" the oil out of the tubing, through the production choke into the topside process equipment (separator). This is the type of oil well, called gas-lifted well, we will consider herein. A problem with these type of wells, is that they can become (open loop) unstable, characterized by highly oscillatory well flow, known as *casing heading*. The flow regime of the well (tubing) in this case is denoted *slug flow*. The two main factors that induce casing heading, is high compressibility of gas in the annulus, and gravity dominated pressure drop in the two-phase flow in the tubing.

The oil production for a typical oscillating well can be seen in Figure 2. This slug flow is undesirable since it creates operational problems for downstream processing equipment. Further, stabilizing the slug flow in the well leads to increased production, as illustrated in Figure 3. The casing heading problem is industrially important, as a considerable amount of such wells exhibit slug flow. This (or similar) control problems are considered in e.g. [12,5].

For simplicity, we will assume that the reservoir contains only oil, which is a good approximation if the fraction of gas and water is low. We assume realistic boundary conditions, that is, constant separator pressure (downstream the production choke), constant gas injection pressure (upstream the gas injection choke) and constant reservoir pressure (far from the well). The (vertical) well is 2km deep and the high fidelity model is modelled in OLGA 2000 dividing both the tubing and the annulus into 25 volumes.

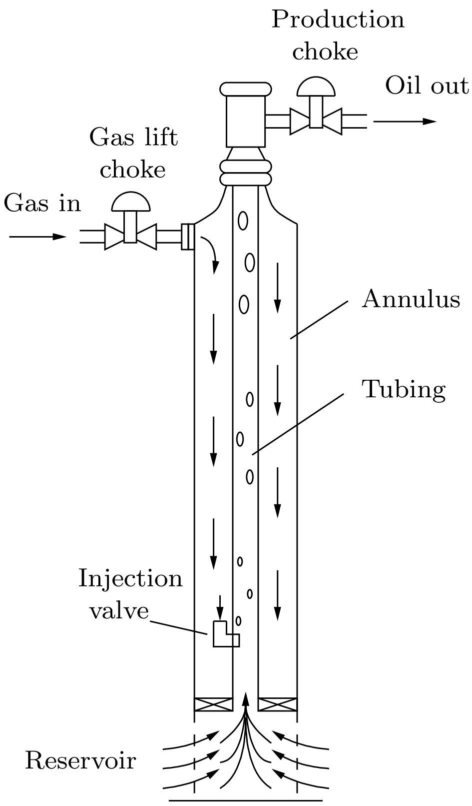


Fig. 1. A gas-lifted oil well.

5.2 A simple model of a gas-lifted oil well

As discussed above, the mechanisms that make the well produce in slugs, are related to the mass of gas in the annulus (compressibility) and the mass of fluid in the tubing (gravity). Consequently, it is reasonable to believe that an ODE based on mass balances will give a good description of the dynamic behavior of the well,

$$\begin{aligned}
\dot{x}_1 &= -w_{iv}(x) + w_{gc}(x, u_1) && \text{mass of gas, annulus} \\
\dot{x}_2 &= w_{iv}(x) - w_{pg}(x, u_2) && \text{mass of gas, tubing} \\
\dot{x}_3 &= w_r(x) - w_{po}(x, u_2) && \text{mass of oil, tubing}
\end{aligned}$$

where w_{gc} is the flow of gas through the gas injection choke, w_{iv} is the flow of gas through the injection valve, w_{pg} and w_{po} are the flow of gas and oil through the production choke and w_r is the inflow of oil from the reservoir. The challenge in making such a model, is to find the relation between the system masses (x) and the pressures in the system that determines the flows (w) based on valve-type equations. To keep the presentation short, we do not go into this, but refer to [10], and note that the three state model gives a reasonable approximation to the OLGA model as shown in Figure 2.

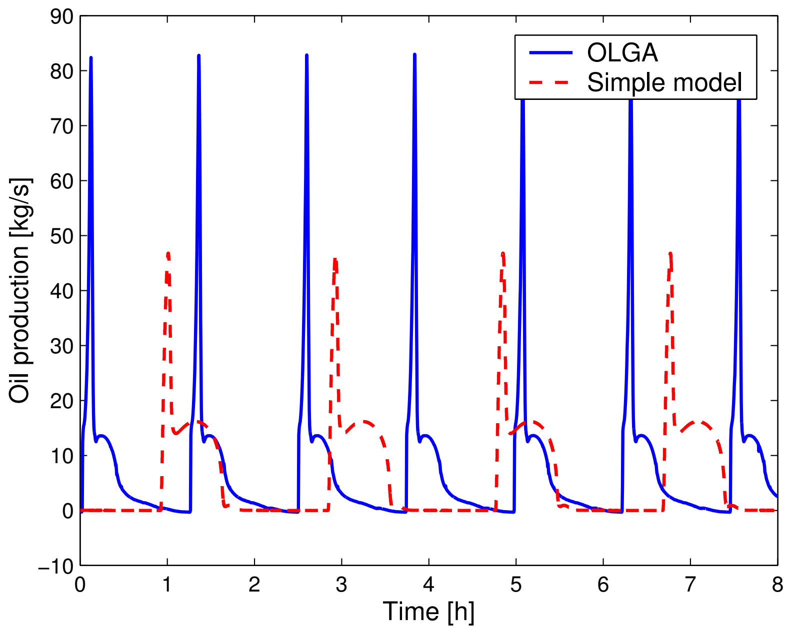


Fig. 2. Comparison of open loop (gas-lift choke is 50% open, production choke is 80% open) behavior between simple model and the rigouros multiphase flow simulator OLGA2000 [3,14].

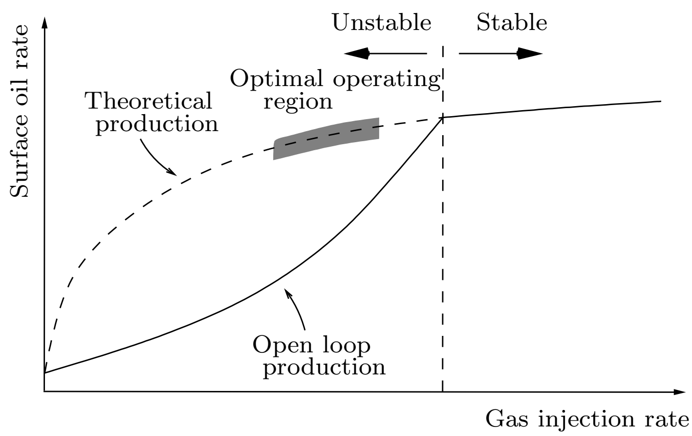


Fig. 3. Conceptual figure showing oil production as a function of gas injection rate. The dotted line is based on steady state calculations, while the solid line is based on dynamic simulations.

5.3 State feedback control

The system written as above can fulfill (a slightly modified) Assumption A2. However, as the expressions for the flows are rather inaccurate (especially for the multiphase flow through the production choke) we will assume that the flow of gas through the gas-lift choke and the flow of oil through the production choke are measured, and that fast control loops control these measured variables. The setpoints for these loops will be the new manipulated variables.

This will, in addition to being a more sensible "engineering approach", simplify the equations. It also allows us to include rate saturations on the opening of the chokes in the simulations.

The dynamic model with the manipulated flows as inputs, is

$$\begin{aligned}\dot{x}_1 &= -w_{iv}(x) + v_1\\ \dot{x}_2 &= w_{iv}(x) - w_{pg}(x, u_2(x))\\ \dot{x}_3 &= w_r(x) - v_2\end{aligned}$$

We choose as phases the sum of gas in the tubing and annulus ($x_1 + x_2$, phase 1 being inflow controlled) and the oil in the tubing (x_3, phase 2 being outflow controlled). The upper saturations on both v_1 and v_2 (the maximum flows through the gas-lift choke and the production choke) depend on the state (through the pressures). Noting that the maximum flows are always obtained when the chokes are maximally open, Assumption A3 can be checked for these saturations. Denote the maximum flows as $\bar{v}_1(x)$ and $\bar{v}_2(x)$, which are given by inserting $u_1 = u_2 = 1$ into the expressions for $w_{iv}(x, u_1)$ and $w_{po}(x, u_2)$.

Then, for $j \in \{1, 2\}$, the controller is given by

$$v_j(x) = \begin{cases} 0 & \text{if } \tilde{v}_j(x) < 0 \\ \tilde{v}_j(x) & \text{if } 0 \le \tilde{v}_j(x) \le \bar{v}_j(x) \\ \bar{v}_j(x) & \text{if } \tilde{v}_j(x) > \bar{v}_j(x) \end{cases}$$

where

$$\begin{aligned}\tilde{v}_1(x) &= w_{pg}(x, u_2(x)) + \lambda_1(M_g^* - x_1 - x_2)\\ \tilde{v}_2(x) &= w_r(x) - \lambda_2(M_o^* - x_3).\end{aligned}$$

For a detailed analysis of stability, and some notes on performance, we refer to [10]. Here, we briefly note that for the simple mass balance model of the oil well, asymptotic stability of an equilibrium follows from Theorem 1 and 2 for $M_g^* = 4400$ kg and $M_o^* = 4600$ kg, and with the set D chosen as

$$3640 \le x_1 \le 4240, \;\; 510 \le x_2 \le 590, \;\; 4550 \le x_3 \le 4650.$$

Simulations (on the simple model) show that the real region of attraction is considerably larger than the one found above, but not global. For instance, if the system is started in a "no production" state (tubing filled with oil – $x_2 = 0$), the system must be brought to a producing condition before the controller is turned on. This is due to the saturation of the chokes. If the tubing is filled with oil, the casing can be filled with enough gas such that $x_1 + x_2 = M_g^*$, without gas being inserted into the tubing. The "oil controller" tries to decrease the amount of oil, but is unsuccessful since the well cannot produce oil with no gas inserted. Increasing M_g^* (temporarily) might be a solution in this case.

5.4 OLGA simulations

Using the OSI [4] link between OLGA and Matlab, the controller, implemented in Matlab, was used on a well modeled in OLGA. The simulation results are shown in Figure 5 and 4. Note that these are state feedback simulation results, the masses and flows were assumed measured.

In the simulations, the well is operated in open loop the two first hours. In this period, the well is stabilized by using a high opening of the gas-lift choke ($u_1 = 0.7$) and a low opening of the production choke ($u_2 = 0.4$). Then, the controller (with $M_g^* = 3450$ kg and $M_o^* = 9400$ kg) is switched on, and remains on for three hours. We see that the controller stabilizes the well at a higher production, and with a significantly lower use of injection gas (in this case, the production increases approximately 2% while the use of injection gas is reduced with 40%). The controller is switched off after 5 hours, keeping the inputs constant. It is seen that the new operating point is open loop unstable. In Figure 5, we see that the controller does not quite reach the mass setpoints. This is mainly due to the flashing phenomena, meaning that there is mass leaving the oil phase entering the gas phase, which is not accounted for in the simple model (and hence the controller). This can be interpreted as errors in the external flows, which the controller has some robustness towards, as discussed in Section 3. The influence is more pronounced in the gas phase, since the total external flow in the oil phase is larger than in the gas phase. Simulations indicate that larger λ's ($\lambda_1 = \lambda_2 = 0.001\text{s}^{-1}$ was used in the simulations shown) reduce the steady state error. Choosing too high λ's leads to problems with saturations, and also numerical problems in Olga may occur. Another remedy for reducing this offset is including an estimate of the flashing in the equations.

5.5 Lab trial

A slightly different controller structure was tried on a lab setup at TU Delft in The Netherlands [5] . The laboratory setup is shown in Figure 6.

[4] OLGA Server Interface (OSI) toolbox, for use with Matlab, developed by ABB Corporate Research.

[5] The experimental setup is designed and implemented by Shell International Exploration and Production B.V., Rijswijk, and is now located in the Kramers Laboratorium voor Fysische Technologie, Faculty of Applied Sciences, Delft University of Technology.

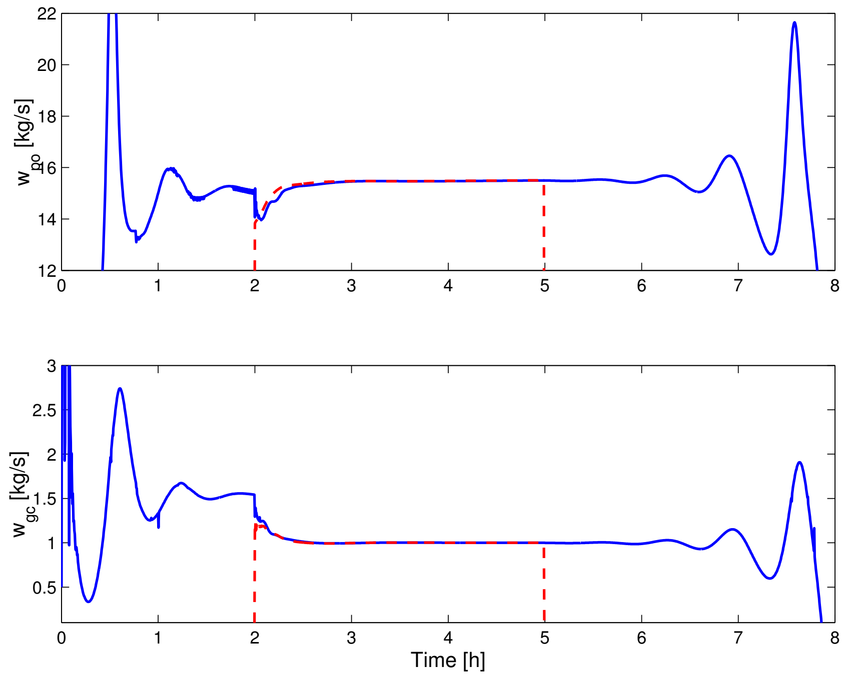


Fig. 4. Desired oil production and gas injection calculated by controller (- -) and the "real" values (-), OLGA simulation

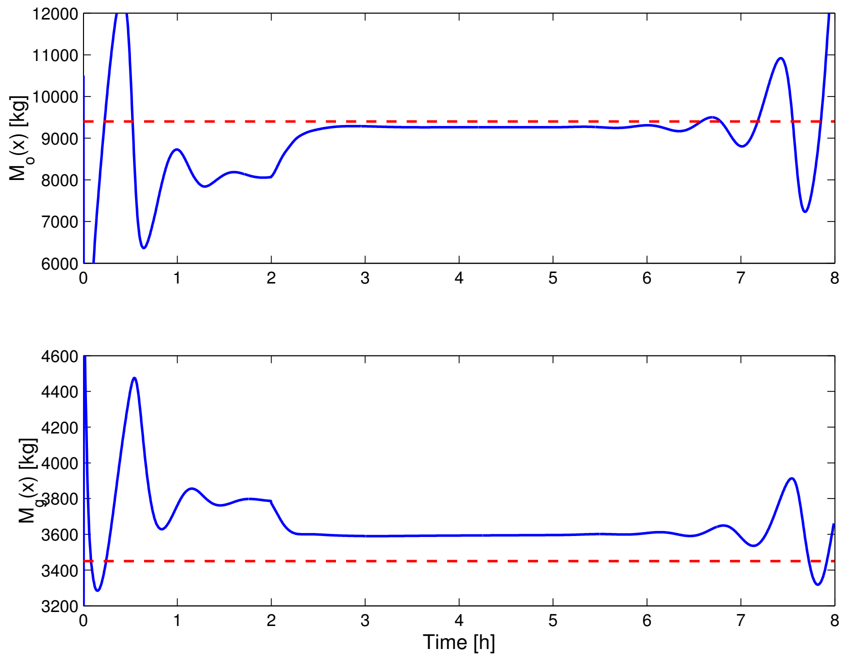
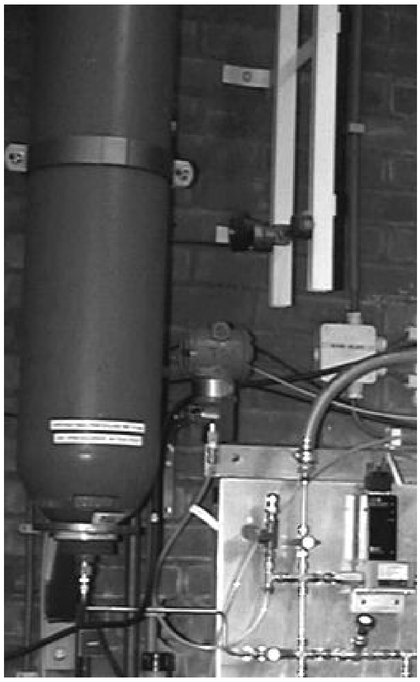


Fig. 5. Mass of phase 1 (gas) and 2 (oil) vs. setpoint, OLGA simulation

The laboratory installation represents a gas-lifted well, using compressed air as lift-gas and water as produced fluid. The production tube is transparent, facilitating visual inspection of the flow phenomena occurring as control is applied. The production tube measures 18m in height and with an inner diameter of 20mm, see Figure 6a. The fluid reservoir is represented by a tube of the same height and an inner diameter of 80mm. The reservoir pressure is given by the static height of the fluid in the reservoir tube. A 30 liter gas bottle represents the annulus, see Figure 6b, with the gas injection point located at the bottom of the production tube. In the experiments run in this study, gas is fed into the annulus from a constant pressure source, giving approximately a rate of 9 L/min (atmospheric conditions). Input and output signals to and from the installation are handled by a microcomputer system, see Figure 6c, to which a laptop computer is interfaced for running the control algorithm and presenting output.

a) The production tube and the reservoir tube.



b) The annulus volume

c) The microcomputer

Fig. 6. The gas-lift laboratory

Having only one control input available (the production choke), we treated the oil in the tubing and gas in annulus and tubing as one phase. This gives us less control freedom than in the simulations in the previous section, but can in many cases be more realistic - for instance there can be situations where the gas-lift choke is not available for control, for example if the amount of available lift gas topside is given by production constraints. An advantage of this structure is that the controller is independent (and hence robust) to mass transfer between the oil and gas phase in the tubing, and that tuning (in terms of total mass setpoint) should be significantly easier. The expected disadvantages are a smaller region of attraction, and that the achievable performance of the well (the oil production) is lower.

The input (flow through production choke) for the system with one phase is then given by (2) and

$$\tilde{v}_2(x) = w_r(x) + w_{gc}(x) - \lambda_2(M_o^* - x_1 - x_2 - x_3).$$

OLGA simulations of this controller (using all available measurements from OLGA) with a PI-controller ensuring the correct mass flow through the production choke, is shown in Figure 7.

The high frequency oscillations seen in Figure 7 are due either to errors in the way OLGA was setup for these simulations, or internal problems in OLGA, possibly due to the small physical size of this laboratory well. Since they are much faster than the interesting dynamics, they should not have an influence on the results and conclusions.

We see that the controller takes the system to a state with higher production. When the controller is turned off, the system starts to oscillate which shows that this operating point is open loop unstable.

Since we in this case have good knowledge of the in- and outflows, there is no steady-state error in the total mass. The fact that we do not know the flashing, does no longer give steady state error in total mass, since the flashing is now an “interconnection flow” (no longer an “external flow”) which the controller is robust against.

There is an interesting phenomena apparent in the OLGA simulations (Figure 7): After the (controlled) mass has converged, the inflow (and the choke opening) continues to move (drift) slowly, before a steady state is reached after around 10 minutes. An explanation for this in terms of the theory, is that there must be a slow dynamic mode in Ω, the set of masses where the total mass is constant. This slow dynamic mode was not observed with the simple model, but it was apparent in the lab trials, as we will see. Even though this mode should not have a direct impact on stability, it makes tuning hard and time consuming, and it obscures the relation between total mass setpoint and well production (production choke opening).

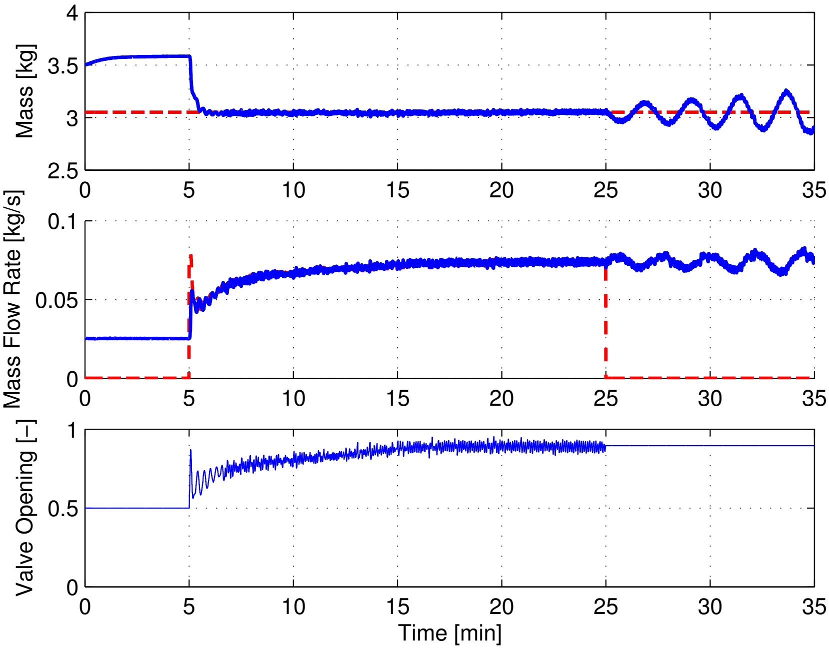


Fig. 7. Total mass vs setpoint, flow through production choke vs setpoint calculated by mass controller, and production choke opening. The controller is turned on after 5 minutes, and turned off after 25 minutes.

The same controller was implemented on the lab. Some limitations in the lab setup posed challenges to the trial. First, there were no available (multiphase) flow measurements of the flow through the production choke. Thus, instead of letting the controller compute the flow through the production choke, we used a simple correlation to compute the desired differential pressure (dp) over the production choke. A (noisy) dp measurement was then used to obtain the desired dp. Second, the phase masses in the lab setup were not measured, necessitating a state observer. The development of a state observer is reported in [6].

The controller proved hard to tune to satisfactory (or even stable) operation, compared with the corresponding simulations on the OLGA simulator model of the lab. The reason for this we believe is partly the bad performance of the inner control loop (due to both the noisy dp measurement and errors in the correlation between the dp measurement and the corresponding multiphase flow). In addition (and probably equally important) comes the slow dynamic mode, as explained above for the OLGA simulations.

In Figure 8 we see the results of a trial, where the well is started rather close to the total mass setpoint. The first transient is due to the convergence of the observer, and then the controller is turned on after two minutes. The controller keeps the mass (and hence the production) stable, until the controller is turned off after 30 minutes. Thereafter, we see that the well starts oscillating. The drift in production choke opening from the controller is turned on to about 25 minutes is clearly seen in the lowermost plot.

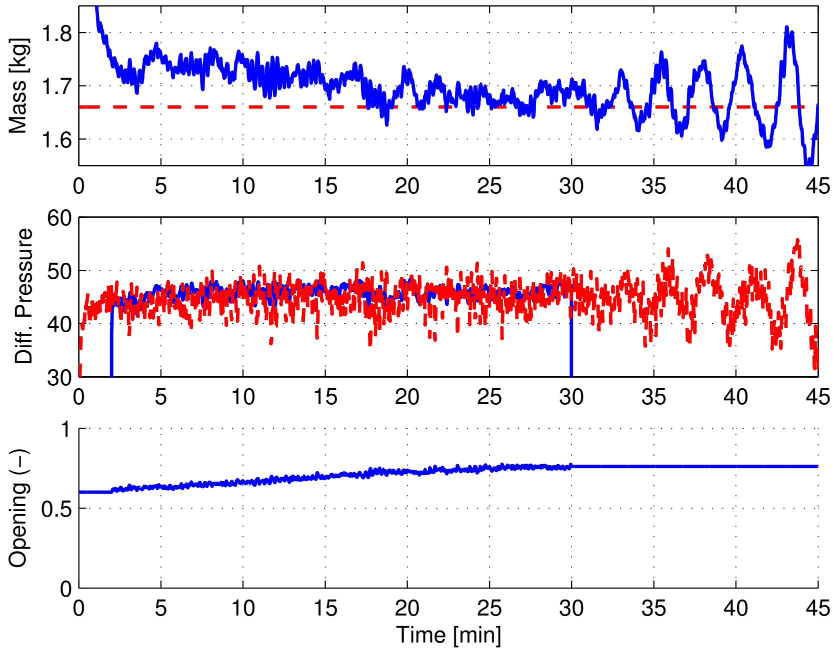


Fig. 8. Estimated mass vs setpoint, differential pressure measurement vs setpoint calculated by mass controller, and production choke opening. The controller is turned on after 2 minutes, and turned off after 30 minutes.

6 Concluding remarks

A controller for a class of positive systems is proposed, leading to closed loop convergence to (or, stability of) a set. The system class could potentially have high applicability, as illustrated by the range of examples studied herein and in [10]. The main restrictive assumption is Assumption A3, ensuring that the "Lyapunov function" used in the proof of the main result is decreasing when the input saturates. The way these assumptions are used in the proof of Theorem 1, along with experience from the examples, indicate that it should be possible to get less conservative conditions, at least for a specific system.

The closed loop system has some robustness properties, most importantly robustness towards unmodeled interconnection terms. These are often the terms that are hardest to model, as for instance in the gas-lift case in Section 5. The closed loop convergence holds independently of the rate-of-convergence parameter λ_j. This means that this parameter (at least nominally) can be used to shape the closed loop performance in terms of the convergence of the mass of each phase, without affecting stability.

The suggested approach leads to stability of a compact set. The approach does not give any guarantees pertaining to the behavior *on* this set, apart from boundedness. However, if the set contains an equilibrium that is asymptotically stable with respect to the set, the equilibrium is also asymptotically stable in the original state space.

The controller was applied to stabilization of gas-lifted wells. Both analysis on the simple model, simulations using the multiphase flow simulator OLGA, and the lab trials confirm that the developed controller was able to stabilize the flow in the gas-lifted well at an open-loop unstable operating point with increased oil production and reduced use of lift gas.

An important feature of the controller is that the developed state feedback controller is independent of the flow through the injection valve, $w_{iv}(x)$, and hence is robust to modeling errors in this flow. This is in contrast to the fact that the system can be open loop stabilized (or destabilized) by the characteristics of this valve. In some cases this valve is designed to always be in a critical flow condition, effectively decoupling the annulus dynamics from the tubing dynamics. Even though this takes care of the instability problem, operational degrees of freedom are lost compared to the approach herein since it implies a constant, given at the design stage, gas injection into the tubing.

Experience from both OLGA simulations and lab trials have shown us that the controller is hard to tune to satisfactory performance - especially for the case where only production choke was used as control. The are several reasons for this - most importantly, perhaps, the difficulty in finding a good relation between mass setpoints and resulting well production. Contributing to this are the slow mode apparent in Ω, and also (and probably related) that for some wells (including the lab setup) the fact that large variations in production gives only rather small variations in mass.

Acknowledgments

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Paper VI

Stabilization of Gas-Distribution Instability in Single-Point Dual Gas Lift Wells

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Stabilization of Gas-Distribution Instability in Single-Point Dual Gas Lift Wells

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Summary

While casing-heading instability in single gas lift wells has attracted a lot of attention, gas-distribution instability in *dual* gas lift wells has not. In this paper, we present a simple, nonlinear dynamic model that is shown to capture the essential dynamics of the gas-distribution instability despite the complex nature of two-phase flow. Using the model, stability maps are generated showing regions of stable and unstable settings for the production valves governing the produced flows from the two tubings. Optimal steady-state production is shown to lie well within the unstable region, corresponding to a gas distribution between the production tubings that cannot be sustained without automatic control. A simple control structure is suggested that successfully stabilizes the gas-distribution instability in simulations and, more importantly, in laboratory experiments.

Introduction

Artificial lift is a common technique to increase tail-end production from mature fields, and injection of gas (gas lift) rates among the most widely used of such methods. Gas lift can induce severe production flow oscillations because of casing-heading instability, a phenomenon that originates from dynamic interaction between injection gas in the casing and the multiphase fluid in the tubing. The fluctuating flow typically has an oscillation period of a few hours and is distinctly different from short-term oscillations caused by hydrodynamic slugging. The casing-heading instability introduces two production-related challenges. Average production is decreased compared to a stable flow regime, and the highly oscillatory flow puts strain on downstream equipment.

Reports from industry as well as academia suggest that automatic control (feedback control) is a powerful tool to eliminate casing-heading instability and increase production from gas lift wells (Kinderen et al. 1998; Jansen et al. 1999; Dalsmo et al. 2002; Boisard et al. 2002; Hu and Golan 2003; Eikrem et al. 2003; Aamo et al. 2005). Automatic control may or may not require downhole measurements. If downhole information is needed by the controller, the use of soft-sensing techniques may alleviate the need for downhole measurements. In Aamo et al. (2005), downhole pressure is estimated on line using a simple dynamic model and measurements at the wellhead only. The estimated pressure is in turn used in a controller for stabilizing the casing-heading instability.

Understanding and predicting under which conditions a gas lift well will exhibit flow instability is important in every production-planning situation. This problem has been addressed by several authors by constructing stability maps [i.e., a 2D diagram that shows the regions of stable and unstable production of a well (Poblano et al. 2005; Fairuzov et al. 2004)]. The axes define the operating conditions in terms of the gas-injection rate and, for instance, the production-choke opening or wellhead pressure.

A dual gas lift well is a well with two independent tubings producing from two different hydrocarbon-bearing layers and sharing a common lift gas supply. The injection gas is supplied through a common casing and injected into the tubings through two individual gas lift valves. A sketch of a typical system is shown in **Fig. 1.** The dual gas lift well introduces a new instability phenomenon: the gas-distribution instability. This relates to the fact that under certain operating conditions, it is impossible to sustain the feed of injected gas into both tubings. Instead, all the injected gas will eventually be routed through one of the gas lift valves. As a consequence, the second tubing produces poorly or not at all, decreasing the total production substantially. There are few reports, if any, on automatic control of dual gas lift wells, although Boisard et al. (2002) briefly mentions an application.

In this paper, we present a simple, nonlinear dynamic model that captures the essential dynamics of the gas-distribution instability. It is an extension of the model for a single gas lift well presented in Eikrem et al. (2003) and Aamo et al. (2005). Using the model, we generate a stability map for a single-point dual gas lift well, and present a control structure for stabilizing the system at open-loop, unstable setpoints. The performance of the controller is demonstrated in simulations using the model, but more importantly, stabilization is also achieved in laboratory experiments.

This paper is organized as follows. In the Mathematical Model section, a nonlinear dynamic model applicable to dual gas lift wells is presented, followed by a discussion on instability mechanisms and the generation of stability maps in the Instability Mechanisms and Control section. The stability analysis is based on computing eigenvalues for the linearized model, accompanied by simulations using the nonlinear model. The proposed control structure is presented in the Automatic Control segment of that section, and experimental results using a gas lift laboratory located at the U. of Technology—Delft are shown in the Laboratory Experiments section. The paper ends with discussion and conclusions in the Conclusions section.

Mathematical Model

The process described in the Introduction and sketched in Fig. 1 is modeled mathematically by five states: x_1 is the mass of gas in the annulus; x_2 is the mass of gas in tubing 1; x_3 is the mass of oil above the gas-injection point in tubing 1; x_4 is the mass of gas in tubing 2; and x_5 is the mass of oil above the gas-injection point in tubing 2. Looking at Fig. 1, we have

$$\dot{x}_1 = w_{gc} - w_{iv,1} - w_{iv,2}, \quad \text{(1)}$$

$$\dot{x}_2 = w_{iv,1} + w_{rg,1} - w_{pg,1}, \quad \text{(2)}$$

$$\dot{x}_3 = w_{ro,1} - w_{po,1}, \quad \text{(3)}$$

$$\dot{x}_4 = w_{iv,2} + w_{rg,2} - w_{pg,2}, \quad \text{(4)}$$

and

$$\dot{x}_5 = w_{ro,2} - w_{po,2}. \quad \text{(5)}$$

where $\dot{}$ denotes differentiation with respect to time; w_{gc} is a constant mass flow rate of lift gas into the annulus; $w_{iv,k}$ is the mass flow rate of lift gas from the annulus into tubing k; $w_{rg,k}$ is the gas mass flow rate from the reservoir into tubing k; $w_{pg,k}$ is the mass flow rate of gas through production choke k; $w_{ro,k}$ is the oil mass flow rate from the reservoir into tubing k; and $w_{po,k}$ is the mass flow rate of produced oil through production choke k ($k \in \{1,2\}$). The flows are modeled by

$$w_{gc} = \text{constant flow rate of lift gas,} \quad (6)$$

$$w_{iv,1} = C_{iv,1}\sqrt{\rho_{a,i}\max\{0, p_{a,i} - p_{wi,1}\}}, \quad (7)$$

$$w_{iv,2} = C_{iv,2}\sqrt{\rho_{a,i}\max\{0, p_{a,i} - p_{wi,2}\}}, \quad (8)$$

$$w_{pc,1} = C_{pc,1}\sqrt{\rho_{m,1}\max\{0, p_{wh,1} - p_s\}}f_{pc,1}(u_1), \quad (9)$$

$$w_{pc,2} = C_{pc,2}\sqrt{\rho_{m,2}\max\{0, p_{t,2} - p_s\}}f_{pc,2}(u_2), \quad (10)$$

$$w_{pg,1} = \frac{x_2}{x_2 + x_3} w_{pc,1}, \quad (11)$$

$$w_{po,1} = \frac{x_3}{x_2 + x_3} w_{pc,1}, \quad (12)$$

$$w_{pg,2} = \frac{x_4}{x_4 + x_5} w_{pc,2}, \quad (13)$$

$$w_{po,2} = \frac{x_5}{x_4 + x_5} w_{pc,2}, \quad (14)$$

$$w_{ro,1} = f_{r,1}(p_{r,1} - p_{wb,1}), \quad (15)$$

$$w_{ro,2} = f_{r,2}(p_{r,2} - p_{wb,2}), \quad (16)$$

$$w_{rg,1} = r_{go,1} w_{ro,1}, \quad (17)$$

and

$$w_{rg,2} = r_{go,2} w_{ro,2}. \quad (18)$$

$C_{iv,k}$ and $C_{pc,k}$ are constants; u_k is the production choke setting [$u_k(t) \in (0,1)$]; $\rho_{a,i}$ is the density of gas in the annulus at the injection point; $p_{a,i}$ is the pressure in the annulus at the injection point; $\rho_{m,k}$ is the density of the oil/gas mixture at the wellhead; $p_{wh,k}$ is the pressure at the wellhead; $p_{wi,k}$ is the pressure in the tubing at the gas-injection point; $p_{wb,k}$ is the pressure at the wellbore; p_s is the pressure in the manifold; $p_{r,k}$ is the reservoir pressure far from the well; and $r_{go,k}$ is the gas-to-oil ratio (based on mass flows) of the flow from the reservoir. The function $f_{pc,k}$ is valve specific and represents a possibly nonlinear scaling of the flow as a function of the choke setting u_k. $f_{r,k}$ is a case-specific, possibly nonlinear, mapping from the pressure difference between the reservoir and the wellbore to the fluid flow from the reservoir. The manifold pressure, p_s, is assumed to be held constant by a control system, and the reservoir pressure, $p_{r,k}$, and gas-to-oil-ratio, $r_{go,k}$, are assumed to be slowly varying and are therefore treated as constant. Note that flow rates through the valves are restricted to be positive. The densities are modeled as follows:

$$\rho_{a,i} = \frac{M}{RT_a} p_{a,i}, \quad (19)$$

$$\rho_{m,k} = \frac{x_{1+k} + x_{2+k}}{L_{w,k} A_{w,k}}; \quad (20)$$

and the pressures as follows:

$$p_{a,i} = \left(\frac{RT_a}{V_a M} + \frac{gL_a}{V_a} \right) x_1 , \qquad (21)$$

$$p_{wh,k} = \frac{RT_{w,k}}{M} \frac{x_{1+k}}{L_{w,k} A_{w,k} - \nu_o x_{2+k}} , \qquad (22)$$

$$p_{wi,k} = p_{wh,k} + \frac{g}{A_{w,k}} \left(x_{1+k} + x_{2+k} \right), \qquad (23)$$

$$p_{wb,k} = p_{wi,k} + \rho_o g L_{r,k} . \qquad (24)$$

M is the molar weight of the gas; R is the universal gas constant; T_a is the temperature in the annulus; $T_{w,k}$ is the temperature in the tubing; V_a is the volume of the annulus; L_a is the length of the annulus; $L_{w,k}$ is the length of the tubing; $A_{w,k}$ is the cross-sectional area of the tubing above the injection point; $L_{r,k}$ is the length from the reservoir to the gas-injection point; $A_{r,k}$ is the cross-sectional area of the tubing below the injection point; g is the gravity constant; ρ_o is the density of the oil; and v_o is the specific volume of the oil. The oil is considered incompressible, so $\rho_o = 1/v_o$ is constant. The temperatures T_a and $T_{w,k}$ are slowly varying and, therefore, treated as constant. This model is an extension to a dual well from the single-well model presented in Eikrem et al. (2003) and Aamo et al. (2005).

Instability Mechanisms and Control

Casing-Heading Instability. The dynamics of highly oscillatory flow in single-point-injection gas lift wells can be described as follows:

- Gas from the annulus starts to flow into the tubing. As gas enters the tubing, the pressure in the tubing falls, accelerating the inflow of lift gas.
- If there is uncontrolled gas passage between the annulus and tubing, the gas pushes the major part of the liquid out of the tubing while the pressure in the annulus falls dramatically.
- The annulus is practically empty, leading to a negative pressure difference over the injection orifice, blocking the gas flow into the tubing. Because of the blockage, the tubing becomes filled with liquid and the annulus with gas.
- Eventually, the pressure in the annulus becomes high enough for gas to penetrate into the tubing, and a new cycle begins.

For more information on this type of instability, often termed severe slugging, refer to Xu and Golan (1989). The oscillating production associated with severe slugging causes problems for downstream processing equipment and is unacceptable in operations. The traditional remedy is to choke back to obtain a nonoscillating flow. As mentioned in the Introduction, automatic control is a powerful approach to eliminate oscillations; moreover, reports also show that this technology increases production (Kinderen et al. 1998; Jansen et al. 1999; Dalsmo et al. 2002; Boisard et al. 2002; Hu and Golan 2003; Eikrem et al. 2003; Aamo et al. 2005). Another approach is to fit a gas lift valve, which secures critical flow. This decouples the dynamics of the casing and tubing volumes and thereby eliminates casing-heading instabilities. Because the topic of this paper is a different kind of instability present in dual gas lift wells, refer to Kinderen et al. (1998), Jansen et al. (1999), Dalsmo et al. (2002), Boisard et al. (2002), Hu and Golan (2003) Eikrem et al. (2003), Aamo et al. (2005), and Xu and Golan (1989) for more details concerning stabilization of casing-heading instabilities.

Gas-Distribution Instability. In single-point dual gas lift oil wells another instability mechanism occurs that is related to the distribution of lift gas between the two tubings. The following statements assume subcritical flow between the annulus and the two tubings. Suppose each tubing is steadily drawing 50% of the lift gas. If one tubing momentarily draws more, the hydrostatic pressure drop in the tubing decreases, resulting in a larger pressure drop across the gas-injection orifice. This in turn accelerates the flow of gas, and the tubing draws even more lift gas. On the other hand, because the gas flow into the second tubing decreases, the hydrostatic pressure drop in the second tubing increases. Thus, the pressure drop across the gas-injection orifice decreases and, as a consequence, less gas is routed through the second tubing. Eventually, all lift gas will be routed through one tubing, which could impact total oil production.

We will now analyze gas-distribution instability using the relatively simple model (1)–(5), applied to the gas lift laboratory used in the experiments of the Laboratory Experiments section. For the laboratory, we have

$$f_{pc,k}(u_k) = 50^{u_k - 1} \quad \text{...(25)}$$

and

$$f_{r,k}(p_{r,k} - p_{wb,k}) = C_{r,k}\sqrt{\rho_o \max\{0, p_{r,k} - p_{wb,k}\}},$$

$$\text{...(26)}$$

$k \in \{1,2\}$, where $C_{r,1}$ and $C_{r,2}$ are constants. **Table 1** summarizes the numerical coefficients used for this case. Given a pair of production-valve openings (u_1 and u_2 for the long and short tubing, respectively), we look for steady-state solutions by setting the time derivatives in (1)–(5) to zero. Not all choices of u_1 and u_2 are feasible with respect to obtaining production from both tubings. The yellow *and* black dots in **Fig. 2** represent the pair u_1 and u_2 whose steady-state solution corresponds to production from both tubings. Other choices will give production from one tubing only and are not of interest to us. For the pairs of interest, we linearize system (1)–(5) around the steady-state solution to study linear stability. The black dots in Fig. 2 represent (linearly) unstable settings. Roughly speaking, there is a region $(u_1, u_2) \in (0, 0.50) \times (0, 0.38)$ of linearly stable settings, while the rest are unstable settings. **Fig. 3** shows the steady-state total production as a function of (u_1, u_2). Clearly, production is higher for large values of u_1 and u_2. In fact, the optimum is located at approximately (1.00, 0.83), which corresponds to an unstable setting and a steady-state production substantially larger than what can be achieved in the linearly stable region. The gas distribution at steady state as a function of (u_1, u_2) is shown in **Fig. 4.**

For u_1 = 0.90 and u_2 = 0.83, the steady-state solution is unstable, with the largest real part of the eigenvalues of the linearized system being strictly positive $\{\max_j \left[\mathrm{Re}(\lambda_j)\right] = 0.03\}$. Selecting the initial condition equal to the steady-state solution, only slightly perturbed, and simulating system (1)–(5), we obtain the result shown in **Figs. 5 and 6.** Fig. 5 shows the gas distribution between the two tubings as a function of time. At approximately steady state, five-sixths of the gas flows through the long tubing, while one-sixth of the gas flows through the short tubing. After approximately 2 minutes, the instability becomes visible in this graph and the gas starts to redistribute. After approximately 13 minutes, all the gas is routed through the short tubing. Fig. 6 shows the corresponding fluid-production curves. The long tubing has a substantial drop in production as a result of losing its lift gas, while the short tubing produces a little more. The total production drops approximately 20%.

Automatic Control. To optimize production, the instability needs to be dealt with. Motivated by the success of the controller used to stabilize the casing-heading instability, the control structure in **Fig. 7** is proposed. It consists of two independent feedback loops regulating the pressure at the injection points of each tubing. More precisely, two productivity-index (PI) controllers (proportional gain plus integral action) are employed, producing the incremental control signals:

$$\Delta u_k(j) = K_{c,k}\left[e_k(j) - e_k(j-1) + \frac{\Delta t}{\tau_{I,k}} e_k(j)\right], \text{......(27)}$$

$k \in \{1,2\}$, where

$$e_k(j) = p_{wi,k}(j) - p^*_{wi,k}. \text{ ....(28)}$$

$K_{c,k}$ and $\tau_{I,k}$ are the proportional gains and integral times, respectively; Δt is the sampling time; and j denotes the time index. $p^*_{wi,k}$, $k \in \{1,2\}$, are appropriate setpoints for the pressure. Repeating the simulation from Figs. 5 and 6, and closing the control loops at t = 10 minutes, we obtain the result in **Figs. 8 and 9.** Fig. 8 shows the gas distribution between the two tubings. At the time of initiation of control (t = 10 minutes), the gas has been considerably redistributed, but the control effectively drives the system back to the steady-state solution. Fig. 9 shows the corresponding fluid production. The control inputs that achieve this result are shown in **Fig. 10.**

Laboratory Experiments

Realistic tests of control structures for gas lift wells are performed using the gas lift well laboratory setup at Delft U. of Technology.* Prior laboratory experiments have verified that the PI controller (27)–(28) successfully stabilizes the casing-heading instability in single gas lift wells (Eikrem et al. 2003). Motivated by that result, the same control structure is tested experimentally for stabilization of the gas-distribution instability in dual gas lift wells.

Experimental Setup. The laboratory installation represents a dual gas lift well, using compressed air as lift gas and water as produced fluid. It is sketched in Fig. 7. The two production tubes are transparent, facilitating visual inspection of the flow phenomena occurring as control is applied. The long tubing measures 18 m in height and has an inner diameter

* The experimental setup is designed and implemented by Shell Intl. E&P, B.V., Rijswijk, and is now located in the Kramers Laboratorium voor Fysische Technologie, Faculty of Applied Sciences, Delft U. of Technology.

of 20 mm, while the short tubing measures 14 m in height and has an inner diameter of 32 mm; see **Fig. 11a.** Each tubing has its own fluid reservoir represented by a tube of the same height, but with the substantially larger inner diameters of 80 mm and 101 mm, respectively. The reservoir pressures are given by the static height of the fluid in the reservoir tubes. The top of the tubings are aligned, which implies that the long tubing stretches 4 m deeper than the short one. A gas bottle represents the annulus (see Fig. 11b) with the gas-injection points located at the same level in both tubings and aligned with the bottom of the short production tube; see Fig. 7. In the experiments run in this study, gas is fed into the annulus at a constant rate of 0.6×10^{-3} kg/s. Input and output signals to and from the installation are handled by a microcomputer system (see Fig. 11c) to which a laptop computer is interfaced for running the control algorithm and presenting output.

Experimental Results. For the prescribed rate of lift gas, the two PI control loops sketched in Fig. 7 are incapable of stabilizing the gas-distribution instability from an arbitrary initial condition and, in particular, initial conditions for which only one tubing is producing are not feasible. Therefore, a simple startup procedure consisting of the following steps was used to bring the system into a state from which the controller is able to stabilize:

1. Set u_1 and u_2 close to the expected steady-state values. This does not have to be very accurate.
2. Momentarily increase the rate of lift gas beyond the nominal rate (w_{gc}) such that both tubings draw gas and produce in open loop.
3. While both tubings draw lift gas, close the control loops.
4. Gradually decrease the rate of lift gas to its nominal value.

In the experiments, the coefficients for the controllers were set to $K_{c,1} = -1.2$, $K_{c,2} = -1.5$, and $\tau_{I,1} = \tau_{I,2} = 50$ seconds, while the sampling time was Δt = 1.5 seconds. The setpoints for $p_{wi,1}$ and $p_{wi,2}$ were set equal to $p^*_{wi,1} = p^*_{wi,2} = 1$ barg (2 bara, 2×10^5 Pa), and the pressure deviations (28) were computed in barg (not in Pa). **Figs. 12 and 13** show the controlled downhole pressures $p_{wi,1}$ and $p_{wi,2}$ as functions of time, along with the setpoints $p^*_{wi,1}$ and $p^*_{wi,2}$. The two PI control loops gradually drive $p_{wi,1}$

and $p_{wi,2}$ toward their respective setpoints, reaching them in approximately 8 minutes. The commanded production-valve openings achieving this result are shown in **Figs. 14 and 15.** The valve openings are approximately 75 and 82%, respectively, when regulation to setpoint is achieved. At t = 10 minutes, the control is turned off to demonstrate that the setpoints are indeed open-loop unstable. Figs. 12 and 13 show that the pressures diverge rapidly from their setpoints after t = 10 minutes, confirming open-loop instability. **Fig. 16** shows the gas distribution between the two tubings. During regulation, in the period between t = 8 and t = 10 minutes, approximately one-third of the gas is routed through the short tubing while two-thirds are routed through the long tubing. The uneven gas distribution for this case of identical setpoints ($p^*_{wi,1} = p^*_{wi,2}$) is caused by the difference in valve characteristics between the two gas-injection valves (see $C_{iv,1}$ and $C_{iv,2}$ in Table 1). Total production during regulation is approximately 10 kg/min, as shown in **Fig. 17.** The effect of the gas-distribution instability is evident as control is turned off in the interval t = 10 to t = 15 minutes in Figs. 16 and 17. The gas quickly redistributes, with 100% being routed through the short tubing and nothing through the long tubing. As a consequence, the long tubing stops producing, while the short tubing produces a little more. The total production drops by approximately 28%, making a strong case for applying automatic control. Comparing the interval $t \in (10,15)$ in Figs. 16 and 17 to Figs. 5 and 6, the qualitative resemblance is striking when considering the highly complex nature of two-phase flow and the simplicity of the model from the Mathematical Model section. While part of the difference between simulations and the experiments is caused by modeling error, the fact that simulations and experiments are performed at different setpoints is also a source of difference in this comparison. Although the model was set up for the laboratory case in this paper, it can easily be modified for real cases by changing parameters and reservoir flow relationships. In particular, $f_{r,1}(\cdot)$, $f_{r,2}(\cdot)$, $r_{go,1}$, and $r_{go,2}$ must be modified to model flows from a real reservoir. Typically, reservoir oil flow is modeled proportional (PI) to the pressure difference $p_{r,k}-p_{wb,k}$, while the gas-to-oil ratio is usually treated as constant.

Additional experiments were run to determine whether just one of the control loops is sufficient for stabilization of the gas-distribution instability. The experiments were unsuccessful, from which we conclude that both control loops are required. It is a drawback that the controllers rely on downhole measurements because such measurements may not be available or may be unreliable. The use of soft-sensing techniques may alleviate the need for downhole measurements, as demonstrated in Aamo et al. (2005). In that reference the downhole pressure was estimated on line from measurements at the wellhead only and the downhole-pressure estimates were employed for stabilization of the casing-heading instability.

Conclusions

In this paper, we have presented a simple scheme for stabilization of the gas-distribution instability in dual gas lift oil wells with a common lift gas supply. A simple nonlinear dynamic model consisting of only five states was shown to successfully capture the essential dynamics of the gas-distribution instability, despite the complex nature of two-phase flow. Using the model, stability maps were generated showing regions of linearly stable and unstable settings for the production valves governing the produced flows from the two tubings. Accompanying plots of total production indicated that optimal steady-state production lies at large valve openings and well within the unstable region. A simple control structure was suggested that successfully stabilizes the gas-distribution instability in simulations and, more importantly, in laboratory experiments. For the settings used in the laboratory, total production dropped 28% when automatic control was switched off! Comparing simulation results with experiments, the predictive capability of the model is evident.

The results of this paper show that the problem of gas-distribution instability in dual gas lift oil wells may be analyzed and counteracted by simple methods and that there is a potential for significantly increasing production by installing a simple, inexpensive control system.

Acknowledgments

We gratefully acknowledge the support from Shell Intl. E&P, B.V., and Kramers Laboratorium voor Fysische Technologie, Faculty of Applied Sciences, Delft U. of Technology. In particular, we would like to thank Dr. Richard Fernandes (Shell) and Dr. R.V.A. Oliemans (Delft U. of Technology).

Nomenclature

$A_{r,1}$ = cross-sectional area of tubing 1 below the gas-injection point, [L^2], m^2

$A_{r,2}$ = cross-sectional area of tubing 2 below the gas-injection point, [L^2], m^2

$A_{w,1}$ = cross-sectional area of tubing 1 above the gas-injection point, [L^2], m^2

$A_{w,2}$ = cross-sectional area of tubing 2 above the gas-injection point, [L^2], m^2

$C_{iv,1}$ = valve constant for gas-injection valve 1, [L^2], m^2

$C_{iv,2}$ = valve constant for gas-injection valve 2, [L^2], m^2

$C_{pc,1}$ = valve constant for production valve 1, [L^2], m^2

$C_{pc,2}$ = valve constant for production valve 2, [L^2], m^2

$C_{r,1}$ = valve constant for reservoir valve 1, [L^2], m^2

$C_{r,2}$ = valve constant for reservoir valve 2, [L^2], m^2

e_k = regulation error, [m/Lt^2], Pa

$f_{pc,1}$ = valve characteristic function

$f_{pc,2}$ = valve characteristic function

$f_{r,1}$ = production function, [m/t], kg/sec

$f_{r,2}$ = production function, [m/t], kg/sec

g = acceleration of gravity, [L/t^2], m/sec^2

j = time index

$K_{c,1}$ = controller gain

$K_{c,2}$ = controller gain

L_a = length of annulus, [L], m

$L_{r,1}$ = length of tubing 1 below gas-injection point, [L], m

$L_{r,2}$ = length of tubing 2 below gas-injection point, [L], m

$L_{w,1}$ = length of tubing 1 above gas-injection point, [L], m

$L_{w,2}$ = length of tubing 2 above gas-injection point, [L], m

M = molar weight of gas, [m/n], kg/mol

p_a = pressure at the gas-injection point in the annulus, [m/Lt^2], Pa

$p_{r,1}$ = pressure in reservoir 1, [m/Lt^2], Pa

$p_{r,2}$ = pressure in reservoir 2, [m/Lt^2], Pa

p_s = pressure in the manifold, [m/Lt^2], Pa

$p_{wh,1}$ = pressure at wellhead 1, [m/Lt^2], Pa

$p_{wh,2}$ = pressure at wellhead 2, [m/Lt^2], Pa

$p_{wb,1}$ = pressure at wellbore 1, [m/Lt^2], Pa

$p_{wb,2}$ = pressure at wellbore 2, [m/Lt^2], Pa

$p_{wi,1}$ = pressure at gas-injection point in tubing 1, [m/Lt^2], Pa

$p_{wi,2}$ = pressure at gas-injection point in tubing 2, [m/Lt^2], Pa

R = universal gas constant, [mL^2/nTt^2], J/Kmol

$r_{go,1}$ = gas-to-oil ratio in flow from reservoir 1

$r_{go,2}$ = gas-to-oil ratio in flow from reservoir 2

t = time, [t], seconds

T_a = temperature in annulus, [T], K

$T_{w,1}$= temperature in tubing 1, [T], K

$T_{w,2}$= temperature in tubing 2, [T], K

u_1 = setting of production valve 1

u_2 = setting of production valve 2

v_o = specific volume of oil, [L^3/m], m^3/kg

V_a = volume of annulus, [L^3], m^3

w_{gc} = flow of gas into annulus, [m/t], kg/sec

$w_{iv,1}$ = flow of gas from annulus into tubing 1, [m/t], kg/sec

$w_{iv,2}$= flow of gas from annulus into tubing 2, [m/t], kg/sec

$w_{pc,1}$ = flow of mixture from tubing 1, [m/t], kg/sec

$w_{pc,2}$ = flow of mixture from tubing 2, [m/t], kg/sec

$w_{pg,1}$ = flow of gas from tubing 1, [m/t], kg/sec

$w_{pg,2}$ = flow of gas from tubing 2, [m/t], kg/sec

$w_{po,1}$ = flow of oil from tubing 1, [m/t], kg/sec

$w_{po,2}$ = flow of oil from tubing 2, [m/t], kg/sec

$w_{rg,1}$ = flow of gas from reservoir into tubing 1, [m/t], kg/sec

$w_{rg,2}$ = flow of gas from reservoir into tubing 2, [m/t], kg/sec

$w_{ro,1}$= flow of oil from reservoir into tubing 1, [m/t], kg/sec

$w_{ro,2}$ = flow of oil from reservoir into tubing 2, [m/t], kg/sec

x_1 = mass of gas in annulus, [m], kg

x_2= mass of gas in tubing 1, [m], kg

x_3= mass of oil in tubing 1, [m], kg

x_4= mass of gas in tubing 2, [m], kg

x_5= mass of oil in tubing 2, [m], kg

Δt = timestep, [t], seconds

$\Delta u_k(j)$ = valve opening change, [-]

$\rho_{a,i}$ = density of gas at injection point in annulus, [m/L^3], kg/m^3

$\rho_{m,1}$ = density of mixture at wellhead 1, [m/L^3], kg/m^3

$\rho_{m,2}$ = density of mixture at wellhead 2, [m/L^3], kg/m^3

ρ_o = density of oil, [m/L^3], kg/m^3

$\tau_{I,k}$ = integral time, [t], sec

λ_j = eigenvalue

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SI Metric Conversion Factors

bar	×	1.0*	E+05=	Pa
bbl	×	1.589 873	E–01=	m^3
Btu	×	1.055 056	E+00=	kJ
ft	×	3.048*	E–01=	m
ft^2	×	9.290 304*	E–02=	m^2
ft^3	×	2.831 685	E–02=	m^3
°F		(°F+459.67)/1.8	=	K
kg/ m^3	×	1.601 846	E+01=	lbm/ ft^3
lbm	×	4.535 924	E–01=	kg

*Conversion factor is exact.

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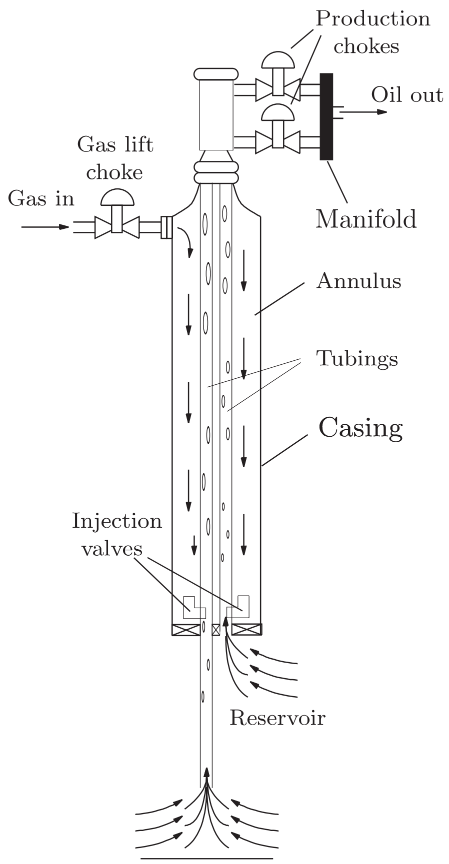


Fig. 1—A single-point dual gas lift oil well.

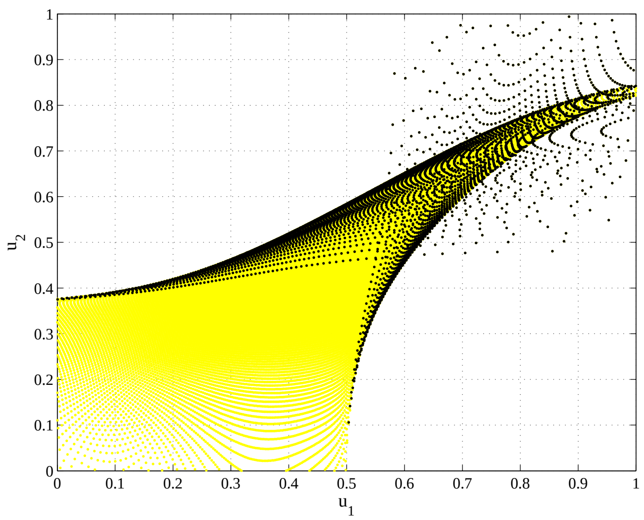


Fig. 2—Feasible choke settings for production from both tubings (yellow *and* black area) and choke settings for which open-loop production is unstable (black area).

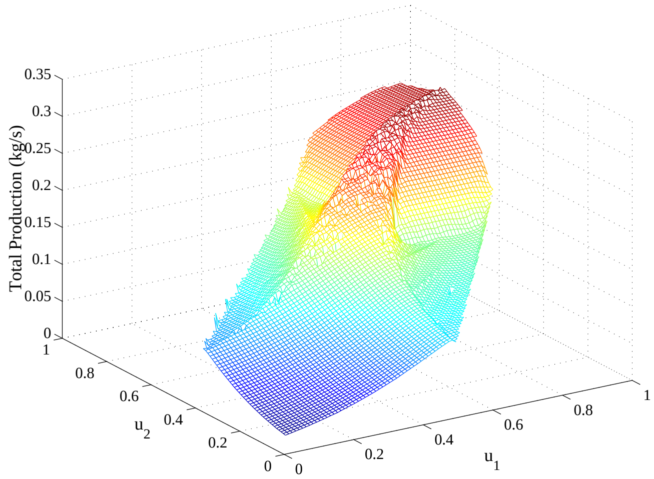


Fig. 3—Total fluid production as a function of u_1 and u_2.

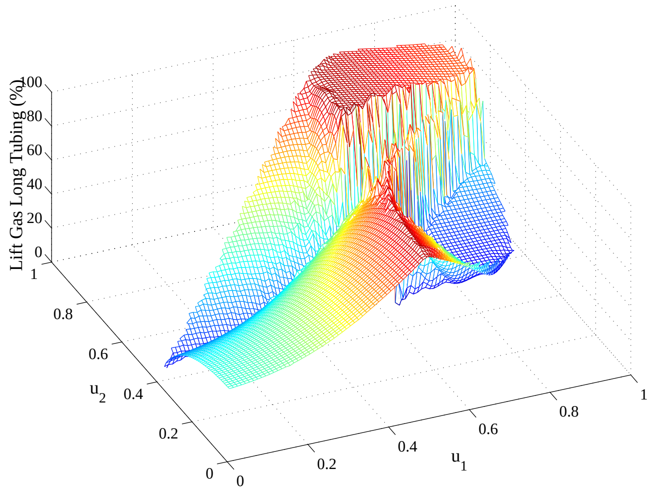


Fig. 4—Gas distribution as a function of u_1 and u_2.

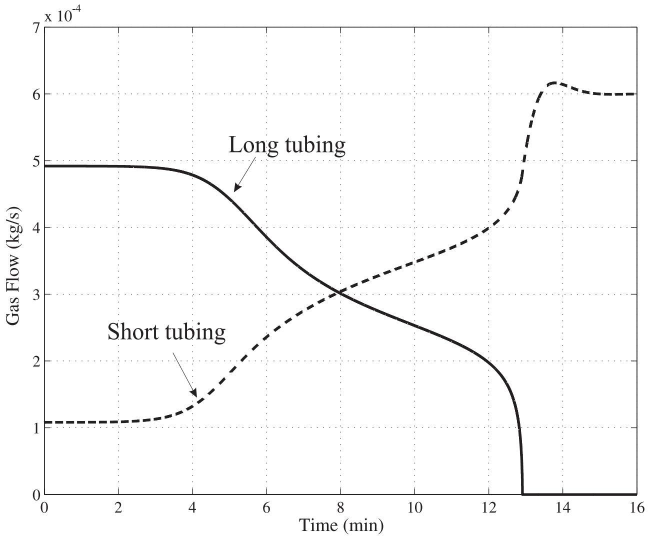


Fig. 5—Gas distribution as a function of time in the uncontrolled case.

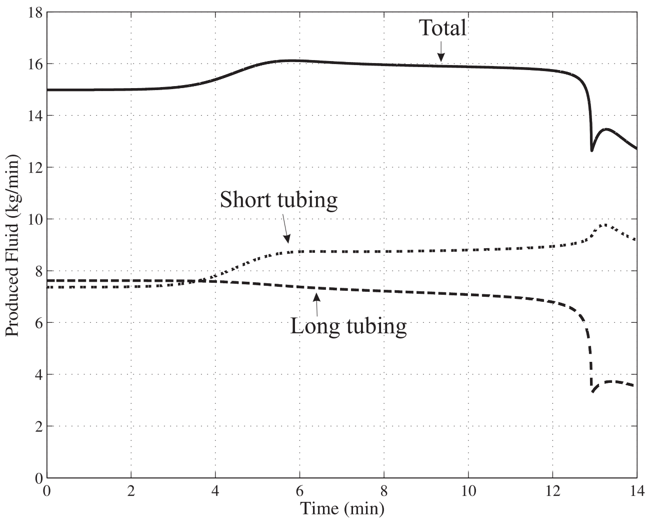


Fig. 6—Fluid production as a function of time in the uncontrolled case.

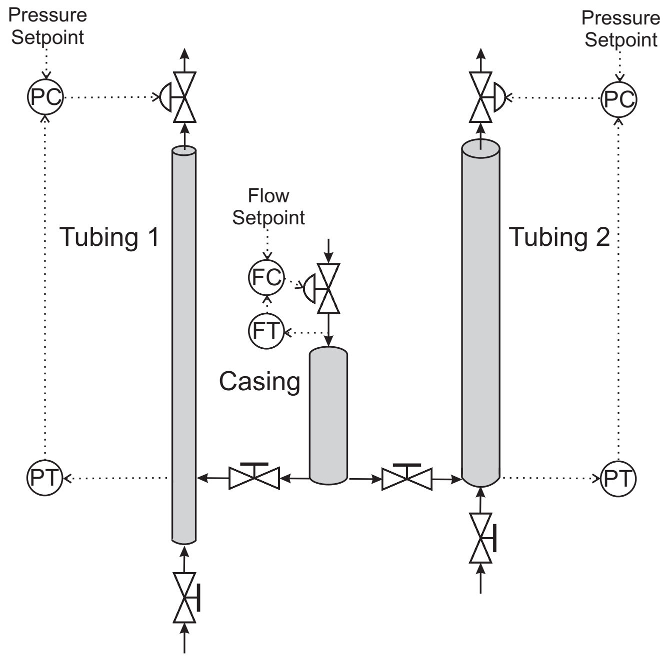


Fig. 7—Controller structure for stabilization of the gas-distribution instability.

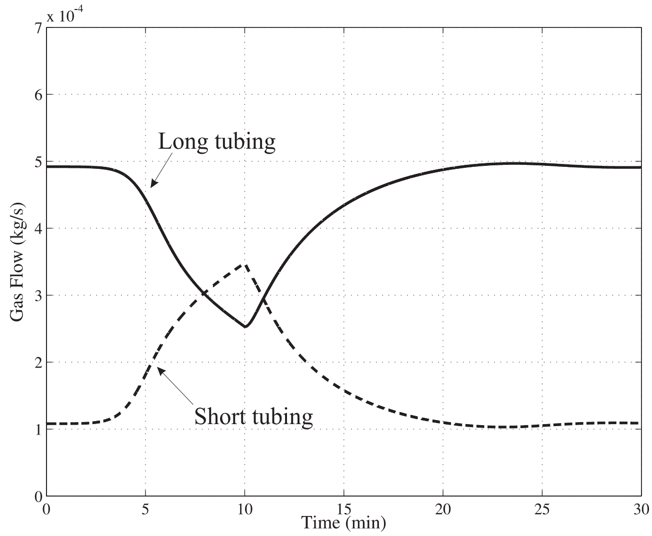


Fig. 8—Gas distribution as a function of time in the controlled case. Control is turned on at *t* = 10 minutes.

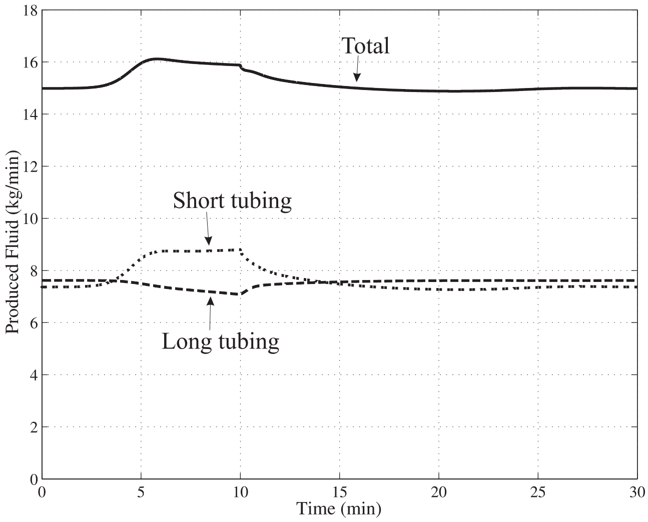


Fig. 9—Fluid production as a function of time in the controlled case. Control is turned on at *t* = 10 minutes.

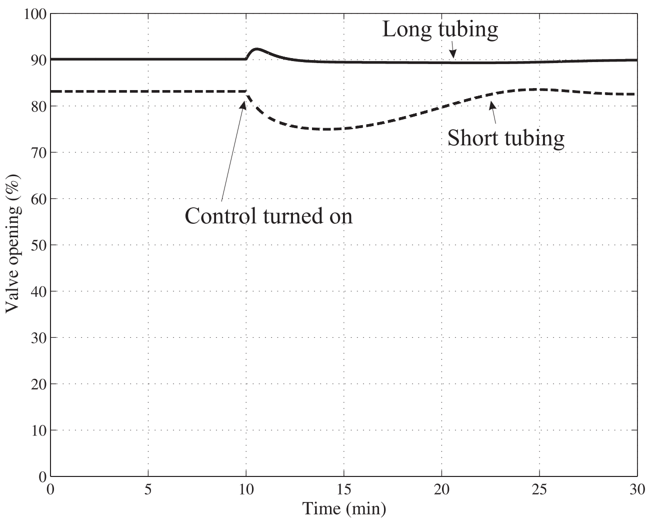
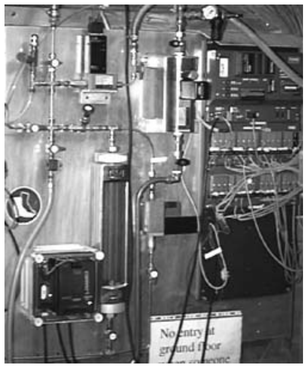


Fig. 10—Production-valve openings as functions of time. Control is turned on at *t* = 10 minutes.

a) The production tubes.

b) The annulus volume.



c) The microcomputer.

Fig. 11—The gas lift laboratory.

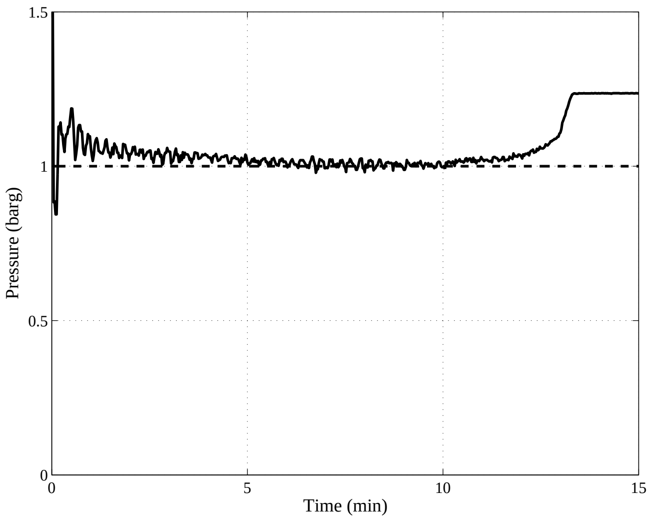


Fig. 12—Pressure at the location of gas injection in the long tubing. Control is turned off at *t* = 10 minutes. The dashed line specifies the setpoint $p^*_{wi,1}$.

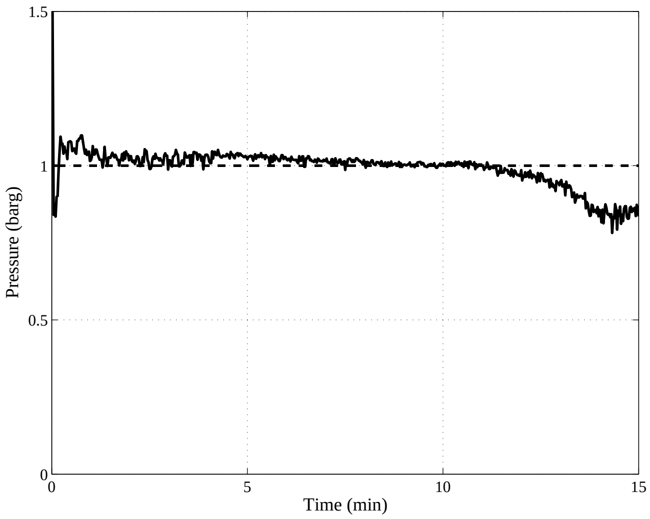


Fig. 13—Pressure at the location of gas injection in the short tubing. Control is turned off at *t* = 10 minutes. The dashed line specifies the setpoint $p^*_{wi,2}$.

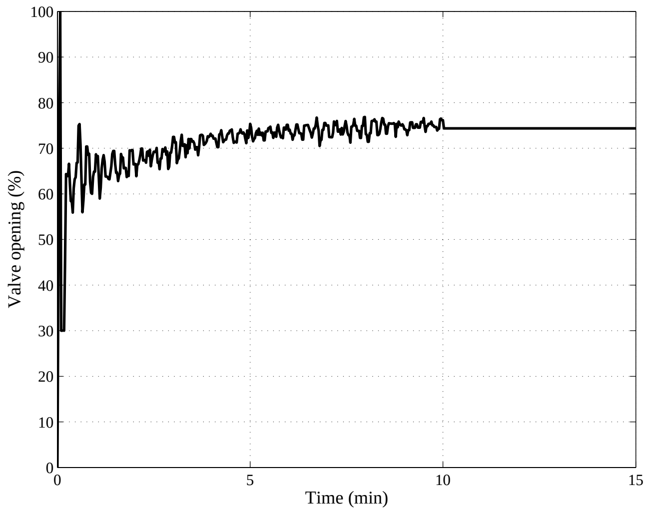


Fig. 14—Production-valve opening for the long tubing. Control is turned off at *t* = 10 minutes, keeping the valve opening at the last controlled value.

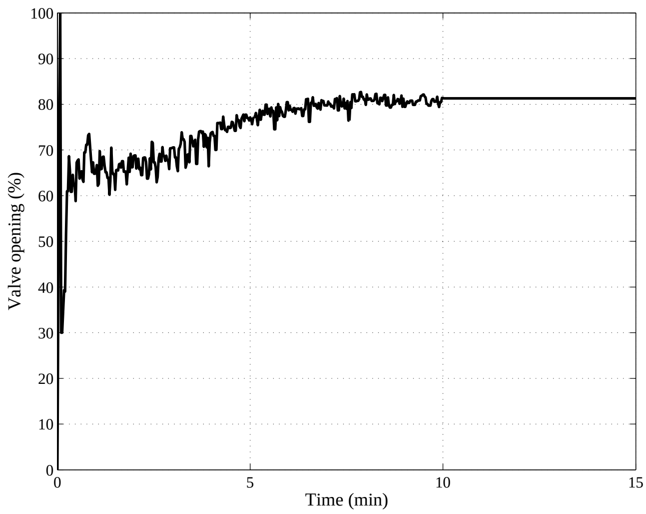


Fig. 15—Production-valve opening for the short tubing. Control is turned off at *t* = 10 minutes, keeping the valve opening at the last controlled value.

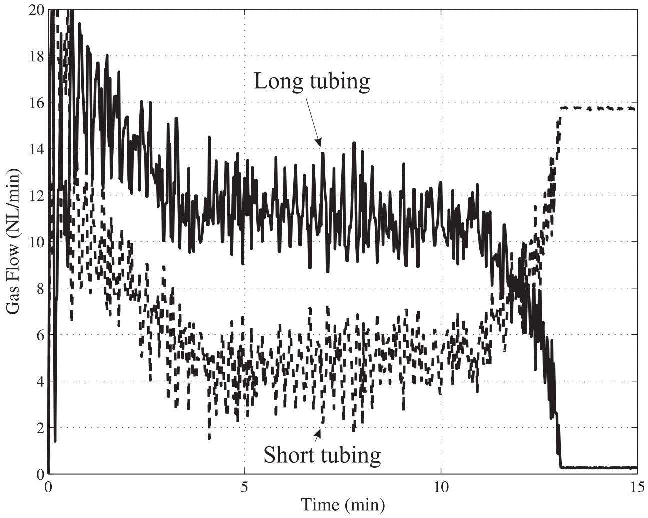


Fig. 16—Gas distribution as a function of time. Control is turned off at *t* = 10 minutes.

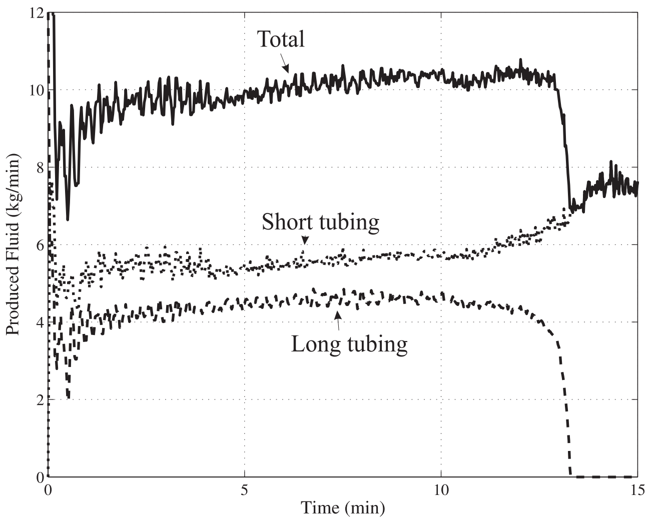


Fig. 17—Fluid production as a function of time. Control is turned off at *t* = 10 minutes.